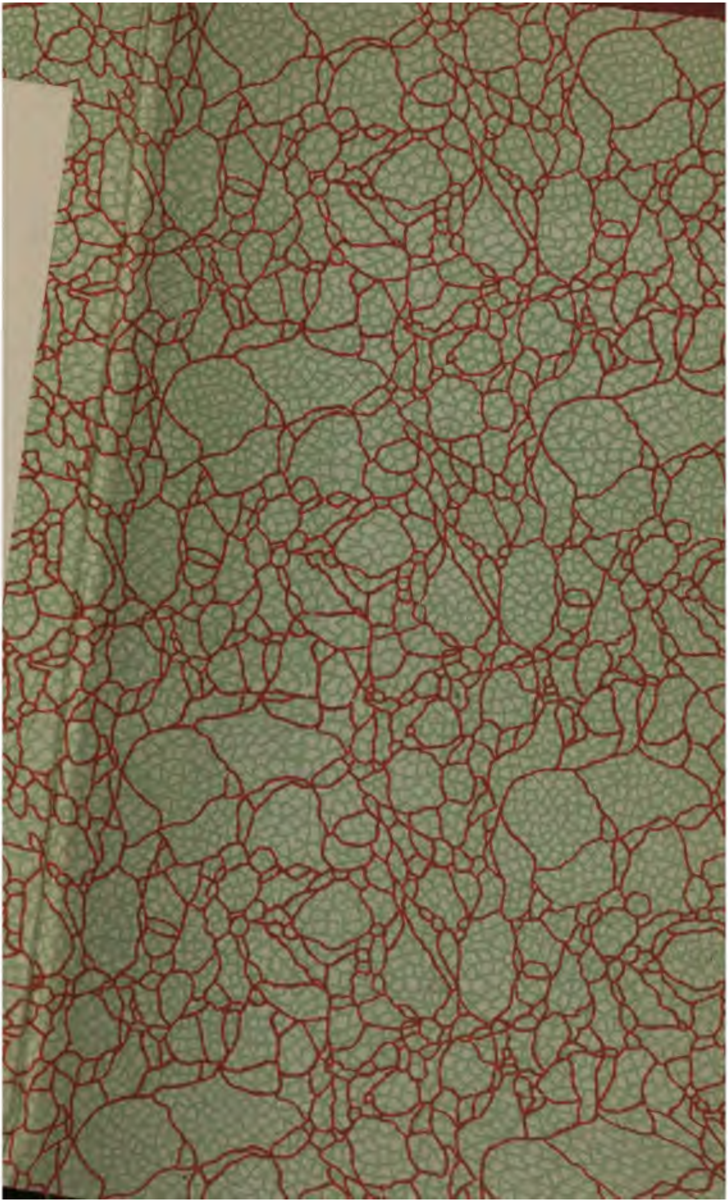
CENTRAL STATION RATES IN THEORY AND PRACTICE

H. E. EISENMENGER



Central Station Rates in Theory and Practice

BY

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With a Foreword by

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and a Contribution by

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> *Vive, vale, si quid novisti rectius istis*
> *Candidus imperti; si non, his utere mecum.*
> —*Horace*

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To MR. F. S. TERRY
in remembrance
of the many years
I worked
in his organization.

PREFACE

All the text of this book was published in the latter half of last year in the form of a series of twenty-five consecutive articles in the ELECTRICAL REVIEW. The author was pleased to note the widespread interest with which these articles were read and the many requests for putting them into the more permanent and compact form of a book. Hence the book. In revising the text opportunity was taken to rearrange the entire matter in more logical order and also to correct a number of errors.

The author's intention has been to produce a book which should be useful to every student of electric rates, to the beginner as well as to the expert. In consequence of this some parts of the text will be found too elementary for the advanced student, whereas other portions will be of interest to those only who wish to penetrate more deeply into the subject of rates.

These last named portions frequently are of such a nature that the use of mathematics, sometimes even of the differential calculus, is indispensable for the solution of the problems arising. Wherever the use of mathematics seemed unavoidable the respective portion has been separated from the rest of the text, so that the nonmathematical reader can skip the portion in which he is not interested without losing the context. Care has also been taken in those cases to make the mathematical deductions as plain and as easily intelligible as possible, even at the expense of brevity, because experience has shown that the practical engineer frequently allows his mathematics to get rusty.

To make this book smooth reading matter for the beginner as well as the advanced student, both the very elementary and the most advanced portions of the work have been detached from the main text and segregated in appendices, so that each reader can pick out those portions which are suited for him and skip the rest.

Thus Appendices I, II, VII, etc., are written for the beginner, and moreover for the nontechnical reader. The engineer and the advanced student need lose no time in reading the explanations of the terms horsepower, kilowatt, etc. (Appendix I) and similar elementary matters.

On the other hand, Appendix VI, for instance, is a new study hitherto unpublished prior to the series of 25 articles of last year; it is an investigation about the correct basis of the demand cost, inasmuch as it is recognized that the "peak responsibility" (customer's demand at central-station's peak-load time) is *not* the correct basis in case of a diversity between the customers. This is not for the beginner, who will be satisfied to assume the peak responsibility or even the maximum demand as a correct basis. Likewise, Appendix IX is a mathematical theory (also hitherto unpublished) of the relation between prices, sales and earnings with particular consideration to the value-of-service principle. Differential calculus is indispensable at least for the deductions of this appendix, although the results are simple and are subsequently stated in the appendix, as well as in the main text, without any reference to higher mathematics for those readers who do not care, or are not prepared, to follow the mathematical deduction. The results of the mathematical investigations of Appendix VIII can also be applied by anybody with a knowledge of ordinary algebra or even only arithmetic, whereas for the deductions and the proof higher mathematics (in this case the theory of determinants) is indispensable.

A third group of appendices, such as for instance V, XI, XIII, etc., are simply details, amplifications, explanations, etc., of the main text which have been removed from the latter in order not to disturb the continuity. It should be kept in mind, however, that all the appendices are an essential part of the book and are *not* merely unrelated supplements.

The author wishes to express his thanks to Mr. S. E. Doane for the Foreword that he has kindly contributed; also to Mrs. S. F. Walker for her contribution of Part VI, which is based on her extended experience on the staffs of the Wisconsin Railroad Commission and of *Rate Research*. Thanks are also due to Mr. F. H. Bernhard for painstaking editing of the original articles and their re-editing in the form of this book.

New York, May, 1921. H. E. E.

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FOREWORD.

BY S. E. DOANE.

In the following pages an effort has been made to cover in a clear and comprehensive way a difficult and intricate subject. Mr. Eisenmenger is well qualified through much experience and study to deal most instructively with this subject.

It has been my privilege to profit much by the patient and careful study which Mr. Eisenmenger made of this problem as my assistant.

As manufacturers of incandescent electric lamps, we were first directed toward a study of the structure of central-station rates in connection with the economic use of our product under the usual rate conditions prevalent in this country. Mr. Eisenmenger undertook the study of these conditions. As the work progressed it became apparent, that, while in general our rate systems were like those of Europe, there were some striking cases of divergence in rate practice between this country and the foreign countries.

As soon as opportunity offered, Mr. Eisenmenger went to Europe and studied the situation in England, Germany, Austria and Italy. During the latter part of his stay I had the pleasure of spending nearly three months with him on the ground, observing the conditions in which the various rates were applied.

Mr. Eisenmenger has asked me to state in this foreword some of the broad conclusions which impressed me as my knowledge of the subject developed. I will start by saying that the very name "public service corporations" is a descriptive name which is commonly applied to those corporations which operate in the zone between community effort and private effort. The history of civilization shows a continual progression in the amount of community effort.

Once upon a time many of our roads between cities were toll roads. The toll roads have nearly all disappeared, but even as long ago as their hey-day the public was fully cognizant of their character and because of their natural monopolistic features they were regulated by popular rule. Even in those days the question of fair rates for the service rendered was discussed and settled by public authorities. There has been much development of the theory of proper rates to cover utility charges, and with the development of the theory has grown a great complexity in the application of the comparatively simple underlying theory.

The requirements of rates are rather simple. Both the public and the utility are entitled to equity, which means (a) the public must pay a fair price for the service; (b) the utility is entitled to a rate which will bring a fair return on its capital and its effort; and (c) the amount of return permitted on the capital is influenced by the risk of its employment by the utility by which the rates are fixed. The mentalities controlling the use of this capital are entitled to a greater or less return dependent upon the skill with which the investment is employed.

The public is entitled to the greatest possible use of this invested capital and its controlled minds which can be delivered to them for what they should properly pay.

Rates should never be so high that the prospective business is unable to make use of the facilities. Rates must always be less than the price for which each user can serve himself. Rates can not be apportioned on an average basis to fit all needs. There must be shrewd intelligence employed in the making of rate schedules which will bring to the public the maximum returns for every dollar it spends without denying the utility the maximum return to which it is entitled for the clever use of the equipment in which it has made its investment.

There seems to be a general agreement in all countries to specifically include as public service corporations those corporations which serve the public principally through an investment, the largest part of which is specific to the particular enterprise, and which also is physically so fixed that it cannot be transferred or applied elsewhere. Thus, the poles and lines of the telephone companies are specific for the purpose intended in the particular locality intended and cannot be economically transferred to use in other localities.

There are corporations operating stage lines, small steamers, automobiles, etc., which, while they might be included in the broad term, are frequently not specifically considered as public utility corporations. The reason is, of course, physical. The rates of stage lines, steamers and similar utility evidences are subject to the natural laws of competition, so that it would appear that it is customary only to consider as public utility corporations those corporations which must so invest their money that it is not for the public good that competition shall be permitted.

It seems to have been settled by usage that rates for public utility corporations shall be set on the basis of barter. The money one pays out of pocket always falls into two general classes: (a) Payments for voluntary purchases; (b) payments for purchases which are involuntary in the individual sense.

In class (a) one pays a price which has no relation to the cost of the article but rather what it is worth to him. This is the basis of barter. The price paid for any article purchased on this basis may be below cost or above cost, but at all times it must be what the purchaser is willing to pay, and depends upon the article and the purchaser's desire for the article and in no sense upon the cost of the article or the ability of the purchaser to pay.

Money paid out of pocket for taxation is not paid on the basis of the value of the article to the payor but is paid for on the basis of the wealth of the payor and again bears no relation to the cost of the article.

There are schools of thought which would assign the utility some portion of the money raised by the general taxation to offset in part its expenses. This cult is small in numbers and is decreasing as the years pass.

There seems to be a fairly clear distinction between the public utility corporations which must sell their product by the laws of trade with some governing restrictions, and that class of service which the community prefers for its advantages should be paid for by taxation. In this latter class falls the administration of law courts, the police, the schools, roads, sewers, the lighting of public places, etc.

It has been my privilege to discuss these subjects at great length with the author of this volume, whose introduction I am writing.

There is a third class of payment which is neither barter nor taxation, but which is administrative or penal. Such, for example, is the practice in some communities of charging a customer an increased water assessment if he has more than his allotted number of outlets, even though the water is also needed. This administrative or penal charge is obviously not a fixed charge and is apparently for the practical purpose of decreasing the customer's use of water.

I will close the introduction by paying a high tribute to the personnel of the public service commissions which have grown up in this country within recent years. As a class they have been faithful to their trust and the public owes them much. In this connection, I cannot refrain from emphasizing the necessity of appointing highly able men to these places of such great responsibility and of giving such men ample time and service and ample compensation and opportunity to permit them to qualify themselves to the fullest possible extent, so that they may use the highest possible intelligence in the public behalf.

S. E. Doane,
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CENTRAL STATION RATES IN THEORY AND PRACTICE

PART I

THE COST OF ELECTRIC SERVICE

1. The Cost of Commodities in General.

A. THE COST OF A CERTAIN QUANTITY. TOTAL COST AND UNIT COST.

1. The term "cost" of a certain commodity[1] is by no means a well defined one and we must distinguish a variety of interpretations.

In the first place, the term "cost" always refers to some given quantity of the commodity and it is usually—though not always—understood from the context what quantity we mean. It may be the total quantity produced (sold, consumed, etc.) in a certain enterprise, for instance per year, or the quantity purchased or consumed by a certain consumer, or the quantity of a certain shipment, etc.

Primarily, as a rule, we have given the cost of the total quantity produced, mostly per year, and desire to know the cost of the quantities purchased by the various consumers, or of other quantities of the com-

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[1] The term "commodity" is meant throughout this book to designate anything that is being sold, whether physical goods, or the right to use them, or work, labor, service, etc.

modity. For this purpose it is necessary to reduce the cost to some unit quantity.[2]

The term "cost" means therefore the cost of a given quantity, which may or may not be the unit quantity. Thus the cost of coal may be said to be $1000, which may mean per year, or per shipment, or per voyage of a steamer, etc., or the same cost may be said to be $4, which would mean per ton.

To avoid such ambiguities, the cost of a certain quantity, other than unity (especially the quantity produced, sold, consumed, etc., by a certain enterprise during a year or other unit of time) will hereafter in this book be called the "total cost" where necessary to distinguish it clearly from the "unit cost," that is, the cost per unit quantity.

Where we have a number of constituent quantities, for instance several classes of service, the cost of producing several of these quantities simultaneously will be called their "combined cost."[3] We can thus distinguish a total combined cost and a unit combined cost. If the constituent quantities comprise all parts of the quantity produced by the manufacturing enterprise, for instance in a year, the combined total cost will be called the "aggregate cost" in this book. This is the entire (annual, monthly, etc.) cost of the enterprise.

The same distinctions as to the quantity of the commodity must be made in case of income, profit and price, so that we distinguish a total income, profit and price and a unit income, profit and price.[4]

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[2] The unit to be chosen depends on the nature of the commodity. It may be, for instance, a unit of weight, length, area, cubic capacity, numbers (pieces, pairs, dozens), time, power, energy, weight-distance (ton-miles in transportation service), etc.

[3] The "combined cost" is not equal to the sum of the costs of the constituent quantities (see later).

[4] If the commodity consists in a service, the price per unit is frequently called the "rate"; electric service, for instance, may be charged "at the rate of" 10 cents per kilowatt-hour, just as we say, a train is running "at the rate of" 50 miles *per hour*.

B. Cost Including or Excluding Capital Expenses.

2. We can further distinguish different kinds of cost for the same commodity, or different meanings for the term "cost" as explained in the following:

The total annual cost necessary to maintain and run any enterprise, especially a manufacturing establishment, such as a central station, consists of two parts:

(a) The direct running expenses per annum which generally stop as soon as the enterprise shuts down, such as salaries, wages, fuel, raw materials, etc.

(b) Expenses which are the result of a certain capital being invested in the property of the enterprise. This capital calls not only for interest but also for annual payments into funds for depreciation, etc.[5] These expenses remain the same whether the plant is working or not. They are not reduced if the plant reduces its output, but they are increased if the plant increases its output beyond its present capacity.

We can now call "cost" the aggregate of all of the above expenses under (a) and (b), assuming a fixed percentage of net return on the capital invested. This method is the obvious one where the capital has been raised entirely by bonds and therefore bears a fixed interest, as in the case of municipal plants. With this meaning applied to the term "cost," it is only necessary to cover the "cost" by the revenue, and no profit[6] is required to yield the expected return on the capital invested. If the gross revenue exceeds the total cost, the balance representing the profit must be applied to purposes not in direct connection with the enterprise, unless we can reinvest it for enlargements, etc.

Or we may understand the term "cost" in such a

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[5] Interest, depreciation, and other capital expenses will be treated more fully in Sections 18 to 23.

[6] Profit is the excess of income over cost and the meaning of this term varies therefore as we apply the different meanings of the term "cost," as explained in this section.

manner that it includes the running expenses (a) and only such of the capital expenses (b) as are actually a fixed percentage of the capital invested and independent of the earnings, that is, interest on bonds and on preferred stock, also depreciation charges, etc., but not the interest on the common stock ("dividend"). In that case it is necessary that the enterprise should yield more than the "cost." The variable excess of the income over cost, that is, the profit, is applied to pay the interest on the stock. That interest, the dividend, is therefore a variable percentage.

Or we may finally exclude *all* the capital expenses (b) from the amount covered by the meaning of the term "cost," so that we will have to pay out of the profit, first the fixed capital charges, such as bond interest and depreciation charges, and then apply the balance for the dividend.

Where the term "cost" is applied in these articles without further specification it will always be understood in this last sense.

In all these cases we can again distinguish a total cost of the establishment, for instance per year, and a unit cost as explained under Section 1.

C. Segregate Cost, Increment Cost, and Average Cost.

3. Another lack of definition in the term "cost" is that it may mean what will be called in this volume the "segregate cost," or it may mean the "increment cost" or the "average cost." The segregate cost of a certain portion of the total quantity produced by the enterprise is the amount it would cost to produce that fractional quantity only. The increment cost is the amount by which the aggregate cost of the enterprise is increased in consequence of the fact that the respective quantity is added to the quantity to be produced.[7]

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[7] It is a general, and probably universal, truth that a larger ishment can—ceteris paribus—produce more cheaply per

Both the segregate cost and the increment cost can be reduced to the unit produced.

Supposing, for the sake of an example, that it costs

$ 600 to produce 500 units,
$1000 to produce 1000 units,
$1300 to produce 1500 units,

then the increment cost of 500 units over the cost of the first 500 units is $400, and the increment cost of 500 units over the cost of the first 1000 units is $300. The segregate cost of 500 units, that is, the cost of producing the said quantity by itself without simultaneously producing other units, is $600.

Both the increment and the segregate cost can be reduced to the unit. The increment cost per unit in the above example would therefore be $0.80 or $0.60, respectively,[8] whereas the segregate cost per unit would be $1.20.

We can finally also reduce the aggregate cost (see Section 1) to the unit, that is, we find the "average cost per unit." This would be in the above example in case of an aggregate production

of 500 units............$ 600/ 500 = $1.20
of 1000 units............ 1000/1000 = 1.00
of 1500 units............ 1300/1500 = 0.867

D. BY-PRODUCTS.

4. Conditions frequently are not as simple as assumed so far. The first complication comes in where

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unit than a smaller one and an establishment of given size can produce more economically if it is producing at a rate corresponding to its full capacity than otherwise. If we have, for instance, a railway where the traffic is so small that not more than one or two trains are necessary in each direction per day, we have certain salaries for station masters, etc., to pay, just the same as if we had many more trains; a certain capital must be invested in the roadbed and buildings, independent of the number of trains running daily; the wooden ties will rot in the same time whether many or few, heavy or light trains are running over them, etc.

[8] Frequently the conditions are such that we can assume the increment cost per unit to be constant for a wide range of the quantities of previously produced goods.

more than one commodity is produced by the respective enterprise. The same processes, activities, capitals invested, etc., which are necessary to produce one commodity, frequently automatically produce at the same time another one or several others. In the coal-gas business, for instance, the result of the process i not only gas, but also coke, tar, ammonia, naphthaline etc. If one of the commodities is of paramount importance we can speak of a *main product* and of one or more *by-products,* in the sense of subordinating the latter to the main product. Or, if all the products are more or less of the same importance among one another, we can call them all by-products, meaning by-products to each other, in the sense of mutua co-ordination.

Now, the question arises: What portion of the total cost, or of any part of the total cost, is to be charged to every one of the by-products?

What portion of the annual cost of coal, for instance, is to be charged to the cost of the gas and what portion to the cost of the coke?

We cannot distribute the cost according to the number of the units produced of each commodity, because gas and coal are not measurable in the same units, and even if they were, it would not be possible to logically justify such a distribution of the cost. We might distribute the total cost according to the selling value of the totals produced of gas and coke and then if the unit cost is required, further distribute each one of these subdivisions to the unit of gas or coal respectively. Or we may prefer some other way. Whichever method we choose, there will always be an arbitrary element about it.

It may be anticipated here that this does not apply to the *prices,* which are determined according to the value-of-service principle. This will be discussed in a later section on prices.

E. PRODUCTS TO BE MEASURED IN MORE THAN ONE UNIT.

1. *General Explanation.*

5. Another complication in determining the cost per unit is encountered where the total cost consists of several portions or items in such a manner that the amount of each portion is determined by some other factor or element and each of these factors is measured in another unit. For instance, one portion or item may depend on the weight of the commodity produced (expressed in pounds or tons, etc.) and another one on the length of the same product (for instance in feet). Instead of theoretical explanations an example will make clearer what this means.

6. This example is the letter carrying service as furnished by the Post Office Department. The annual cost of the letter mailing service (excluding the charges on the capital invested) is composed of the two following main portions or items:

(a) The annual cost of transporting the letters, whether by rail, or street-vehicle, or on the shoulders of the postman.

(b) The annual cost of assorting and distributing the letters in the post offices.

The first item, like the cost of any other transportation service, can be assumed to be proportional to the number of ton-miles required per year, and in reducing the cost to the unit we have to use the ton-mile (or ounce-mile or some such unit). This item will therefore be constant *per ton-mile* (or ounce-mile). The second item is plainly the larger, the larger the *number* of letters is that is being handled per year and has practically nothing to do with the weight of the letters or the distance over which they are being carried, etc. The proper unit is the letter or a certain standard number (say, 100, 1000) of letters and this item *will be constant per letter.*

The total cost for any volume of service rendered during any period of time is therefore the sum of two items, one of which is the product of a constant figure × the number of ounce-miles required, whereas the other is the product of another constant figure × the number of letters carried. This applies to any volume of service, to individual letters as well as to a given mail bag filled with letters, etc. The result of the computation will be correct, no matter whether we have big, fat manuscripts or thin, little notes and whether they travel from New York to San Francisco or into the neighboring town only.

2. The Use of a Smaller Number of Elements Simplifies the Computation, but Reduces the Accuracy.

7. The question of the average unit cost of letter service will therefore be answered as:. So much per ounce-mile plus so much per letter. We can, however, also refer the total cost of letter service to one of the two above elements only and ask: How much is the average cost per letter? Or: How much is it per ounce-mile? This is easily figured out, but if we try to apply it to a given letter or portion of service (mail bag) it will furnish correct results only if in that letter (bag of letters, etc.) the ratio of the ounce-miles to number of letters happens to be the same as in the entire service over the whole year. If we apply, for instance, the average figure per letter to a very thin letter traveling over a short distance, the computation will furnish too great a value because we figure the cost as if the letter would require a greater number of ounce-miles than it actually does. Conversely, if we apply to such a letter the average figure per ounce-mile the result will be too small, etc.

3. The Use of a Greater Number of Elements Enhances the Accuracy, but Complicates the Computation.

8. We have now obviously a certain inaccuracy,

not only if we reduce the cost to a single one of the items, but also in the case where we use both of the above items (cost of transportation and cost of handling) since these items are not really the only ones (as has been assumed so far) which have a bearing on the cost of service, even though they are the most important ones. We have to correct, therefore, the statement made above in Section 6 and say: "The first item (a) is *practically* constant," or "nearly constant," or "does not vary much" per ounce-mile, and the same correction applies to item (b) per letter or per 1000 letters, etc.

We can, therefore, increase the accuracy by increasing the number of "elements" or "items" (cost of transportation and cost of handling in the above example) which we choose to consider for the computation of the cost. We might use, for instance, three items: (a) the cost of transportation between railway stations or ports (per ounce-mile); (b) the cost of transportation within the distance of a given post office, that is, from the letter box to the post office, between the post office and the railway station and from the post office to the addressee (per ounce-mile); and (c) the cost of handling the letter in the post office.

Whether we select from the indefinitely large number of these elements one, two, three, or four, etc., and where we draw the line between the most important ones and those of lesser importance, is to a certain degree a matter or abitrary judgment, and the decision will have to be guided by considerations of practicability. *The larger the number of elements we choose, the greater will be the accuracy and the smaller will be the deviations in the individual case.* At the same time the computation will become more complicated. We have to strike in this case (as in so many others) a happy medium, and steer a safe course between the Scylla of complication and the Charybdis of inaccuracy.

4. Increasing the Accuracy With a Given Number of Elements by Subdividing the Customers Into Classes.

9. We can, however, with a given number of elements chosen, increase the accuracy with less complication in the following way. We subdivide all the customers or all the cases in which the respective business is transacted, into a number of classes or groups, selected in such a manner that the unit cost of each one of the considered items—that is, of ounce-mile and letter in our case—is most nearly constant for all customers or parts of business within each group, but varies from one group to another. For instance, we might classify the letters according to the distance over which they are carried into local, interurban or domestic, and international letters. Then the answer to the questions, "How large is the cost per letter?" or "How large is the cost of transportation per letter?" will be more accurate, that is the individual deviations from the average will be much smaller. The average cost per local letter will be one figure, per interurban another, and per international a third one. Or we might classify the letters according to their weight; or according to the location of the addressee into the three classes of rural, suburban and business district letters. The transportation charges for the rural letters will be higher than in the other classes because the letter-carrier has to walk a longer distance from the post office and between the houses, etc. We may, of course, also combine two or more of these classification systems and differentiate heavy and light rural letters, heavy and light suburban letters, etc.

Employing this principle, we can increase the accuracy with less inconvenience than when we employ an additional item.

II. The Cost of Electric Service in Particular.

A. The Three Elements of Cost.

10. The problem of properly expressing the unit cost of electric service can be solved in an entirely analogous manner to that previously described.

Just as we can say, the cost of letter service is so much per ounce of letter, we can also say electric service costs the central station, for instance, 3 cents per kilowatt-hour. That would mean that it costs 3 cents to furnish the current for burning a 100-watt lamp for ten hours or for obtaining one horsepower during one hour at 74.6% motor efficiency, etc. This is certainly a very clear and simple statement of the unit costs, but unfortunately it does not tell us much, unless we know the exact conditions under which the current is being furnished. It tells us no more than if we hear that it costs the Post Office Department one cent, or some such figure, to transport, handle and deliver a letter or one ounce of letter, without any statement about the distance over which the letter is to be carried or about the weight of the individual letter, etc. Yet we frequently read statements of the cost of electric power in terms of kw-hr. They are worth anything only if the special conditions of the case are known. They give over-all averages only. It may cost the same central station ½ cent per kw-hr. to serve one customer and 10 cents or more to serve another.

In accordance with what has been shown in the example of the letter service, we would have to have a large number of elements which have a bearing on the cost of the commodity selected, in such a way that a certain portion of the total annual cost of the central station is proportional to the number of units per annum of one of the elements, another portion to the number of units of another element, and so on. In order to avoid too great a complication and to get practicable results, we generally limit the number of these elements to three, and more or less neglect the rest as they are of lesser importance. More accurately speaking, we average the small remaining part of the total cost which is not proportional to any of the three items of cost, *somehow* into the three items.

(The reader should keep in mind that we are at present dealing with the cost to the central station and not yet with the systems of charging the consumer.)

The three items for the computation of the cost are:

(1) An item proportional to the number of kilowatt-hours generated or sold.[1] This item is therefore constant per kilowatt-hour. It is called the kilowatt-hour cost or *energy cost.*

(2) An item proportional to the maximum number of kilowatts[2] loading the central station. This item is therefore constant per kilowatt; since the maximum power in kilowatts is called the maximum demand, this item is called the *demand cost.*[3]

(3) An item which is caused by the mere fact the consumer *is* a customer of the central station.

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[1] Not all the kilowatt-hours generated are sold. Some of them are used for the home consumption of the central station and others are lost in the transmission to the consumers. But the kilowatt-hours generated and sold can be assumed to be proportional to each other and it does not make any difference whether we assume the energy generated or sold to be the basis of the computation of cost and, in the first case, whether we include the home consumption or not, as long as the units are consistently understood in the same manner. Where it comes to the computation of cost as a basis for the price to be charged to the costumers it is evident that the energy sold is to be taken into consideration only.

[2] The non-technical reader whose ideas about the difference between the meanings of the terms "kilowatt" and "kilowatt-hour" are not quite clear is referred to Appendix I.

[3] Some of the readers who are not familiar with electrical engineering may find it useful to have the difference made clear between the first and second item in plain though quite unscientific terms as follows: Imagine a lighting installation with lamps all of the same size. Item (1) of the cost, the energy cost, is proportional to the number of kilowatt-hours, that is, in this case to the number of lamp-hours. (The term "lamp-hours" means the sum of the burning hours of the individual lamps.) Item (2), the demand cost, is proportional to the maximum number of kilowatts (or, what amounts to

This item is independent of the amount of service required by the customer, as the second item in the example of the letter-mailing-service cost (assorting the letters) was independent of the weight of the letter and of the distance over which it had to travel. This item of central-station cost is proportional to the number of customers and is constant per customer. It is called the *customer cost.*

I. The Energy Cost.

11. Every kilowatt-hour generated or delivered requires a certain amount of steam—at least in case of steam prime movers—and with that of fuel and water. In other words, the total costs of fuel and water increase with the number of kilowatt-hours generated and can be roughly set proportional to the latter so that we have a fixed unit cost of fuel and of water per kilowatt-hour. The items of fuel and water do not appear in hydroelectric plants; but in all central stations, whatever their prime motive power be, we have the cost of lubricating oil which increases with the size of the generator running, with the number of generators running and with the number of hours they are running. In other words, for a given central station the cost of the lubricating oil increases with the number of kilowatt-hours generated and can be roughly set proportional to the latter so that here too we have a fixed cost per kilowatt-hour. The same applies to the cost of attendance to the generators and the switchboards, to repairs and a number of other items.

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the same thing, to the maximum number of watts) drawn by the consumer at any time, that is, in our hypothetical case proportional to the maximum number of lamps burning simultaneously. Thus, 100 lamps burning for 20 hours would contribute five times as much to the demand cost as 20 lamps burning for 100 hours, but they would contribute the same amount to the energy cost. On the other hand, 100 lamps burning for 80 hours would cause the same demand cost as 100 lamps burning for 20 hours only, but they would cause fou *times the energy* cost.

This part of the cost, the energy cost, is also called sometimes, though not entirely correctly, the operating cost. We will see that a part of the real operating cost may belong to the "demand cost" (to be discussed in Section 24) and, conversely, the kilowatt-hour cost includes costs which are not operating cost. But, on the whole, roughly speaking, the kilowatt-hour cost and the operating cost are more or less identical.

12. As indicated previously in the example of the letter carrying service, we can establish a distinction in the amount of the energy cost per kilowatt-hour between various classes of customers. The most important one of the possible subdivisions into classes would probably be according to the distance of the customer's location from the generating station, inasmuch as generally a larger portion of the energy is lost in transmission if the energy has to be transmitted over a greater distance. We would be justified in saying that those customers who are located further away from the generating station cause a greater energy cost per kilowatt-hour than those who are located nearer the station and we might find the numerical values by methods similar to those to be shown later. But this distinction according to the location is rarely, if ever, made in practice.

Other variations in the amount of the energy cost per kilowatt-hour will be mentioned later (especially in Section 53) after the necessary fundamentals (load-factor, diversity-factor, etc.) have been discussed.

13. Let it be stated here—although that statement ought to be unnecessary for the careful reader—that the "energy cost" or "kilowatt-hour cost" is something entirely different from the "cost per kilowatt-hour." The cost per kilowatt-hour is the total annual cost of the central station divided by the number of kilowatt-hours generated (or delivered, or sold, as the case may be) anually, and is given in *dollars per kw-hr.* or *cents per kw-hr.* The kilowatt-hour cost (or energy cost), on the other hand, is a certain portion of the central station's total annual cost, corresponding roughly to the annual operating cost. It is given in *dollars per year*. Usually we reduce the kilowatt-hour cost to the kw-hr. generated or sold annually and thus arrive at the *kw-hr. cost per kw-hr.* or *energy cost per kw-hr.*, which is given in dollars per kw-hr. or cents per kw-hr. The kw-hr. cost per kw-hr. is a fraction of the total cost per kw-hr. By multiplying the kw-hr. cost per kw-hr. by the number of kw-hr. consumed annually by a certain consumer, we get the annual kw-hr. cost of that customer as a part of his total annual cost.

It is important to keep these distinctions between the kw-hr. cost and cost per kw-hr. in mind to avoid confusion.

2. The Demand Cost.

14. The second item to be discussed is the *demand cost,* that is, the part of the total cost which is practically proportional to the maximum demand in kilowatts or watts.

A. THE TOTAL DEMAND COST OF THE PLANT.

1. Capital Invested.

15. Suppose, for the sake of a simple example, that a central station has to deliver 24,000 kw-hr. a day with constant load on the generators all day; that means 1000 kw-hr. in any one-hour period of the day. To supply this, our generators, apparatus, lines, etc., have to have a capacity of 1000 kw. The load-factor[1] is 100%. If, however, these 24,000 kw-hr. would have to be delivered in six hours, for instance in the time between 4:00 p. m. and 10:00 p. m., again with constant load during these hours, the load of the central

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[1] Readers who are not thoroughly familiar with the meaning of the terms "load-factor" and "load curve" should first read Appendix II, which gives an explanation of these terms so essential for the understanding of the following sections.

station will reach 4000 kw., because 4000 kw. × 6 hr. = 24,000 kw-hr. The capacity of the central station will, therefore, have to be four times as great as in the first case, or 4000 kw. instead of 1000.[2] Now a central station of 4000 kw. will cost, though not four times as much, still a good deal more than a 1000-kw. station. The same applies to the transmission and distribution lines, transformers, etc.

16. As a rough approximation we can say that the capital invested is proportional to the capacity of the central station in kilowatts. It should not be forgotten that this is an approximaion. It is a well known fact that as the central station grows, the cost of the equipment per kilowatt goes down. Larger plants can use larger generating units than smaller plants and such units are not only cheaper to manufacture per kilowatt, but they occupy also less floor space per kilowatt, thereby reducing the cost of the building construction, etc., per kilowatt. The same applies to switching apparatus, etc. Moreover, even generating units of the same capacity have been becoming cheaper as the engineering sciences advance.

But as long as we remain within certain upper and lower limits of size near the present plant capacity, also when we do not apply our approximation over a period of too many years, we can assume the capital invested in the generating plant to be proportional to the capacity. If the central-station capacity has grown very much, especially if the manufacturing conditions have changed in the meanwhile, the factor of proportionality will have to be changed; there will be less capital invested per kilowatt in the generating station.

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[2] This is not strictly correct, since there must be a certain reserve capacity for breakdowns of a generator set, and the reserve capacity will obviously in general be a greater percentage of the total capacity where we have a small number of generating units than where we have a large one, that is, it will be in general a greater percentatige for smaller capacities. But, as a whole, the approximating assumption that the capacity of a central station is proportional to the maximum load of the central station is justifiable.

17. Similar considerations as have been explained for the cost of the power plant will also apply to the transmission lines, transformers, etc., so that we may say with a certain degree of approximation that the cost of these parts also is proportional to the peak load of the central station.[3]

We thus arrive by a series of approximating assumptions at the result that the total capital invested in the generating plant, lines and transformers is proportional to the peak load of the central station.[4]

2. *The Constituents of the Demand Cost.*

a. Capital Charges.

1. Net Return (Interest, Dividend).

18. The capital invested in a commercial enterprise has been paid in by its owners because they have the belief that they will get a satisfactory return, whether that be a certain prearranged fixed percentage, as on bonds, or a variable one in accordance with the degree of success of the business. (See Appendix III-A.)

This return on the capital is the *raison d'être* of any business enterprise and therefore it has been mentioned first. As a matter of fact it is the last one of the capital charges inasmuch as all the other charges

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[3] This implies an additional simplifying assumption, because it is evident that the cost of the lines and transformers depends also on local conditions, that is, on the location of the customers, etc., but, on the whole, in a given locality we can say that the variable portion of the cost of the lines and transformers will increase roughly in the same percentage as the maximum load on the central station.

[4] These assumptions are, briefly recapitulated, the following:

1. The capacity of the power house is proportional to the peak load.
2. The capital invested in the power house is proportional to the capacity of the power house.
3. The capital invested in the lines, transformers, etc., is proportional to the peak load.

These and the other approximations, to be discussed in following, are put together in diagrammatical form at th *beginning of* Appendix V—"Approximations."

to be mentioned hereinafter (depreciation, repayment, taxes, insurance, etc.) must be, or should be, considered first. We can thus distinguish a gross return and a net return, both of them being based on the ***net*** income. The gross return includes all capital charges, or at least depreciation and possibly repayment[1]; after deducting all those charges from the gross return we get the net return which is paid to the owners of the capital as bond interest and stock dividend.

We have therefore to distinguish the following four items:

1. Gross income.
2. Net income = gross income — cost.
3. Gross return $= \frac{\text{net income}}{\text{capital}}$, usually given in per cent.
4. Net return = percentage gross return — percentage depreciation, etc.

2. Depreciation.[2]

19. For practically all construction work, whether it be machinery, or buildings, or electric lines, etc., we can figure that after a certain number of years it has to be replaced because it is worn out and useless. How great this number of years is depends largely on the nature of the construction. A building may last 50 years or longer, a pole of an overhead line may be too decayed for further use after 10 years or less, etc. The value will decrease year after year and this is called the depreciation of the construction.

Moreover, some parts of construction, especially machinery, must not infrequently be discarded long before they are physically unfit for further use because new inventions have been made and it has become either more economical or otherwise advisable

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[1] Taxes and insurance are frequently counted as cost, although from the present point of view they should rather be classified as capital ch [illegible]

[2] *See* [illegible] *6-249.*

to install machinery which embodies the new improvements rather than to continue employing the old obsolete machinery. This has, for instance, been frequently done in the last few years with reciprocating steam engines, which, although still in their prime of life and perfectly serviceable, had to be exchanged for the newly invented steam turbine because the latter consumes so much less steam (and requires so much less space) that the whole interest and other investment charges on the steam-turbine set would be less than the cost of the steam saved[2] annually. The reciprocating engines were kept as reserve capacity in case of breakdowns or they had to be sold at what one could get for them—that is, the so-called scrap value, which generally is only a small percentage of the price paid originally.

We see therefore that the value of the capital invested in an electric plant is not, without further measures, safe and secure for all times, but that on the contrary this value is incessantly and automatically being reduced and we must take steps to counterbalance this reduction of value by creating a fund into which the equivalent of the depreciation is deposited out of the annual earnings before we apply the latter for profit.

Suppose, for instance, we have bought a generator set for $100,000 and after 12 years it is worn out and has to be scrapped, the scrap value to be $10,000. The balance of $90,000 has disappeared out of the generator set and if we do not want to lose that amount we must have earned so much over and above the interest that the aggregate of these yearly excess earnings with compound interest has accrued after 12 years to make up the $90,000 which has disappeared. We have to establish a "depreciation fund" into which regular payments are made every year and invested

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[2] And *capital* charges on the floor space saved, which can b *utilized for* extensions of the capacity.

at compound interest. That means, for instance, if we invest the money accumulating in the fund at 5% compound interest, we have to pay in every year about $5650 for depreciation, or 5.6% of the original capital of $100,000; then at the end of 12 years we will have $90,000 accumulated in the fund which, together with the expected proceeds of $10,000 from the sale of the scrapped generator set, will just about suffice to buy a new generator set in place of the old one.

Of course, all these calculations have to be rather crude ones as the figures we are dealing with are all known only very approximately. We do not know what the actual useful life of the generator will be. We can be guided from past experience, which is laid down in tables for various classes of machinery and other constructions, but these tables give, of course, only average values and, moreover, when it comes to obsolescence, due to future new inventions, we are entirely at sea. Even apart from obsolescence, we do not know how great the scrap value will be and what the price of the new generator set will be in about 12 years from now, etc. But something near that figure (5 or 6%) will have to be put aside every year for depreciation in the assumed simple case.

20. It is obvious that in this hypothetical simple example the money in the depreciation fund rises from zero at the beginning of the first year to 100% of the net replacement value at the end of the twelfth year and then drops suddenly to zero again as the new generator is bought. This 12-year cycle is periodically repeated an indefinite number of times. As the value disappears from the physical property by depreciation it turns up in the depreciation fund and then by purchase of a new equipment flows back into the physical property, thus fluctuating in a 12-year cycle to and fro between the two. In practice, *however. the fluctuations* of the fund are not as large but much smaller, for several reasons, which are briefly explained in Appendix III-B.

3. Repayment (Sinking Fund).

21. Where, as usually, a portion of the company's capital has been raised by bonds, these bonds mature within a certain time, for instance, after 20 or 30 years; this means that the principal (face value of the bonds) has to be paid back at that time according to the terms of the loan. It is then most commonly raised by a new issue of bonds. An alternative is the creation of a fund for the payment of the bond indebtedness ("sinking fund"). A certain amount must be paid into this fund every year of such size that at the time when the bond issue matures the accumulations in the fund with compound interest aggregate to just the face value of the maturing bond issue.

In some cases the franchise granted to the company stipulates that the franchise will expire after several decades, perhaps after 30 or 50 years, and that the equipment of the company has to pass at that time into the hands of the city, or some such body, without any compensation whatever to the company. If we want to be very exact and careful, we will have to create another sinking fund in such cases of such size that this fund, together with the liquid amount contained in the depreciation fund at the end of those 30 or 50 years (or whatever the term is), amounts to just the capital invested. The depreciation fund and the sinking fund together will then, at the time of the termination of the franchise, serve to pay back the face value of all stocks and bonds. Now, it is obvious that under these conditions any company will endeavor to avoid replacements of machinery, etc., during the last years of its franchise, as it knows that in the *near future* it will have to give up, without ar compensation, its physical property. Consequently, at the end of the period the depreciation fund will be as nearly 100% filled as it can be. The plant will be run down and an unusually large portion of its value will have passed over into the depreciation fund. For this reason and because, in view of the great length of time, the annual payments into a repayment fund of that kind would be a very small percentage, these payments are sometimes neglected.

4. Other Capital Charges.

22. Interest, depreciation and possibly repayment are not the only annual capital or investment charges. *Taxes* are mostly in accordance with the value of the property—that is, in first approximation, with the capital invested. The cost of *insurance* also is the higher, the costlier the plant. These two items may therefore in first approximation be regarded as proportional to the capital invested.

23. We come to the conclusion that we can, as an approximation, assume every one of the capital charges, whether interest, depreciation, repayment or other charges, as proportional to the capital invested and therefore we can also assume the sum of these items, that is, the total capital charges, to be approximately proportional to the capital invested. The capital charges are therefore proportional to the plant capacity and in further approximation to the plant's peak load.

b. Demand Cost Other Than Capital Charges.

24. In addition to the above expenses we have certain other expenses which are not capital charges but which also depend on the size of the plant only, and may be assumed to be proportional to the latter and consequently to the peak load.

Thus, a certain portion of the fuel and lubricating cost and of the cost of maintenance and repairs, etc., may depend on the size of the plant. To explain this, let us assume, for instance, that we have to generate 24,000 kw-hr. in 24 hours by a 1000-kw. generator set, running at full load. Let us assume further that we have in another instance to generate the same number of 24,000 kw-hr. in the same total time of 24 hours, but no longer at uniform load, but with a 4-hour uniform peak period of 5000 kw. This would, of course, require an equipment of larger capacity, for instance, a 5000-kw. set instead of the 1000-kw. set. During these four peak hours the generator would furnish $4 \times 5000 = 20{,}000$ kw-hr., leaving 4000 to be generated during the remainder of the day —that is, during 20 hours. The average load during these 20 hours will therefore be only 200 kw.—that is, 4% of the capacity of the 5000-kw. generator.

Now, large generating units are more efficient than small ones; that means they are consuming fuel more economically per kilowatt-hour output than small ones running at the same percentage of their normal (full) load. The same refers to consumption of lubricants. On the other hand, there is a certain economic load for every generator set at which the latter is most efficient; this load is near the rated or normal load of the set. As the load decreases below that amount the specific fuel and oil consumption *per kilowatt-hour* is getting larger and larger, and for small percentages of full load it will become quite high.

In the assumed example the 5000-kw. generator will therefore work more efficiently than the small one during the four hours of its full load, generating in that time 20,000 kw-hr. During the remaining 20-hour period the large generator will work less efficiently than during the peak load period and, in fact, it will work quite wastefully, as it is running at only 4% of its normal load. The average efficiency at which the large generator supplies the energy is therefore in general different from the efficiency of the 1000-kw. set. It may be higher or lower, depending on the way in which the efficiency varies with the size of the generator and with the percentage of the load. In practice, these figures are such that an increase of the demand will under the given conditions probably always result in a decrease of the average 24-hour efficiency and therefore in an increase of the fuel and oil consumption.[8] We see from this how the peak load of the central station influences the cost of fuel and lubricants in such a manner that under the given assumptions a part of these costs may be assumed to be proportional to the peak load.

Similar considerations prevail for the maintenance and the repairs of the equipment. As regards the total cost of attendance, it will of course always be larger for a larger machine than for a smaller one, etc.

25. We see in this manner that there are certain costs which are not investment charges, but still are dependent on the peak load and can be assumed to be proportional to the peak load. Together with the capital charges which are, as shown, also approximately proportional to the peak load they make up the *"demand cost,"* sometimes called *"fixed cost."* The demand cost of a central-station plant can therefore be assumed as being proportional to the central station's peak load. This involves, as we have seen, certain approximating assumptions which are recapitulated and discussed more in detail in Appendix V.

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[8] We would, of course, for the very reason of improving the economy of generation not install (as assumed) one unit of 5000-kw. in the above example, if we could choose the size and the number of the generating units in such a manner that a better 24-hour efficiency can be obtained. It is then conceivable that the change of the load curve (that is, the increase of the peak load) may bring about a reduction of the fuel and oil cost, that is, that the percentage of the fuel and lubricant expenses (which is proportional to the peak load) is negative. The explanation of this apparent contradiction with what has been found above is given in Appendix IV—"The Influence of the Central Station's Capacity (Peak Load) on the Operating Expenses."

B. THE APPORTIONMENT OF THE DEMAND COST BETWEEN THE CUSTOMERS.

1. *The Peak Responsibility.*

26. The most important object of the computation of the cost of service is the ultimate apportionment of that cost between the consumers, that is to say, the determination of the share of the cost which is caused either by an individual consumer or by a certain group of consumers, for instance by the power consumers. Unless we go into theoretical details of little practical importance, this apportionment is simple and easy in case of the energy cost, where it has to be made simply according to the number of kilowatt-hours, but it is less simple, theoretically as well as practically, in case of the demand cost.

Since the total demand cost is proportional to the central station's peak load, the demand of a group of consumers or of an individual consumer is apparently proportional to the share which is contributed by that group or individual to the peak load of the central station.

Now, the peak load of the central station is built up of the sum of the amounts of power required by the individual consumers at the central station's peak-load time, enhanced by the line and transformer losses and the power demand of the central station for its own operating purposes. If a consumer is using 100 kw. at the central station's peak-load time, the central station has to keep 100 kw. (plus a percentage enhancement for the above-named losses, etc.) of capacity ready for his use and for *his* use only, regardless of whether he is using these 100 kw. during the peak-load hours only or for 365 24-hour days a year[1]. If the total capacity of the central station and lines,

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[1] This statement will have to be revised and slightly modified in the following paragraphs (see also Appendix VI), but for the *purpose of* a first introduction it is of sufficient accuracy.

etc., is, for instance, 10,000 kw., the demand cost of that consumer will be 1% of the total demand cost of the central station; if he uses 200 kw. at the central station's peak-load time, his demand cost will be 2%, and so forth; in short, it will be proportional to the capacity used by him at the central station's peak-load time.

The demand of a customer at the time of the central station's annual peak load is therefore a very important factor for the theory of the demand cost. It defines the measure in which the consumer is responsible for the building up of the central station's peak and it has therefore very properly been called the *"Peak Responsibility."* This very descriptive name will be used hereinafter for the consumer's demand at the central station's peak-load time. Thus, if the lower curve in Fig. 1 would represent the load curve of an individual consumer or the aggregate load curve of a certain group of consumers, and if the upper curve would be the total load curve of the central station, $d_p = M'k$ would be the "Peak Responsibility" of that consumer or group of consumers.

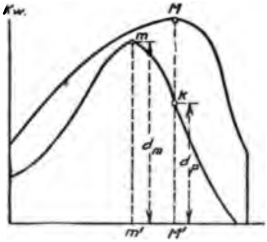

Fig. 1.

2. *The Theoretically Exact Basis.*

27. If we give further thought to the problem of the demand cost, we will soon find that it is not the peak responsibility alone which has a bearing on

the demand cost of a consumer. The following hypothetical example will explain the general idea.

Assume a 1000-kw. central station to have two consumers or classes of service only: "*A*" using 500 kw. from 5 to 6 o'clock every afternoon and nothing outside of that time, and "*B*" using 500 kw. for 24 hours every day. The peak load of the central station occurs then during the hour from 5 to 6 p. m., and the peak responsibility of each one of the two consumers is 500 kw. If we were to judge by the peak responsibility alone, the central station's demand charges would have to be equally divided between the two consumers. It is clear, however, that the central station could take on new consumers without increasing its total capacity and demand cost, if these new consumers never exceed an aggregate demand of 500 kw. and if all their demand occurs outside of the hour from 5 to 6 o'clock. They are then making use, so to speak, of the 500 kw. which are "charged" to *A*. (It should be remembered that we are dealing here with the cost and not with the price, so that the term "charge" is here not used in the meaning of "price-making" but only in the book-keeping sense of apportioning the cost.) The shape of consumer *B*'s load curve does not permit any other consumer to use the portion of the central station's peak load for which *B* is "charged," because, according to the assumption, *B* never releases that portion for the use of others. It seems logical that *B* should be "charged" for a larger portion of the demand cost than *A*, although each has the same peak responsibility.

We will always find corresponding conditions wherever there is a diversity between the consumers[2], in other words, where the load curve of the respective consumer is not entirely similar in shape to the central station's load curve.

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[2] For the explanation of the term "diversity" (diversity-factor) see Appendix VI and later Sections 42 to 49.

28. The solution of the problem to find the real value of the consumer's demand cost requires a rather complicated, though by no means difficult, mathematical analysis, which is given in Appendix VI, together with the resulting full solution of the problem. A synopsis of the results of this mathematical analysis is given near the close of Appendix VI. Condensing these results still more and expressing them in non-mathematical terms with a corresponding loss of preciseness, we can say that a consumer's demand cost is not determined exclusively by his load at the central station's peak-load time but also by the amount of his load at such times when the central station has to carry a load which is nearly as large as the peak. The demand cost of a consumer is influenced by the amounts of his load *at every single moment of time,* but the influence decreases steadily according to a certain law as we proceed from the element of time at which the central station's load is a maximum (peak load) to such elements at which the central station's load is smaller. The influence of the consumer's momentary load at such moments of time when the central station's load is near its maximum is paramount, whereas at such times when the central station's load is not a large percentage of the peak load that influence becomes insignificant or practically imperceptible.

29. It follows from this that, if the peak of the central station's load curve is more or less a sharp point, the peak responsibility of a consumer is of greater relative influence on his demand cost than if the central station's load curve has a well rounded-off top which extends over a comparatively long period.*

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*The reader who, for one or the other reason, does not care to go into the mathematics of Appendix VI may check this statement by considering the obvious facts in the following extreme example: If we assume the load curve of the central station to be a horizontal straight line (100% load-factor), then obviously we have the extreme case of the rounded-off peak and the demand cost of every consumer in that particular case is evidently proportional to his kilowatt-hour consumption because there *is no particular moment* discernible at which we could assume

3. *Central Station's Peak Extending Over a Certain Period of Time.*[1]

30. The exact method of determining the demand cost, as mentioned in the last few sections, and as set forth in detail in Appendix VI, can of course be used in practice only for larger groups of consumers (for instance, for the power consumers where we can separate the load curves for light and power). An application to individual consumers is entirely out of the question, unless we have to deal with consumers of exceptional magnitude, for instance, a railway company supplied with traction power. For smaller individual consumers the method would be far too complicated.

31. Even if we are satisfied with the consumer's peak responsibility as exclusive basis for the demand—assuming for the moment that this scheme would be commercially feasible—we would discover at the first attempt that it is impossible to determine the time of the central station's peak load exactly to the second or even to the minute. The maximum of the central station's load will extend over a period of perhaps 10 or 20 minutes, or more, at practically constant load, and this peak may even be repeated on several days, during the year. The changes of the central-station load within these peak periods will be so small that the eye cannot detect them on the chart, whereas the load of the consumer during these periods may vary between zero and 100%.

32.. But even if it were possible to discern a certain moment of time of one or a few seconds' dura-

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the peak responsibility to take place. The influence of the consumer's load at any particular moment is therefore in that case not larger than of that at any other moment.

[1] As the peak responsibility is generally not made the basis for the demand cost in practice, the principles deduced in this and the following sections are not practically applicable to the peak responsibility, but they are reflected in the practical methods of determining the basis for the consumers' demand cost (*maximum* demand, see Section 38 et seq.).

tion during which the load of the central station is larger than at any other moment of time and disregarding the statements of Sections 28 and 29, we would still have to consider the loads in the neighboring moments of time because they influence the necessary size of the plant on account of the overload capacity of the equipment.[2]

33. For these two reasons we have to extend the theoretical moment at which the peak-load responsibility takes place over a certain length of time, 2 minutes, or 10 minutes, or 30 minutes, etc. Just how long this period should be chosen depends on the shape of the highest portion of the load curve of the central station (short and steep or round and flat). This taking into account of the "near-peak load" is at the same time an approach to the fulfillment of the postulates of the exact theory as set forth in Appendix VI. (Compare also Sections 28 and 29.)

34. During the assumed duration of the central station's peak load the load on the central station will be practically constant. But the load of the respective individual consumer or group of consumers during that interval may vary within wide limits, in fact from 0 to 100%. The question arises, then, which one of the loads of the consumer is to be taken as peak responsibility. Evidently we have to assume some average of these loads during the interval.

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[2] The load which electrical machinery, cables and other apparatus is able to carry is in general determined by the temperature rise, that is, by the heating effect of the current being generated in, or passing through, the respective equipment. We can therefore put a heavier load than normal (24-hour load or "continuous rating") on electrical equipment without doing any damage, provided that the overload does not last longer than until the maximum permissible temperature is reached. The heavier the overload, the shorter is the period during which it may be applied without damage. For instance, electrical machinery will generally be able to carry a momentary overload of 50% of the continuous rating, but it will stand only about 25% over the continuous rating for half an hour without overheating. Now, it is clear that if we have overloaded the equipment, for instance for 29 minutes, with 25% it will no longer be able to safely withstand immediately afterwards an *overload of, let us say,* 15% as long as it would otherwise.

35. If the central station's equipment would not have any time element involved in its overload capacity we might simply take the arithmetical average of the consumer's loads, that is, the number of kilowatt-hours consumed by him during the central station's peak-load interval, divided by the duration of that interval (in hours, or fractions of an hour).

36. But as stated above, the overload capacity of a large portion of the equipment (generators, transformers, cables and other electrical apparatus) depends on the temperature rise of the equipment, that is, on the combination of the heating effect of the current passing through, or generated in, the equipment and the cooling effect of convection, radiation, etc. This fact suggests the choice of another average for the value of the consumer's peak responsibility such as would be furnished by the readings of an instrument constructed to register the maximum temperature rise of an element which is heated by a conductor connected in series with the consumer's load. (Wright demand meter, see Appendix XVI.) The instrument would thus reproduce to a certain degree the heating effect developed in the electrical machinery.

It must not be left out of consideration, however, that the heating and cooling effect of such an instrument may be—and generally is—different from the corresponding effect on the electrical machinery, not only in size but also in its variation with time, although they follow similar laws.[3] Also the heating curves of the various parts of the electrical equipment are different from one another.

The question which one of the two averages to choose is further complicated by the fact that the temperature rise determines the overload capacity for a certain portion only of the central station's equip-

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[3] See "Rates and Rate Making," P. M. Lincoln, Transactions A. I. E. E., 1915, page 2279.

ment, whereas the permissible overload of other parts of the equipment is determined by other factors without any time elements entering into consideration.[4]

37. Neither of the two averages will, therefore, give an entirely correct solution and we can hardly say that one solution comes much nearer the theoretically true value than the other. We have, moreover, moved so far from the exact theoretical conditions of the problem by the previous simplifying assumptions[5] that these nicer distinctions and details of the influence of the duration of the loads and overloads on the demand cost are entirely obliterated and swallowed in the inaccuracy introduced by these approximations. For all these reasons we cannot say that we ought to give the preference to either method of averaging the instantaneous demands during the peak-load period.

4. *The Consumer's Maximum Demand and Substitutes Therefor.*

38. Even allowing all the approximations for the computation of the consumer's demand cost which have been made so far (see footnote[5] to preceding section), conditions would still be too complicated for practical application to the individual consumers, except to those of the very largest size. In order to compute the demand cost of every consumer on the basis of his peak responsibility it would be necessary to know every consumer's peak responsibility. We would have to place a curve-drawing wattmeter or

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[4] Thus the boilers and prime movers (this term means the non-electrical machinery driving the dynamos) respond to overloads by a reduction in efficiency and an increase of wear and tear; the distribution lines do not allow of any overload at all, as this would result in an excessive drop of voltage which should be avoided even for the shortest period of time.

[5] Mentioned in the second footnote of Section 17; also, the further assumption that the demand cost is proportional to the peak responsibility is an approximation. In addition to these the assumption to be made later for practical application that the consumer's maximum demand is proportional to his peak *responsibility.*

ammeter (which are very expensive instruments) on every consumer's premises. At the end of the year we would have to find out from a similar instrument in the central station just at what date and what hour the peak load of the central station has occurred and then determine from every individual consumer's chart how large his average peak responsibility has been during the period of the central station's peak load. All this is obviously a commercial impossibility, on account of the high cost not only of the necessary instruments but also of the handling of their record charts. For this reason we have to apply a further approximation to determine the consumer's share of the demand cost, as follows.

39. We do not determine the consumer's kilowatt demand at the time of the central station's peak load, but rather his demand at the time of his own peak load, that is, his "maximum demand" in kilowatts (or watts) regardless of the time when it occurs, the assumption being that the consumer's maximum demand is proportional to his peak responsibility. This is rather a bold assumption, inasmuch as it seems to amount to nothing less than that the shapes of all the load curves of all the consumers are similar amongst each other and thereby also to the central station's load curve. But, as will be shown later (Section 42), the error introduced by this assumption can be materially reduced by classifying the consumers into groups of such character that the curves of all customers within each group are liable to be similar to each other (compare also Section 9); the choice of the consumer's maximum demand instead of his peak responsibility becomes then entirely justifiable.

40. A consumer's maximum demand is much more easily and cheaply determined than his peak responsibility. We can use comparatively inexpensive instruments for that purpose, the so-called maximum demand meters or demand indicators. These instruments indicate only the maximum demand during any chosen period, just as a maximum thermometer records the maximum temperature during any chosen period. And just as a maximum thermometer is very much cheaper than a curve-drawing thermometer, so a maximum-demand indicator is much cheaper than a curve-drawing wattmeter, not to speak of the cost of the handling of the charts.

Or we can use another type of instrument, the so-called demand limiter, which automatically limits the amount of the maximum demand of the consumer by interrupting his current and making his lights flicker as soon as he exceeds the given amount of demand. He can, therefore, never cause a greater demand cost than the adjustment of the instrument will permit.

With a corresponding sacrifice of accuracy we can even go so far as to abandon all such instruments. We can simply estimate the maximum demand to be expected from the size and character of the installation. This will be discussed more fully when we come to the systems of charging, that is of making the rates or prices (Part II of this book).

41. It has been shown above that where the consumer's peak responsibility is made the basis of the demand cost we have to take the consumer's demands over a certain period of time and then take an average of all these instantaneous demands. Similarly, we have in this case also to take not the instantaneous demand of a given moment but the average of all the instantaneous demands during a certain period of time when the demands are at and near their maximum, including the elements of time shortly before and after the real maximum demand occurs. The average may again be the arithmetical mean or the heating equivalent, as has been explained in Section 36. The instruments employed to determine this *maximum* demand are described in Appendix XVI.

5. *The Diversity-Factor.*

42. If the load curves of all customers would have the same shape amongst each other, the central station would have to have a load curve of the same shape (disregarding losses and home consumption of the central station) and the peak load of each customer would occur at the time of the central station's peak load. Then the sum of the customers' peak loads would be equal to the central station's peak load and the error introduced by replacing the peak responsibility with the consumer's maximum demand would be nil.

Actually, however, the shapes of the various consumers' load curves are different from one another. We can now classify the consumers into various groups or classes in such a manner that the curves within each class are liable to have similar shapes. For instance, we might group the lighting customers in one class and the power customers in another one. Lighting consumers will draw almost all their current during the evening, whereas most power consumers will consume practically all their current during the regular working hours, that is, mostly in daytime during week days. Then we may subdivide each one of these classes. For instance, the lighting consumers might be subdivided into stores, offices, factories, restaurants, theaters, churches, residences, street lighting and others. If necessary the subdivision may be carried further, for instance, the stores may be subdivided according to what is sold therein, etc. Various central stations have different practices in this respect. Likewise, the power consumers may be divided into a number of classes.

The error will then not be so very great that is introduced by the assumption that all the customers of the same class have the same shape of load curve. Gross deviations from the average load curve will occur *only* in individual cases and need not be given individual attention. In fact, they cannot be given individual attention.

It is not difficult to picture to ourselves how the peaks of various classes occur at various times. The stores will, for instance, have their peak-load time of the year at 5 p. m. or soon afterwards, around Christmas time when the days are shortest. Shortly after that hour the rush hours for the street railway will set in when the people are returning from work to their homes. When they arrive at home their residences will be lit up and we have a peak of the residence load which may last until 9 or 10 o'clock. The theaters will have their peaks beginning at about 8 p. m. The peak of summer amusement parks comes, of course, in the summer evenings after sunset. Similar diversities occur between the peaks of the various classes of power load. Where we have an irrigation load for agricultural purposes the peak of that load will even come in the daytime of summer.

43. We might now from actual measurements in a number of representative cases, or by guesswork, obtain the load curve of every class or group of customers and from this we can obtain the ratio of the group's peak responsibility to the group's maximum demand. Then we would have to multiply by that ratio the known cost per kilowatt of peak responsibility to get the cost per kilowatt of that group's maximum demand. This first step would be only another way of basing the demand cost of the class or group on the peak responsibility of the group. Suppose, for instance, the group's load curve shows a peak four times the amount of its peak responsibility, then the cost per kilowatt demand of the group's peak load would be one-fourth of the cost per kilowatt of peak responsibility. If it costs, let us say, $4 of fixed charges per month to keep 1 kilowatt in readiness at the central station, then we must figure only $1 per *month as fixed cost per* kilowatt of maximum demand of that class of customers. Since the group's maximum demand can, of course, never be smaller than the group's peak responsibility, this correction to the kilowatt cost of the group's maximum demand will never be an increase of unit demand cost, but generally a reduction.

44. This fact, that the peaks of various classes or groups of consumers do not occur at the same time, is called *diversity* of these classes. To make it quite clear what this means we can look at it in another way. Taking a simple case, let us suppose we have two classes of consumers only, *A* and *B*, each using 1000 kw. throughout the day, but raising their respective demand to 3000 kw. each for one hour during the day. Now, if these two peaks of 3000 kw. coincide, we would need a plant with a capacity of 6000 kw. to satisfy the aggregate demand. If, however, the peaks occur one after the other a plant of 4000 kw. will be large enough; 1000 kw. for one class and 3000 for the other. We can use a portion of the capacity of the plant twice, so to speak, first for *A* and then for *B* (or vice versa) on account of the diversity which exists between the two demands. If we had *A* alone, that class would have to be charged with the demand cost of 3000 kw. as the plant would have to have the capacity of 3000 kw. and the same would apply if we had *B* alone. As it is, with both classes on the lines and with the diversity between them as stated, we have to provide 4000 kw. for the two, and each one will be charged as an average with the demand cost of 2000 kw. only[1], because, as explained above, on account of the diversity we are able to use a part of the capacity of the first plant for one class of consumers and then for the other one.

45. The degree of diversity is measured by the so-called *diversity-factor,* that is, the ratio of the sum of the maximum demands of the various classes

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[1] *Compare also* **Appendix VI.**

divided by the sum of their peak responsibilities; in other words, the diversity-factor is the weighted average[2] of the ratios $\frac{\text{maximum demand}}{\text{peak responsibility}}$ the meaning of which ratios (or, to be exact, of their reciprocals) has been explained in Section 43. As the sum of the peak responsibilities is equal to the peak load, we arrive at the definition of the diversity-factor as being the ratio of the sum of the maximum demands to the peak load[3]. In our example the sum of the maximum demands is 3000 + 3000 = 6000 kw. and the actual demand (or the sum of the peak responsibilities) is 4000 kw., so that the diversity-factor is 6000/4000 = 1.5. Obviously the diversity-factor can never be smaller than unity.

The diversity-factor is of great importance in central-station economics. It indicates, so to speak, how many times the capacity of the plant can be used.[4]

46. We have a diversity not only between the various groups or classes of load which in their aggregate build up the central station's load, but also between the consumers of the same class amongst one another. If the consumers of every class would actually have the same load curve, as presupposed so

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[2] Readers who are not familiar with the term "weighted average" will find an explanation in Appendix VII.

[3] A more complete and general definition of the term "diversity-factor" will be given later (Section 49).

[4] In our above example we have for instance a plant of 4000 kw., whose capacity is distributed in such a manner that
at *A*'s peak-load time: 3000 kw. are used by *A*
+ 1000 kw. by *B* = 4000 kw.;
at *B*'s peak-load time: 1000 kw. are used by *A*
+ 3000 kw. by *B* = 4000 kw.

Assuming from the first one of these two conditions a subdivision of the plant's capacity in such a manner that 3000 kw. are set aside for *A*, and 1000 for *B*, we must conclude that 2000 kw. of *A*'s 3000 can be used over again for *B* at the time of *B*'s peak load and, as these 2000 kw. are one-half of the total plant capacity, this capacity can be used 1½ times. This way of looking at the meaning of the diversity-factor becomes still clearer if we assume more than two (for instance three) customers with the described load curves and the individual *peaks occurring one after* the other.

far (Section 42), there would exist no diversity amongst them; in other words, the diversity-factor amongst them would be unity. As a matter of fact, however, there are differences in the load curves and consequently there is a diversity between the consumers of the same class; therefore, the diversity-factor between these consumers will be greater than unity.

An example will make this clear. Measurements have shown[5] in a certain city block of apartments containing 189 residential consumers that the individual maximum demands in these installations, if added up on paper, gave the sum of 68.5 kw. But the actual maximum output of the transformer supplying these 189 installations (and no other load besides them) was only 20 kw. The reason for the difference between these two figures is, of course, that the maxima in the various installations did not occur at the same time. Customer *A* might be entertaining one night in his residence with all his lights ablaze while *B* might that same night happen to use very little light; perhaps he is attending *A*'s party and all the lights in his residence are shut off. *C*'s demand may be a maximum one hour later than *D*'s demand, etc. The diversity-factor in this example would be 68.5/20 = 3.4. If the demand cost of every kilowatt of transformer's maximum demand would be, for instance, $3.40 per month, there would be a cost of only $1 chargeable to every kilowatt of the consumer's maximum demand. (Of course, owing to the diversity between the various transformers of the system, the cost of the kilowatt of transformer's maximum demand will again be lower than the cost of the kilowatt of central station's peak load. Assuming for illustration that the diversity-factor between the various transformers of that station equals 2, then the demand cost per kilowatt of

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[5] Insull, "Central-Station Generation." Transactions A. I. E. E., 1912, page 246.

central station's peak load would be 2 × \$3.40 = \$6.80 per month, as against \$1 per kilowatt of the maximum demand of the class of consumers in question.)

47. We have so far discussed only two examples of diversity-factor: among the residence consumers to the transformers and among the transformers to the central station. Evidently we can take any group of consumers (or loads) and speak of a diversity-factor among the constituents of that group to the total group. The constituents may again be subdivided into smaller groups. We may, for instance, choose the following successive divisions: Central station, substation, feeder, transformer, consumer, lamp, and then we might distinguish a diversity-factor among the elements of any one of these divisions to any other higher division, as follows:

Among substations to central station

Among feeders to substations;
among feeders to central station.

Among transformers to feeder;
among transformers to substation;
among transformers to central station.

Among consumers to transformer;
among consumers to feeder;
among consumers to substation;
among consumers to central station.

Among lamps to consumer;
among lamps to transformer;
among lamps to feeder;
among lamps to substation;
among lamps to central station.

In every one of these cases[6] we can again distinguish between the *individual* diversity-factor and the *average* diversity-factor. For instance, choosing the diversity-factor among the consumers of the same transformer, we can distinguish a diversity-factor of an individual transformer, or an average diversity-

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[6] Except in case of the diversity-factors taken directly to the central station.

factor of all transformers on the same feeder, or of all transformers on the same substation, etc. (always meaning the diversity-factor among the consumers to the transformer).

48. Given a certain classification into divisions, for instance as chosen above in Section 47, we can distinguish a diversity-factor among the members of one group either to the next higher group (for instance, consumers to transformers) or to any other group above that (for instance, consumers to feeders or to substations). These latter diversity-factors may be designated as being "combined" from the "elementary" diversity-factors. The combined diversity-factor is the product of the weighted averages[7] of the constituent elementary diversity-factors.

If we take, for instance, the diversity-factor among consumers to a certain feeder as the combined diversity-factor, and choosing the diversity-factor among consumers to transformers, and among transformers to feeder, as the elementary constituents, then we have the following relations:

(a) Individual diversity-factor among the consumers connected to one certain transformer =

$$\frac{\text{sum of maximum demands of all consumers connected to transformer}}{\text{maximum demand of the respective transformer}}$$

(b) Weighted average over the whole feeder of the diversity-factors among the consumers to the transformers $= \frac{\text{sum of the numerators in (a)}}{\text{sum of the denominators in (a)}} =$

$$\frac{\text{sum of maximum demands of all consumers connected to feeder}}{\text{sum of maximum demands of all transformers connected to feeder.}}$$

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[7] Reference is here made again to Appendix VII for those readers who are not familiar with the term "weighted average" (see footnote to Section 45).

(c) Diversity-factor among the transformers connected to one feeder=

$$\frac{\text{sum of maximum demands of all transformers connected to feeder}}{\text{maximum demand of the feeder}}$$

(d) Diversity-factor among the consumers connected to a certain feeder =

$$\frac{\text{sum of maximum demands of all consumers connected to feeder}}{\text{maximum demand of that feeder}}$$

By multiplying the fraction under (b) with that under (c), the denominator of (b) cancels against the numerator of (c) and we get the fraction under (d); this means:

The weighted average over the whole feeder of the diversity-factors among the consumers to the transformers × diversity-factor among the transformers connected to one feeder = diversity-factor among the consumers connected to one certain feeder (combined diversity-factor). This is in accordance with the aforesaid contention.

49. Now we can proceed to the general definition of the term "diversity-factor" as standardized by the American Institute of Electrical Engineers. This definition will now be understood more readily than if we had started the explanation of diversity with that definition. "Diversity-factor is the ratio of the sum of the maximum power demands of the subdivisions of any system or parts of a system to the maximum demand of the whole system or of the part of the system under consideration, measured at the point of supply."[8]

In shorter, though less precise, words we can say that the diversity-factor is the ratio between the sum of the maximum demands and the sum of the peak responsibilities or the average ratio between maximum

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[8] *Transactions A. I. E. E.*, 1914, page 1797.

demand and peak responsibility. It determines, therefore, the ratio by which the demand cost must be reduced from peak responsibility to maximum demand and this is the reason for the importance of the diversity-factor.

Statements about the numerical values of diversity-factors will be found in "Diversity-Factor," by H. B. Gear (Transactions A. I. E. E., 1910, page 375) and in a lecture by Professor Ryan (ELECTRICAL REVIEW, April 3, 1915, page 638).

We can also speak of the diversity-factor between groups of consumers which are not yet connected to the same system of supply. We may, for instance, have a number of neighboring towns, each served from its own central station. If we now scrap these generating systems and connect all those towns to one common station, this latter source of power will not have to be quite as large as the previous independent ones taken together, because the peak loads in the different towns will occur at more or less different times.

6. Variations of the Demand Cost per Kilowatt Maximum Demand Between Consumers.

50. It can be seen from the above that the demand cost, if reduced to the maximum demand of the consumer, varies in a large degree, owing to the diversity of the load curves.

There are other variations in the demand cost per kilowatt between the customers, due to other reasons, for instance the following:

51. The installations of small customers, that is, in general of the residential consumers, are generally scattered more widely apart than the large consumers' installations which are crowded together in the factory and business districts. The small installations require, therefore, a greater length of distribution lines, not only per installation but even more per kilowatt installed. The transformers are greater in number and smaller in size for a given total capacity. The residences are generally located on the outskirts of the cities, away from the center of gravity of the system of power supply and consequently the transmission lines will have to be both longer and of larger section, etc. All this means greater investment and consequently greater cost of service due to increased interest and other capital charges.

52. Another element which affects the demand cost is the following: Electric light requires a high degree of constancy of voltage since 1% of voltage fluctuation causes more than 3½% fluctuation of the light, so that small fluctuations of the voltage may prove very annoying to the eye. We may easily notice this in some street cars, especially in interurban service, where the voltage regulation is adapted to the modest needs of the motor service only and not with a view to the exacting needs of lighting. Motor service, as just stated, is not as sensitive to fluctuations of the voltage. If it were not for the lighting service our distribution lines in the cities would contain much less copper and they would consequently be much cheaper. We are justified in saying that the capital charges for the extra copper necessary to reduce the necessary fluctuations from the power-service requirements to the narrower limits required by the lighting service should be charged as cost to the lighting consumers only and not to the power consumers.

This and other minor variations in the demand cost between consumers are generally disregarded when it comes to the computation of the cost and of the rates, but it is well once to get a clear conception that they do exist.

53. In passing, we can return to the kilowatt-hour cost and mention that we have under this heading also variations in the energy cost between customers. To explain these it has been necessary to first explain

what the load curve is and the load-factor and the diversity-factor. Therefore, these variations in the energy cost could not be discussed in their proper place in Section 12 when we were dealing with the energy cost and the other classes of variations of this cost.)

If the load-factor of the customers and their diversity-factor is low, the transformers will run at low load during a large part of the time. Now the efficiency of a transformer decreases if the load on the transformer decreases below full load. Moreover, instantaneous copper losses in the transmission and distribution lines are proportional to the square of the power consumed (at given power-factor[9]); this means that the aggregate losses of energy for a given amount of energy transmitted during a given time are larger if the power is great during a part of the time and small during the rest, than if the energy is transmitted at a uniform rate. In addition to this the power-factor decreases at low transformer loads which increases the average current and therefore further increases the copper losses.

For all these reasons the energy cost will be higher with a poor load-factor and a poor diversity-factor than if these factors are good. But this increase in the energy cost is usually disregarded; taking it into account would make matters too complicated.

We will also see that there are variations in the third part of the cost, the consumer cost or customer cost.

3. The Consumer Cost.

A. CAPITAL CHARGES.

54. There is a certain amount of money that must be invested by the electric light and power company for every consumer individually over and above the capital which is invested for the con-

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[9] *The non-technical reader will find an explanation of the te* *power-factor* in Appendix X.

sumers collectively in the central station and in the distribution lines. A large part of this additional investment is occasioned by the bare fact that the consumer is connected to the lines and this part has practically nothing to do with the amount of electrical energy consumed by the customer or with his maximum demand. It amounts to the same for a large consumer and for a small one. To make this clear, consider the following:

It is the quite general central-station practice that most of the equipment up to and including the consumer's meter is paid for by the electric light company, whereas the interior wiring proper, beginning behind the meter, is paid for by somebody else, usually the house-owner or the consumer. There is, therefore, a certain capital invested by the company for every single consumer individually. This consists in the cost of the so-called service wires or drop wires, that is, the connection from the street lines to the building and into the main service cutout; moreover, it includes the cost of the meter, etc. Up to a certain size of consumer this investment will be practically constant per consumer and above that size we can regard it as composed of two parts: A constant part (cost of average length of service connection, if constructed with minimum size of poles and minimum thickness of service wires; also cost of minimum size of meter, etc.) and another part proportional to the maximum demand of the consumer. Apportioning the capital charges of this latter part to the demand cost, we see that we have a certain investment left which is independent of the size of the consumer and therefore we will also have certain annual charges which are similarly independent of the size of the consumer.

B. OPERATING EXPENSES.

55. Moreover, there are direct annual expenses, *apart from the* expenses caused by the capital investment, which also amount to substantially the same sum per customer, whether he be a large or a small one. Thus the electric light company generally has to send out a meter reader every month to every customer to read the amount of kilowatt-hours consumed during that month and then the company's clerks have to figure out the amount of his bill. They have to write out the bill, send it out to the customer, keep their records whether and when he has paid and more such clerical and bookkeeping work. In addition to this are the costs of maintenance and repairs to the service lines and meters, etc.

56. All these expenses, including the capital expenses for meter, etc., are caused by the mere fact that the respective consumer is connected to the company's lines and they are the same for every consumer, that is, they are *constant per consumer.* They are called the *"consumer cost"* or the *"customer cost."* To give the beginner a rough idea of the order of magnitude of this amount, it may be stated here that different companies figure it somewhere between 50 cents and $1 per month for every customer. This is negligible in case of large consumers, who are paying a hundred or several hundred dollars per month. But it constitutes a very large percentage of the cost of serving the small consumer, whose monthly bill is perhaps somewhere between $1 and $5, or often even lower than $1.

C. INFLUENCE OF THE CONSUMER COST ON THE TOTAL COST PER KILOWATT-HOUR.

57. It has been mentioned before (Section 10) that it is frequent practice to reduce the total cost (and also the price) of serving a customer, to the unit of kilowatt-hours consumed by him, that is to say, the cost (or the price) of electricity is so much *per kilowatt-hour.* It is now clear that the total cost of serving the small consumer, per kilowatt-hour, is very much greater than that of serving the larger consumer in consequence of the fact that the consumer cost is constant per consumer. If the customer cost is, for instance, $1 per month per consumer, the cost per kilowatt-hour caused by a consumer who is using only 10 kw-hr. per month, *must* exceed 10 cents per kw-hr., since 10 cents per kw-hr. are necessary to cover the item of the customer cost alone. If the consumer is using 10,000 kw-hr. per month, the customer cost per kw-hr. will be 0.01 cent, that is, practically negligible, and the cost per kilowatt-hour will be as low as other circumstances will permit.

D. VARIATIONS OF THE CONSUMER COST BETWEEN CONSUMERS.

58. Of course, the consumer cost per consumer although being approximately constant is not quite *strictly* constant, no more than the demand cost per kilowatt, or the energy cost per kilowatt-hour, are *strictly* constant.

If, for instance, a building is located far back from the street (where the distribution lines are run) the service lines from the street to the house will have to be longer and therefore more expensive than the average and perhaps we may even have to put up an extra pole or two for the service line. It has been shown before (Section 51) that the demand cost will be higher if the building is located in a thinly settled part of the town (which is particularly true in case of residences). Likewise, the customer cost in such districts will be higher because the meter readers, trouble men, etc., of the company will have to walk or ride so much longer from one customer to the next and therefore will not be able to cover as many customers in a day. This will apply particularly to residences in poorer districts where not every house is connected to the lines and also in some very fashionable quarters where every house is surrounded by a large garden. Furthermore, in case of residences, these employes of the electric light company will have to call twice or oftener in a certain percentage of the houses because they found the door locked and nobody at home on their first visit. This second visit is, of course, more expensive than the first one because the meterman may have to go quite a distance out of his way to reach the single house where he could not get access the day before. On the other hand, in case of "commercial customers" (this term means business enterprises) the meterman can be sure that he can get access to the premises and the meter at any time of the business day.

These differences between the customer cost of various customers are as a rule only small; an attempt to take all these small variations into consideration would generally hopelessly complicate matters and bring very little gain, just as in some cases of the variations of the energy cost and the demand cost, as already explained.

B. The Determination of the Numerical Values of the Three Elements of Cost.

59. To determine for practical purposes the three items of energy cost, demand cost and customer cost, we can proceed as follows: We see from the company's books how large the expenses have been in a certain year under the various headings, for instance, for fuel, attendants' wages in the power house, salaries of the clerical forces, and dozens of other headings. We also know the value of the various parts of the plant, the life to be expected from them, etc., and consequently we can find the annual capital charges for those various parts. Now we distribute these various items of annual expenses by our best judgment under the three headings of energy cost, demand cost, and customer cost. For instance, 90% of *the fuel* cost may be assumed to go to energy cos

and the remaining 10% to demand cost. The wages of the meter readers will go to customer expense and a certain percentage of the salaries of the clerical force goes to the same item. An intelligent scrutiny will generally give a pretty definite idea just how much of the time and the salaries has been devoted to the making out of the bills and similar work which goes to customer cost. Other expenses will be more or less in the air as far as that is concerned, for instance, the directors' salaries, advertising expenses, etc. It is plain that there is a certain element of arbitrariness in the whole procedure, but this element is smaller than would appear from this description. A large part of the cost can be assigned without much doubt to one or the other of the three items and only a moderate percentage remains questionable and has to be squeezed in somehow. After we have thus determined the total amount of each one of the three items of the cost, it is easy to divide them by the total energy consumption in kilowatt-hours, by the maximum demand in kilowatts and by the number of customers, respectively, to get the three unit costs. Corrections will then have to be made to allow for the diversity-factor, etc., as shown above.

60. As has been pointed out just now, there is in this method a certain element of uncertainty and arbitrariness about the apportionment of certain parts of the cost. We can remove this uncertainty in theory, and under certain conditions greatly reduce it in practice by a method which may be roughly characterized as taking into consideration the changes which the energy consumption, plant capacity (peak load) and number of consumers of the central station have undergone during the last few years and bringing them into connection with the simultaneous changes of the total yearly expenses. In this manner we arrive at the average values which the three headings of cost have had during the period under consideration.

This method, which implies the use of a little simple mathematics, was first proposed by the present author in 1914. An abbreviated reproduction of the original article in which this method was first put forth in this country by the author is contained in Appendix VIII.

61. The preceding sections contain quite a lengthy theory of the cost of electric service and of the methods by which we can arrive at the figures for that cost. The author does not wish to convey the idea thereby that such scientific methods are generally employed by the central-station companies. Many of these companies, especially the smaller ones, are using to this day crude methods to learn what their unit costs are. We still frequently find the statement that the cost to the central station of service is so much per kilowatt-hour. This is a very convenient and intelligible statement, but—except in some special cases, where the governing conditions of service are known—it is not at all adequate. It will not be denied, however, that larger and well managed companies are employing much time and work on ascertaining their unit costs—time and work which are generally well and profitably spent, because the first requisite in setting an intelligent selling price for any service is dependable knowledge of the cost of rendering that service.

PART II

THE PRICE OF ELECTRIC SERVICE

1. General Remarks About the Three Principles of Establishing the Profit Over Cost.

62. The price is cost plus profit; in other words, the price is made by adding a certain percentage of profit to the cost. This percentage may be constant for all units of the commodity[1] being sold and consequently for all customers so that the prices or charges to every customer are proportional to the cost. Or the percentage of profit may be varied between the different units of the commodity sold, in an endeavor to make the customers pay a larger percentage of profit for those units which they value more highly and for which they are therefore willing and able to pay higher prices than for those units which have a smaller monetary equivalent and which possibly would not be bought at all at higher prices. In this case we will charge to different customers prices involving different profits for the same service[2] and, on the other hand, we will charge the same customer prices involving different profits for different kinds of

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[1] This means in case of electric service it is constant for all kilowatt-hours where an energy charge is made and constant for all kilowatts wherever a demand charge is made and also constant for every customer as far as a customer charge is concerned.

[2] Restricting ourselves, from here on, for the sake of convenience of expression to such commodities as are services, although th same deductions apply to all commodities.

service. For instance, we may charge him a different percentage of profit for heating current and for lighting current.

63. These two differentiations—kind of service and customer—go together and intermingle. If, therefore, in the following theoretical discussions sometimes, for the sake of brevity, only the differentiation between customers is mentioned, it must be understood that this always includes differentiation between classes or kinds of service, unless a remark is made to the contrary.

64. The first one of the two principles mentioned above (that of collecting the same percentage of profit from all customers) is called the *"cost-of-service principle,"* because the price is based on the average cost of the service (see Section 3). The second method of price calculation may be embodied either in the principle which is effectively designated as *"charging what the traffic will bear"* ("maximum-earnings principle") or in the so-called *"value-of-service principle."*

65. The term "What the traffic will bear" means: Collect from every customer and for every kind of the service prices as high as the customer is willing and able to pay[3], and not lower, but refuse to sell service in all cases where the prices sought are so low that they do not cover the cost (more exactly speaking, that they reduce the earnings, see Section 72), so that the earnings of the vendor reach a maximum; in other words, extort from the public as much profit as you possibly can.

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[3] Each consumer has, consciously or unconsciously, established certain limiting prices which he is willing to pay for the various parts of the service, even though the line beyond which he is not willing to buy the service may be a more or less hazy one. If he is charged more than this limit for any part of the service, he will forego the use of that part and restrict himself to the remaining parts of the service until, with increasing prices, he will finally drop out as a customer entirely. Thus with an increase of electric lighting rates a customer will refrain from burning certain lamps at certain hours.

The lengthy term "what-the-traffic-will-bear principle" will in the following be replaced mostly by the shorter term, "maximum-earnings principle."

66. The "value-of-service principle" differs from this inasmuch as its aim is not primarily a maximum of profit to the vendor, but it attempts to bring the greatest good—in the form of good and price-worthy service—to the public and to let the greatest number of people partake of the benefit of the respective service. The value-of-service principle charges higher prices for those kinds or classes of the service for which the respective customers are willing and able to pay such prices without reducing their consumption, but it differs from the "what-the-traffic-will-bear" policy inasmuch as it uses the excess income from these higher prices to reduce the prices to those who would otherwise use less service or who would not be customers at all, for instance, to the poorer people.

Under the "maximum-earnings" principle (see Section 65) the earnings can never be high enough. Under the "value-of-service" principle, if the earnings become abnormally high, lower rates are offered. either mainly to the profitable customers, or to the unprofitable ones who heretofore had not been served. or to both groups. We can thus either (a) give to the profitable customers the advantage of lower rates, or (b) extend the benefits of the service to new customers, or (c) we can combine the two advantages, each one in a lesser degree than under (a) and (b), respectively; that is, we grant a lesser degree of the rate reduction to the profitable customers and extend at the same time the service in a lesser degree to new customers (see Sections 88-93).

The value-of-service principle "discriminates," so to speak, between customers by charging different *percentages of profit* from them, whereas the *cost-of-service principle* "discriminates" between customers by charging different percentages of the prices which their valuation of the service would prompt them to pay. The discrimination of the value-of-service principle is of the same order as the discrimination of an income-tax system which does not collect the same sum of money from everybody objectively, but endeavors to make each one bear the same financial burden, measured subjectively at what money is worth to the respective person.

67. With the possible exception of such cases where the valuation of the respective parts of the service is determined by the price of a competitive service of equal quality, we can never hope to have more than a roughly approximated and hazy knowledge of that valuation. Nevertheless, the discussions hereafter of a few theoretical questions in this connection may be of interest and usefulness, although they start from the assumption that the valuations by the customers of the various parts of the service are exactly known in dollars and cents.

In the same manner the theory of structural strength of materials is useful as giving us an exact insight into the conditions, although in practical application to engineering structures invisible and unknown irregularities or defects in the interior of the material and other factors introduce such an element of uncertainty and haziness into the calculations that we have to choose a large factor of safety in practice and cannot go far towards exact application of our theory. Likewise in figuring the voltage drop in transmission and distribution lines we possess very elaborate theories but if we apply them in practice we find that the basis of the computation, the load to be expected, is generally known only very approximately, mostly even only from guesswork. Yet nobody will deny that these theories are useful, because they show us in what way various factors influence the result.

II. THE PRINCIPLE OF CHARGING WHAT THE TRAFFIC WILL BEAR (MAXIMUM-EARNINGS PRINCIPLE).

68. The principle of charging what the traffic will bear ("maximum-earnings principle") takes care of the interests of the producer only, in an entirely one-sided way, to the detriment of the other party concerned; that is, of the consumer who, especially in the case of public service corporations, represents the general public. This principle is therefore unethical and its further examination might be dispensed with if it were not for the fact that the "value-of-service principle" is a development of the maximum-earnings principle with certain corrections to take care of the consumer's interests and, as it is thus based on the maximum-earnings principle, we will have to investigate the latter as an introduction to the study of the value-of-service principle.

69. The sum of the total charges collected from all the customers is the *gross revenue* or *gross income.* Deducting the cost of production from the gross revenue, we get the *net revenue* or *net income.*

In the case where the capital is to bear a fixed percentage of interest (bonds, see Section 2) we will require from the point of view represented by the "what-the-traffic-will-bear" principle that the *net income* becomes a *maximum.* If, however, the percentage of return on the capital is variable (stock dividend), we are not interested in a maximum of the net income, but *in a maximum of return* on the capital.[4]

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[4] The last sentence is in general subject to a certain correction: If the enterprise is prosperous, it will for the following reason be of advantage to grant a reduction of prices within certain limits, even though this will be connected with a reduction of the rate of return. The reduction of prices will result in an increase of the demand for the commodity, a larger amount of the commodity will have to be produced and this requires an increase of the capital invested. If the price reduction remains within certain limits it must of necessity result in an increase of the net inc [illegible] ed in Section

70. The maximum "earnings" to which the "what-the-traffic-will-bear" principle aspires may therefore either be a maximum net income or a maximum rate of net return (dividend). The term "earnings" will hereafter be used to embrace the meanings

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4 of Appendix IX). If now the additional ("increment") net income is large enough in comparison to the additional ("increment") capital, the additional investment will be profitable in itself without any reference to original investment. If we have, for instance, prices which produce 50% net return on a capital of one million and if a reduction of prices will reduce the net return to 40% but at the same time make the investment of another half million necessary for increasing the producing facilities (f. i., [illegible] case of a central station), we will prefer 40% of 1½ million to 50% of one million, the more so, if the money can be raised by bonds. The additional half million will bear

$$\frac{0.40 \times 1{,}500{,}000 - 0.50 \times 1{,}000{,}000}{500{,}000} = 20\%,$$

which is so handsome and attractive a net rate of return that we would not hesitate to reduce the price still further in order to obtain the opportunity for the investment of further capital at a lower percentage than 20%. The rate of return on the additional capital decreases with every price reduction and we will continue the price reduction until the return on the increment capital ceases to be attractive.

By induction from this example we can say: The most desirable price from the point of view of the producer (central station) is found if, starting from the price which furnishes a maximum rate of return, we lower the price so long—and not longer—until a further very small price reduction will furnish an increment income barely large enough to yield a rate of return on the increment capital, which is just at the limit of being attractive to new capital. The total (or "average") net return on the capital is then smaller than the maximum possible net return but larger than the rate of return which is still just attractive to new capital. Likewise, it can be easily seen that the most desirable price is lower than the price which furnishes a maximum rate of return but higher than the price which furnishes a net return barely at the limit of being attractive to new capital. At the same time this price can, of course, never reach down as low as to the price p_n (which furnishes a maximum of the net income n) because the increment net income at that price is zero and therefore the (gross) rate of return on the increment capital will also be zero which after deduction of depreciation and other fixed capital charges would make the net return (dividend) on the increment capital not only not attractive but even negative. (Compare Section 18.)

Wherever the most desirable price is situated, we are safe in the statement that at least as long as a reduction of the price results in an increase of the net return this reduction is advisable from the producer's standpoint. With this understanding the following investigations will assume that the most desirable price is that which brings about a maximum net return, although it is actually lower and situated between the price which produces a maximum net return and that price which produces a maximum net income.

of both "net income" and "rate of return" and for the sake of generality the meaning "gross income" will be included also.

71. The requirements of a maximum net income and of a maximum return are not fulfilled by the same conditions. A change of prices may, for instance, increase the net income, but it may at the same time increase the necessary capital in a greater measure[5] so that the rate of return is reduced. (Compare Section 4 of Appendix IX.)

72. If we want the earnings to become a maximum, we must arrange the rates according to the following two principles: (1) We must attempt to charge for every unit of the commodity just the maximum limit at which it can still be sold under the circumstances and not less, and (2) we must refuse to sell any unit of the commodity at a price which reduces the earnings.

73. The first one of these two points implies that we have not only to make different charges for the same service to different individual customers—or at least to classes of customers—but also to charge the same customer differently for different kinds of service, for instance, for heating and lighting current. We have even to go further than that; we must charge different units (for instance, different kilowatt-hours) of the same service differently to the same customer, as the following example will make clear. A certain customer may use 100 kw-hr. per month for his lighting if he has to pay an energy charge of 6 cents per kw-hr. If he gets a cheaper price, he will generally use more current for his lighting by increasing his illumination, or his burning hours, or by using indirect lighting methods, etc. The additional kilowatt-hours are worth less to him than the original ones; they have more the character of a luxury, as is plainly

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[5] Increasing the quantity of the commodity which can be which therefore is to be produced.

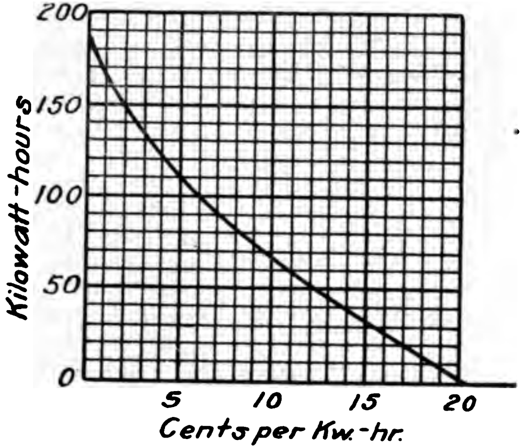

Fig. 2.—Customer's Sales Curve.

demonstrated by the fact that he is not willing to buy them if he has to pay 6 cents, whereas he does buy the first 100 kw-hr. at that price. Let us assume that the number of kilowatt-hours he is willing to purchase per month at various unit prices of the kilowatt-hour with a given constant demand charge is given by curve[6] *m* in Fig. 2, which shows that at 6 cents per kw-hr. he will use 100 kw-hr. and if the charge is lowered to, let us say, 4 cents per kw-hr., his consumption will rise to 120 kw-hr. If we want to sell that latter amount to him, we need not lower the unit price for the first 100 kw-hr. per month because the customer is willing to pay 6 cents for every one of them, but we must lower the unit price to 4 cents per kw-hr. for each one of the next 20 kw-hr.[7] In pursuance of this

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[6] To be called "sales-curve" of the customer, compare Section 1 of Appendix IX.

[7] The first 120 kw-hr. cost then a total of $600 + 80 = 680$ cents, or an average of $680/120 = 5.67$ cents per kw-hr. It might seem on superficial observation that the effect is the same if we charged 5.67 cents per kw-hr. uniformly for every kilowatt-hour throughout, from the first to the 120th. It must

same method of reasoning we find from the curve that (choosing unit prices stepped off by whole cents) we ought to charge a price of 3 cents per kw-hr. for the next 12 kw-hr. (i. e., from the 121st to the 132d), etc., until we reach the limit mentioned in the following section (74).

Extending the same principle to the other (right) side of the curve we will find that instead of charging 6 cents per kw-hr. for the first 100 kw-hr. we should rather charge the customer 20 cents per kw-hr. for his first 2 kw-hr., 19 cents for each of the next 6 (3rd to 8th), 18 cents for the next 6 (9th to 14th), and so on, until we arrive at a unit charge of 7 cents for all kilowatt-hours from the 81st to the 90th and of 6 cents from the 91st to the 100th kw-hr. To be quite exact, we would have to deal with steps smaller than 1 kw-hr. and smaller than 1 cent; strictly speaking, the steps should be of infinitesimal size.

74. The second one of the two points in Section 72, that under the maximum earnings principle we must refuse to sell any unit of the commodity at a price which reduces the earnings, obviously means the following:

(a) If we aim at a maximum *gross income* it is evident that every sale will increase the same. There is no price above zero in existence which reduces the gross income.

(b) If we aim at a maximum *net income* we must avoid any sales of which the gross income does not reach at least the increment cost over the cost of all the commodities already produced before[8], that is,

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not be lost sight of, however, that if we charge uniformly 5.67 cents per kw-hr. the customer will not use 120 kw-hr. but a lower number (somewhere between 100 and 120 kw-hr.); according to the curve he will use only about 104 kw-hr. with a resulting revenue of only 104 × 5.67 = 590 cents instead of 680. On the other hand, if we attempt to make the customer use the whole amount of 120 kw-hr. under a uniform price for every kilowatt-hour we will have to go down as low as 4 cents per kw-hr., which will reduce the income to 480 cents.

[8] See Section 3 of the main text and the footnote to Section 25 of Appendix IX. The increment cost depends not only on the

only such sales must be made of which the *increment net income is positive.* If the increment net income is positive, the *rate of return of the increments* must of necessity be positive also, and *vice versa*; we are designating by this term the rate of return which the increment net income provides for the increment capital.

(c) If we aim at a maximum *rate of return* it is not enough that a positive "rate of return of the increments" result from the respective sale, but it must be *greater than the rate of return* produced by all of the other sales which have been made before the addition of the respective sale.

75. We can also express the contents of Section 74 in the following manner: Under the maximum-earnings principle we must refuse to make any sales at prices which are lower than the cost increment per unit[9], the term "cost" being defined in each case as those portions of the expenses (see Section 2) with which the respective kind of "earnings" (gross income, net income and rate of return, respectively) is concerned in the following way: *Gross income* is not concerned with the cost at all, inasmuch as cost does not play any part in the makeup of the gross income and

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amount of the commodity added but also on the amount of the commodity produced before the addition of the incremental amount of the commodity. The increment cost from 100 to 200 units will generally be greater than the increment cost from 1100 to 1200 and this in turn is greater than the increment cost from 10,100 to 10,200 units. The same applies to the increment capital.

[9] Note the difference between the meanings of the following three terms: (a) increment of (total) cost or cost increment; (b) increment of unit cost and (c) cost increment per unit.

If, for illustration, it costs $500 to produce the first 100 units of a certain commodity (for instance castings) and $600 to produce the first 150 units, we will get the numerical values for the above three terms for an increase of the production from 100 to 150 units as follows:

(a) The cost increment is $600 — $500 = $100.

(b) The increment of the unit cost = unit cost for a production of 150 units minus the unit cost for a production of 100 units = $600/150 — $500/100 = —$1. The increment of the unit cost is therefore a negative amount and it would in this case be more convenient to use the term "decrement of the unit cost."

(c) The cost increment per unit = $100 / (150—100) = $2.

we have to set the cost in this case = 0; as the cost is therefore constant, the cost increment is also = 0 and the lowest price which is permissible under the maximum-earnings system in order to obtain a maximum gross income is zero, that is, any sale, however low the price obtained, will swell the gross income. *Net income* is affected by the cost of labor, material, etc., as explained in Section 2, but it is entirely independent of the capital charges, such as interest, depreciation, etc., and we will therefore define the term "cost" in this case as excluding the capital expenses. Where we are interested in the *rate of return* (dividend) the term "cost" must be understood to include the capital expenses on the increment capital[10] figured at the same rate of interest which all the other aggregate sales yield on the aggregate capital.

We see that, with exception of the case dealing with a maximum of the gross income (which has no practical importance), the question whether or not a certain sale is desirable under the maximum-earnings principle can be answered only if we take into account the sales already made before the addition of the respective sale, because the increment cost and the increment capital depends on the number of units produced. In case we consider the rate of return, the profitability of the other sales has also an important bearing on the question; for instance, a certain sale netting a rate of return of 15% on the increment capital will be desirable, if the other sales together produce an average return of 10%, but it must be avoided if the rate of return of the other sales averages to 20%.

76. If we consider not one single sale but ask which ones of all the possible sales shall be accepted and which ones refused to get a maximum of earnings, we will first pick out those which produce a maximum gross income per unit because they will

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[10] The increment capital to be figured in the same way as the increment cost; see the preceding footnote of this Section.

produce a greater increment net income and "rate of return of the increments," respectively, than any other sales, regardless of the way in which the increment cost and the increment capital, respectively, are figured. This means we start with the sales which bring higher unit prices than any other ones; then we add gradually sales with lower and lower prices until we arrive at prices which begin to reduce the earnings (either a negative increment net income or a "rate of return of the increments" smaller than the average rate of return resulting from the sales considered heretofore).

77. We see from the foregoing *that the average cost per unit has no influence on the price of a certain unit* under the maximum-earnings principle. The price of a certain unit which is sold profitably may be higher or lower than the average cost. The same applies for the value-of-service principle.

III. THE VALUE-OF-SERVICE PRINCIPLE.

78. In comparing the value-of-service principle with the two others previously mentioned as to their relative merits we could dismiss the "what-the-traffic-will-bear" principle (maximum earnings) from the outset as unethical and unfit for public service corporations. We would confine ourselves to a comparison between the cost-of-service principle and the value-of-service principle, but since the latter is nothing but an evolution of the maximum-earnings principle, we will have to pay attention to the latter principle also.

In order to determine the relative advantages of the cost-of-service and the value-of-service principle, both to the consumer and the producer, it is convenient to assume that we have been making our charges first on the cost-of-service basis, that is, we have been charging the same percentage of profit to every customer and for every particle of service. We assume further that the price has been regulated so that the earnings derived from that system of charging are just what is considered fair, neither more nor less.

Then we change over to charging on the value-of-service basis, in such a manner that the earnings (gross income, net income or net rate of return) of the producer remain unchanged, or at least are not reduced below what has been recognized as fair. The producer will therefore experience at least no damage as a consequence of the change-over. We will first investigate under what conditions it is possible to make such a change-over to the value-of-service principle in such a manner that the price is not increased to any consumer[1]. This means that we will begin our investigation with that part of the value-of-service principle which requires only a lowering of the price (namely, for those parts of the service for which the respective consumers are not willing or able to pay the original price), but we will leave out of consideration, for the beginning, an advance of the price for those parts of the service for which the customers would be willing and able to pay more than the original

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[1] If the prices are made lower to some customers than to others, the latter are liable to complain that they are "discriminated against" and to demand that the same prices be charged to every customer. This demand is prompted by a hazy feeling that in that case the resulting price will be the average between the prices charged previously to the various customers, or at any rate lower than the prices charged to the complaining customers, because, it is stated, "the minus of profit from the favored customers must be covered by a plus from the others."

In a well designed rate system this assumption is not justified. The price reduction to certain customers increases the aggregate consumption and therefore the production. A larger production results in a lower unit cost so that the central station can, as a next step, reduce the prices, for instance, to all customers alike by a certain constant percentage, without reducing its own profits below what they were originally. (In fact the profit of the central station may even be left higher than it had been before the first price reduction to the "favored" customers had been made.)

Thus the customer who is grumbling that he has to pay more than some other customer is actually paying less than he would have to pay if he were not "discriminated against."

This problem is treated in detail in the following Sections.

price. The value-of-service principle will then be of indisputable advantage to some of the parties concerned and of disadvantage to nobody. Then we will extend our investigation to the introduction of raised prices, where necessary.

79. The change from the cost-of-service principle to the value-of-service principle means that we will charge no longer one uniform price[2] throughout, but a number of prices in accordance with the valuation of the respective service or part of service by the respective customer, that is, in accordance with the customer's power and willingness to pay for the respective service. This process of judiciously raising or lowering the prices according to the valuation of the service will be called *"price splitting."* If price splitting comprises only price reduction and no increases it will be called *"price splitting downwards"* in contradistinction to *"price splitting upwards."* Of course, price splitting downwards and upwards can be combined, so that some prices are reduced, others raised with respect to the original price.

80. In Appendix IX is given an analysis of the relations between prices and earnings, including an investigation of the conditions which bring about the possibility of increasing the earnings by splitting the prices downwards only, that is by lowering the prices of at least some parts of the service and to at least some customers, but raising them to nobody.

A. ADVANTAGES TO THE PUBLIC AND TO THE PRODUCERS.

The principal results of the investigations in Appendix IX about price splitting are in brief words the following (Sections 81-86):

81. If we determine the prices for *every particl*

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[2] Or, more accurately speaking, prices with a uniform percentage of profit.

of the service separately[1] in accordance with the value which the respective customer attaches to that particle, but with the restriction that the prices shall never be raised above the original price, this operation (price splitting downward) *must always* result in an increase of the earnings (whether gross income, net income or dividend), provided that no particle of service is sold at a price below a certain minimum. The amount of this minimum depends on the kind of earnings we have in view (gross income or net income or rate of return) and it is identical with the values given as lower limits in Section 74 (a), (b) and (c) for the "What-the-traffic-will-bear" system with the three kinds of earnings. These values are of necessity always lower than the original price, provided the latter has been chosen so that it does not result in negative earnings. (The above is proved in Sections 10-13 of Appendix IX.) *Therefore profitable price splitting downwards will always be possible.*

This method of price determination would mean not only that the prices vary between different kinds of service and between different customers, but different particles of the same service would be charged differently to the same customer. For instance, every customer would have to be charged slightly differently for the first kilowatt-hour supplied to him for a certain kind of service than for the second one, and for the second differently from the third, etc. (see Section 73). The same would apply to the kilowatts of demand. The theory would, moreover, require that the kilowatt-hours used for kitchen lighting, for instance, be charged differently from those used for the lighting of the living room, and those used at 6 p. m. differently from those used at midnight because the valuation of these particles by the customer will probably be different.

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[1] *For instance every* kilowatt, every kilowatt-hour and every *individual customer,* respectively.

82. As this theoretical method of charging is obviously impracticable we will in practice try to approximate it as closely as we can. We will combine into one group or class of service, or of customers, large numbers of such units of service of which we expect that there is approximately the same valuation on the part of the customer or customers. We then charge all units in the same class or group at a uniform price. Price splitting downwards will then mean lowering the prices for all units in one or more of the groups in such a manner that the prices remain equal amongst one another for all units in the same group, but they will differ between groups. We have therefore in this practical value-of-service system a hybrid between the theoretical value-of-service system and the cost-of-service system. We may, for illustration, charge all kilowatt-hours for heating service at the same price and differently from the kilowatt-hours for other than heating service. Or we may charge the first 50 kilowatts demand of wholesale power at certain fixed demand charges which are higher than those for the following kilowatts, etc. The more uniform the valuation is by the customer of the units within every group, that is, the more intelligently we select the groups and the larger we choose the number of the groups the more closely will this method and the results therefrom approximate the method of Section 81 and the results of that method.

This method of varying the prices, not between individual units but between groups, will be called "price splitting between groups."

83. The latter part of Appendix IX (beginning with Section 15 of that Appendix) contains a detailed investigation of the conditions under which price splitting downwards between groups is of advantage to the three different kinds of "earnings" (gross income, net income and rate of return).

The principal results of these investigations are the following:

1. The necessary and sufficient condition that price-splitting downwards between groups shall increase the earnings is that the number of units sold in the group (or groups) for which the price is being lowered, increases more rapidly than the "profitable portion of the price" is being diminished by the price reduction[2]. The term "profitable portion of the price" means the excess of the price over the cost increment per unit[3] provided that the term "cost" (see Section 2) is defined in each case in the same way as in Section 75 (where we dealt with the lower limit of the prices permissible under the maximum earnings principle), that is:

Gross income...	Cost = 0; cost increment = 0.
Net income....	Cost excludes capital expenses.
Rate of return..	Cost includes capital expenses; cost increment includes the capital expenses of the increment capital figured at the same rate of gross interest which is obtained for the rest of the capital.

Other interesting and fundamentally different ways of stating the same law are found in Sections 16-19, 29-30 and 37-38 of Appendix IX.

2. It follows from condition 1 of this Section that the prices can in no case be profitably reduced below the "cost" increment (as defined above), although we can find or construct special cases of sales curves[4]

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[2] If, for illustration, we contemplate a price reduction which reduces the profitable portion of the price to 2/3 of its original value, the number of units sold in the respective group must increase to at least 3/2 of the original amount (that is, to the reciprocal of 2/3) to make this price reduction advisable. The profitable portion of the price has then been reduced from 150% to 100% and therefore the amount of the sales must be increased from 100% to at least 150%. If the profitable portion of the price is reduced to 1/2 (200% to 100%), the number of sales must at least be doubled (100% to 200%), etc.

[3] *See the* first footnote of Section 75.

[4] *For explanation of the* term "sales curve" see Section 73.

which bring the range of profitable price reductions as close down to that "cost" increment as we desire. In other words, the "cost" increment is the lower limit to which the price can profitably be reduced under any circumstances. With a given sales curve we can not reduce the price profitably lower than to a certain lowest limit which is greater than the "cost" increment.

3. Condition 1 *must always* be fulfilled (this means price splitting downwards between groups is *always* of advantage to the earnings), if the price of the original cost-of-service system is at least as high as the price which results in a maximum of the respective aggregate earnings of all groups under the cost-of-service system. Condition 1, on the other hand, *may* be fulfilled if the price of the original cost-of-service system is lower than the above limit.

84. Stating the above laws in still shorter terms (which of course implies a corresponding further reduction of the accuracy) we can say: A lowering of the prices will improve the earnings if the price is lowered in those groups for which the sales rise sufficiently rapidly as the prices decrease. This means that price splitting downwards between groups is of advantage if one or certain ones of the groups show a comparatively heavy percentage of increase of sales in the range where the price is lowered. If the term "price splitting downwards" is to be understood in such a manner that the prices are reduced to one or more of the groups, whereas they are kept at the original value in at least one of the groups, we have to add the condition that at least one of the groups increases its sales only comparatively slightly for the price reduction in which the others show a heavy percentage of increase of sales. In other words, the groups must show a good differentiation between the shapes of their "sales curves," at least in the range of prices below the original price in which the price reduction will take effect. An instance of this kind is furnished by the two groups of lighting and heating service. If we reduce the price, the consumption of heating current and the heating demand will rise more rapidly than that for lighting service and therefore a reduction of the rates for heating current below those for lighting current will be proper, a thought which is borne out in a large number of rate schedules.

85. It will be the easier to classify the service and the customers into such well differentiated groups the more groups we have.

At the same time we can expect that under a cost-of-service system of charging, which has been intelligently devised from the producer's point of view, the price will be such as to furnish a maximum of earnings or at least that the price will not be far away from that value. With reference to Section 83, condition 3, we can therefore say that even under a system of price splitting by groups it will always be possible to increase the earnings by means of price splittings downwards.

It should be kept in mind that an ideal analysis of the total service into groups of service will be identical with a separate determination of the price for every particle of service and, as stated in Section 81, and proved in Appendix IX, this will under any practical circumstances result in the possibility of increasing the earnings by price splitting downwards.

86. We can thus say generally: If the original cost-of-service system has been intelligently devised from the producer's point of view and if the classification into groups is intelligently made, *it will practically always be possible to raise the earnings by the introduction of price splitting downwards.*

87. Whereas we have seen (Sections 81 and 83) that price splitting can be carried out with advantage to the earnings as long as the prices do not reach *below a certain* lower limit, this must not be interpreted by any means to imply that we reach a maximum of the earnings by lowering the prices to that limit. On the contrary, carrying the prices down to that level would mean that we have lowered the prices so far that all the advantages gained by the reduction of the prices have been gradually reduced to zero and we are back at the same point—as far as the amount of the earnings is concerned—as before price splitting began. In order to obtain a maximum of the earnings the lowering of the prices must not be carried to that extreme but must be stopped earlier. (The exact amount to which the prices are to be lowered in order to procure the maximum of the earnings is discussed in Appendix IX.) In order to reach a maximum of the earnings the prices should in general be changed to a different level in every group. Where we are interested in price splitting downwards only, we effect a change of prices in those groups only where they should be lowered and in the other groups we leave the prices at their original level.

88. We go back now to our original problem of Section 78 and assume that we have had a system of charging under the cost-of-service system with the prices regulated in such a manner that the earnings are just what is considered fair, not so high that the public might be justified in demanding a reduction of the rates, nor so low that that part of the public which has invested its savings in the public service corporation finds the return for its capital and risks insufficient so that capital would be driven away to other more lucrative enterprises which would result in the ultimate damage to the consuming part of the public as well. Then we change over to the value-of-service principle by price splitting downwards only (and not upwards). According to what has been said in the preceding Sections (see Section 86) it will practically always be possible to do this in such a manner that the earnings are increased thereby[5]. We arrive thus at a system of charging which will be called for the sake of convenience "the first value-of-service system" or for short *"the first* system."

The upper part of Fig. 3 shows how the various consumers are affected by this change of charges. Fig. 3 refers to the sale of a certain particle or group of service, for instance to the first 50 kw-hr. of lighting service, or it might refer to the demand charges, for instance of the first 10 kw. of power service, or to any other reasonable subdivision of the service. This particle of service will be worth different amounts of money to different consumers, which means different consumers will be able and willing to pay different amounts for the respective particle of service. These amounts will be called hereafter the consumer's *"limiting price"* for that particle of service. This "limiting price" (in dollars or cents), which varies for the various consumers between zero and a certain maximum, is stepped off in horizontal direction in the various parts of Fig. 3 from the line *OO* to the right. Every consumer is charged a certain percentage of his "limiting price" and these percentages which the consumers are charged actually under the various systems represented in Fig. 3 and to be discussed presently, are stepped off upwards from the respective horizontal base line in vertical direction. Where that percentage is 100% or less the consumer will purchase the respective particle of service to which the diagram refers, otherwise he will stay away and not avail himself of the benefit of the central station's electric service. For those consumers who are buying the service under the various systems the area of the diagram is shaded, for all the others it is left empty, so that the shading shows at

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[5] It will be shown later (last sentence of Section 92) that this fact, though helpful for the understanding of the evolution the *value-of-service* principle, is not altogether essential.

a glance how far the benefits of the electric service reach the different classes of the population.

In the cost-of-service system (top diagram) one price only is charged to all customers, therefore only one certain class of prospective customers will be charged just their limiting price (100%), all the others are charged either less (to the right of *PP*), or more than 100% of their limiting price (all prospects to the left of *PP*). The latter will stay away and not become customers under this system.

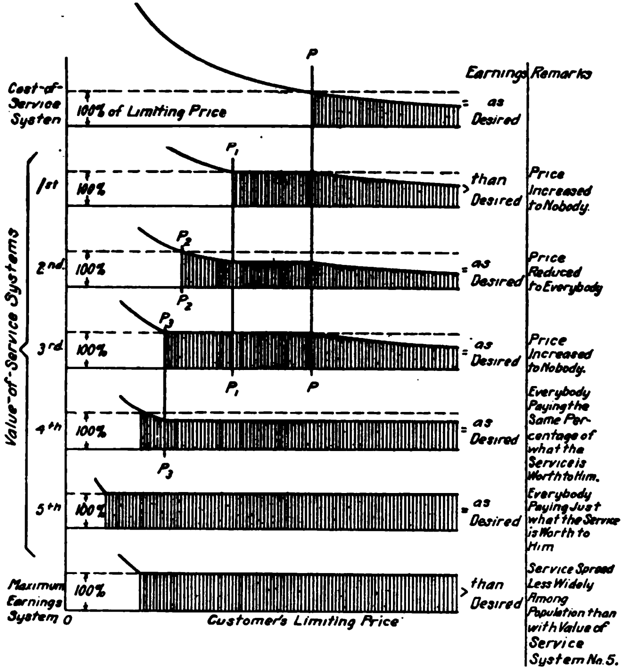

Fig. 3.—Comparison Between Cost-of-Service System, Value-of-Service Systems, and Maximum Earnings System.

If now price splitting is introduced within a certain range from the original price OP downwards, for instance in such a manner that the lowest price to be split off reaches as far down as P_1P_1, all customers whose limiting price is in that range will be charged their limiting price. Thus (see the second diagram from the top in Fig. 3), all customers whose limiting price is large enough as to reach beyond the line P_1P_1 but not large enough to reach beyond the line PP are charged just their limiting price (100%), those to the right of P_1P_1 are charged a smaller percentage, and the prospects to the left of P_1P_1 would be charged more than 100% of their limiting price and will therefore not be customers. Under this system which will be called *"the first value-of-service system"* we have added as customers all prospects whose limiting prices are in the range between P_1P_1 and PP (compare the shaded ranges of the two top diagrams in Fig. 3).

If now the splitting of prices for the "first value-of-service system" has been made in such a manner that the total earnings of the company are increased thereby (see Section 86) and if, on the other hand, we want to preserve the amount or percentage of earnings which has been considered fair before price splitting was put into effect, we will have to modify this "first value-of-service system" by introducing price reductions.

89. We might reduce *all* prices in the same fixed ratio so long until the excess of earnings has disappeared. In that way every one of the consumers would be positively benefited by the introduction of the value-of-service system. Let this be called the *"second system."*

This reduction would mean that certain customers at the lower end of the scale would be added which could not be reached under the former prices. The *customers added will* be those whose limiting price is greater than the lowest one of the new prices, but smaller than the lowest price of the first value-of-service system (see Fig. 3, System 2, shaded portion to the left of P_1P_1).

90. Or *("third system")* we might leave unchanged the prices that are charged to the existing customers under the "first system" and add new lower prices for those customers only who have not been able and willing to pay the prices of the first system for the respective service. This means that those consumers who had been customers under the cost-of-service system (that is, the customers to the right of *PP*) are not benefited by the introduction of this (third) value-of-service system, but neither do they lose. They simply continue to pay the price they were charged under the original cost-of-service system (and under the first value-of-service system). This is the disadvantage of the third system compared to the second where all customers are positively benefited by being allowed a price reduction. The advantage over the second system is that a certain portion of the public can be supplied with service that had to forego the benefits of electric service under the second system[6] (between P_2P_2 and P_3P_3 in Fig. 3).

In this third system all consumers pay for their service just as much as they are able and willing to pay, with the exception of those consumers who are

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[6] That System 3 must actually bring service to such customers as have lower limiting prices than the lowest served under System 2 (in other words that P_3P_3 must be situated to the left of P_2P_2) becomes evident if we imagine System 3 to be developed out of System 2 by raising all prices between P_3P_3 and *PP* to 100% of the limiting price and raising all prices to the right of *PP* accordingly until they reach the corresponding prices of System 3. These increases of prices without any accompanying reductions must necessarily result in an increase of the earnings because none of these price increases has been carried beyond the 100% limit so that all the consumers who have been customers under System 2 will remain customers under System 3. To offset this increase of earnings and to bring the earnings down to the desired normal height we have t add new customers to the left of P_2P_2, by offering prices *them which* are low enough to attract them.

charged the highest price, that is the price of the original cost-of-service system. These customers pay less than 100% of their "limiting price." (See Fig. 3, to the right of *PP*.)

91. Unless we insist that the changeover to the value-of-service system shall avoid raising the price to any customer over the price paid under the original cost-of-service system there is no logical reason why those customers who are reaching up to that arbitrary dividing line *PP* of the original "cost-of-service price" should be exempted from the rule which applies to all other consumers, namely that each consumer shall pay not more and not less than what the service is worth to him. Under the "third system" those customers who would be willing and able to pay the highest prices of all are charged less than their limit (whereas all the others are charged just their limit). We can therefore go another step forward and *raise* the prices to that class of consumers so that every customer has to pay just what the service is worth to him. This permits of a subsequent further reduction of the prices and, if the earnings are, as assumed, to be kept constant, such a reduction is even necessary.

We can again choose two methods for this subsequent reduction, in analogy to the two methods applied in developing the second and the third system. Either we reduce all prices by the same percentage, so that each customer[7] pays the same percentage of what the service is worth to him *("fourth system")* or we add another set of prices at the lower end of the scale extending this set downwards so long until the excess of the earnings has been swallowed *("fifth system")*. This means new consumers are taken on for the service in question for whom even the lowest prices charged heretofore have been prohibitive.

It should be noted that if we carry the value-of-

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[7] With the exception of those added at the lower end of the scale by the last reduction. (See Fig. 3, System 4.) Those consumers pay a higher percentage.

service principle in this manner to its ultimate consequences (fourth and fifth systems) this amounts to a "maximum-earnings system" (see Section 65) with these modifications (compare Fig. 3):

(a) In the fourth system the prices are reduced to all consumers[8] by a certain fixed percentage below the price charged under the maximum-earnings system. This percentage applies alike to all customers so that every customer has to pay the same percentage of what the service is worth to him. The minimum price must not be identical with that of the maximum-earnings system; it may be higher or lower.

(b) In the fifth system every customer has to pay 100% of what the service is worth to him, but the prices reach further down than with the maximum-earnings system. Under the maximum-earnings system we must refuse to make any sales at prices so low that they would reduce the total earnings (see Sections 72 and 74), whereas under the fifth system we go deliberately below that limit with the intention of reducing the earnings to the desired limit.

92. It is of interest that with none of the five systems are we bound in any way by considerations of the cost, as far as the individual price is concerned. The individual price may reach down even below the increment cost; with the fifth system the prices are even bound to reach below that limit. The consumers for whom the service is of higher value make up for the reduced profit or even loss caused by others.

Each one of the six systems (original cost-of-service system and five value-of-service systems) in the order named carries the benefit of electric service further into the population than the preceding system.

We see also from the above deductions and from the diagram Fig. 3 that, unless we want to avoid raising the prices to any customers (Systems 1 to 3)

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[8] *With the exception of those who are paying the very lowest prices (see diagram).*

we arrive at the same ultimate systems (4 or 5) regardless of whether it is possible to employ price splitting downwards with an increase of the earnings, or not.

93. The beginner should clearly understand that the above is a theoretical investigation based on the assumption that we have a full and definite knowledge of how the sales vary with the prices. Its object is to get an insight into the general effect and the possibilities of an intelligent value-of-service system (see Section 67). The five systems are five steps or stopping points, chosen with a certain degree of arbitrariness to explain the evolution and the effects of the value-of-service principle. But in practice the knowledge referred to above is a thing not to be thought of. We can therefore in practice not distinguish between the five systems as clearly as we can in theory. Practical price-making in general is necessarily always based on business instinct and even guesswork, and so is the practical application of price splitting in particular.

All we can do in practice is to grant lower prices to those groups of service which, as for instance heating and cooking service, can be expected to increase their sales with decreasing prices much more rapidly than others, for instance lighting service. At the present average price which is charged for general lighting we have reached a certain degree of saturation of lighting service. Although a reduction of the price for lighting current to one-half or one-quarter of its present average value would undoubtedly increase the use of electric light, yet it would not increase it by nearly as much as a reduction to the same price would increase the use of electric heating because the present *lighting* rates are almost prohibitive for electric heating and cooking, except for some small appliances, largely as a luxury or convenience *than a necessity.* Therefore we will conclude that a reduction of the price—or rather of the percentage profit—for heating and cooking below that charged for lighting service is generally advisable. There are of course other considerations, such as the complication of the rates or the necessity of a separate meter, which may make it undesirable to create a separate rate for heating and cooking (to remain at the example chosen) and all this requires good judgment on the part of the rate maker to decide whether he should use under the given local conditions a separate rate or one of the other methods for the same purpose (Wright rate, number-of-rooms rate, etc.) to be described and discussed in the following Part III of this book, or whether it might not be better to entirely forego a diversification of the profits. The latter practice is found as a rule more in smaller companies where simplicity of the rates is paramount, but as the knowledge of the principles of rate making is spreading this practice has a tendency to disappear.

B. FACTORS DETERMINING THE CUSTOMER'S VALUATION OF THE SERVICE.

94. It has been said above that the value-of-service principle distinguishes between different kinds of service according to the monetary value attached to the service by the customer. This monetary value constitutes the upper limiting value for the price, and the value-of-service principle endeavors to make the charges with due consideration of this limiting value for the various parts of the service and the various customers.

Now this limit of the monetary value beyond which the customer will not purchase the respective part of the service depends on two factors: (1) The value he places on the service, that is the degree in which the service is necessary, useful, profitable or convenient to him; and (2) the value he places on money.

The first factor is determined by a variety of circumstances, among which the possibility of obtaining a similar service from some other source and the quality and price of such competitive service are prominent. This factor can vary for the same customer between different parts of the same service. Thus, for instance, a customer will generally value artificial heating service more in winter than in summer. (In the latter case he will even place a negative value on it as he would pay for having heat removed.)

The second factor is to a large degree determined by the amount of money which the customer is able to spend (and is willing to spend). That is, it depends—apart from the customer's economic inclinations—on the degree in which the customer (or the class of customers) is blessed with worldly goods.

C. Application to Individual Customers and to Classes of Customers.

95. Now obviously we cannot shade prices for each individual customer by determining or estimating the monetary value he attaches to the respective part of a service. This method of applying the value-of-service principle between individual customers is generally considered unfair in commercial practice[1] and respectable retail merchants, for instance, charge the same price for certain goods on a certain day for all customers without exception.[2] Discrimination be-

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[1] There are exceptions, however. For instance, the practice of physicians to make lower charges (or none at all) to indigent people and to indemnify themselves for their time from the pocketbooks of their wealthier patients, is generally not objected against. The amounts expected from the individual as contributions towards charitable and other public purposes (also tips) which, although supposedly voluntary contributions, still frequently are collected and given under a certain pressure, are also in proportion to the individual contributor's assumed financial standing.

[2] The Orient has different ideas on this subject. Haggling y go on for hours until the proper adjustment is found be-

tween individual customers is forbidden to public service corporations by the public utility commissions.

But we can and should make our prices so that different percentage profits are obtained from different *classes* of customers and from different *kinds* of goods or *classes* of service. This kind of differentiation or "discrimination" is recognized as perfectly legitimate by the public utilities commissions and is also probably felt as being fair by the majority of the general public. It is, in fact, fairer than the apparent lack of discrimination, as expressed by charging equal percentages of profit from everybody, whether rich or poor (cost-of-service principle).

In practice we apply the *value-of-service principle* between classes of customers and classes of goods, whereas the strict *cost-of-service principle* is applied to the customers and goods within each class.

As an example of the differentiation of profits between classes of *customers* remember the regular "sales" in department stores on certain *days* of the week. On these days extra low prices or extra high discounts ("double stamp days") are allowed which apply chiefly to those classes of customers to whom the difference in price is important enough so that they are willing to take upon themselves the inconvenience of planning their shopping and delaying their purchases until that particular day arrives, and then of shopping in the crowd. The excursion and holiday rates on railways, etc., are a similar example.

As regards the differentiation between classes of the *commodity* or service we have a well-known example in the discrimination of the post-office charges between letters and printed matter ("first class," "second class" and "third class" mail). It costs the Post Office Department practically the same to trans-

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tween the respective customer's valuation of the goods and his valuation *of his* money, and this is considered perfectly correct.

port and deliver a sealed written letter as it costs for printed matter of the same weight from the same sender to the same addressee and yet the charges are 3 to 12 times as high. The chief reason is that senders of a "first-class" letter are able and willing to pay the higher charges, whereas only a small percentage of the newspapers, printed advertising circulars, etc., which are now being sent by mail would be transported that way if the charges were raised to the same amount as the first-class letters, or even to the average between the first, second and third classes. Another example of discrimination between classes of commodities is furnished by the price reduction to every customer (in sales or permanently) of certain *goods* in department stores and other stores (which is not identical with the reduced prices on certain days mentioned above) when it is found out that these goods are not selling rapidly enough.

D. THE VALUE-OF-SERVICE PRINCIPLE IN CENTRAL-STATION SERVICE.

1. General Principles.

96. In determining the cost of central-station service we have found (see Section 10 et seq.) that the cost of production of service consists of three separate items. The lowering or raising, respectively, of the profits from different customers as required by the value-of-service principle should therefore apply theoretically to every one of these three items of cost. We have seen, however, (see Section 57) that the customer cost is of importance only with the smaller customers and is negligible with the large and even with the medium-sized customers. We might therefore determine the customer charge of the rate by adding the profit to be collected from the small customers and apply it to all sizes of customers alike.

It will be shown, however, (Part III) that the rates do not generally follow the three-charge system, inasmuch as the customer charge is frequently left out altogether. Not even the two-charge system (energy charge and demand charge) is universal, though it is applied in the great majority of rates. We come to the conclusion, therefore, that an exact mathematical application of the value-of-service principle is not feasible—even if we could have the exact foundations for the same in the form of exact data of the valuation of the service on the part of the various customers or classes of customers. We will, in general, have to be satisfied with selecting those of the three charges as we choose to embody in the rates, at such amounts that the average profit is larger or smaller according to what the following investigations (Section 97 et seq.) will show to be desirable.

As the principle of differentiating the percentages of profit refers to classes of consumers or service only and not to individual consumers, the prices must be laid down in hard and fast rules in schedules which leave no room for personal interpretation or for preference to individual customers. If, for instance, we find that the electric light company ought to draw a smaller profit from residential consumers than from commercial lighting consumers (see later, Sections 97 and 98) we will have to have one schedule for residential consumers and another one for commercial lighting.

As regards the raising or lowering of the profits for different classes of consumers and classes of service in the central-station business, we will have to be guided by the same two factors which have previously been mentioned (see Section 94) as being determinative for that discrimination in general business, viz.: (a) the customer's valuation of the service he receives, (b) the customer's valuation of the money he has to give in return for the service. These two factors d *not necessarily* work in the same direction, one fact may indicate a lowering of the profit to a certain class, the other one a raise.

The classification of the customers for a rate diversification in general and for application of the value-of-service principle in particular is made under two main headings, the "size" of the consumer and the purpose to which electricity is being put. The size of the customer is usually determined by the number of kilowatt-hours consumed per month or sometimes by his maximum demand, or by both. The schedules distinguish accordingly wholesale and retail customers, or the character of the business as such brings the respective customer under a certain class of size. For instance, residential customers are naturally small customers, street railways large ones.

2. Classification According to the Size of the Consumer.

a. The Small Consumers.

97. The smallest consumers generally value their dollar higher than the medium-sized and large customers and therefore we must be satisfied with a smaller profit from the smallest customers, otherwise they will turn to other illuminants. This does *not* mean that we will have to charge them lower average *prices* per kilowatt-hour, since the *cost* per kilowatt-hour is very much higher for this class of customers, chiefly on account of the "customer cost" (see Section 57).

b. The Medium-Sized Consumers.

98. Calling medium-sized consumers such consumers as have a connected load somewhere between 5 and 10 kw. as a lower limit and perhaps 30 to 50 kw. as the upper limit, such as medium-sized stores, restaurants, small factories, etc., we find that we cannot *claim* any special reason why this class should pay *specially low profits,* whereas such reasons do exist

for the small and the large consumer. (See Sections 97 and 99.) The natural inference is that this class of customers must pay a larger profit than the average.

Let it be emphasized again that charging prices to the medium-sized customer which involve higher profits than the average will be of advantage not only to the electric light company, but also to the community of electric light users, possibly including the medium-sized customers themselves. The higher profits from the medium-sized customers do not necessarily mean that these customers have to pay for the others. If we would not grant prices with lower profits to the other customers, for instance the small ones, they would drop out and the profit derived from them would be lost. The prices to the remaining, that is the medium-sized, customers would have to be raised because the cost would have to be so much higher owing to the reduced size of the plant.

c. The Large Consumers—Central Station vs. Isolated Plant.

99. Proceeding now to the third class, the large consumers (for instance, large hotels, department stores, office buildings, theaters, etc.) we find that they, too, like many medium-sized customers, would have to use electricity as a matter of necessity, but here another element enters in. The large consumer is in a position to build his own generating plant and under certain conditions may be able to generate his electricity cheaper than what the electric light company would charge him if it applied average profits in making his prices.

The selling price of the service from the central station is therefore in competition with the output cost of the service from the isolated power plant. This is not *necessarily* a disadvantage to the central station, but it may be so. The central station has the advantage of the bigger plant, which reduces the operati expenses per kilowatt-hour and the capital invested per kilowatt capacity of the power station. The central station further has the big advantage of a diversity-factor larger than unity, which means, as we have seen in Section 45, it can use the same kilowatt of power-house capacity several times over for different customers, one after the other. Also the reserve capacity of the isolated plant is generally a larger percentage.[8]

The difference in the quality of service should not be forgotten in this connection. In many of the hotels and other places which are still operating an isolated plant we can, for instance, notice a rhythmical flicker of the electric light, which is not only annoying, but also very injurious to the eyes.

These are the chief (but not all) of the advantages of the central station over the isolated plant. On the other hand, the isolated plant has a number of economic advantages over the central station. It is not burdened with a high-voltage transmission and a distribution system since the house wiring generally is connected directly to the power-plant switchboard. The advantage sometimes quoted that the isolated

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[8] To illustrate these points: If the isolated plant has a peak load of 1000 kw. we would have to install perhaps two 500-kw. generators to carry the load, whereas if we supplant the service from the isolated plant by central-station service the central station will have to increase its capacity for that purpose by perhaps only 700 or 500 kw. or less, according to the size of the diversity-factor, so that the capital invested per kilowatt of maximum demand is so much smaller. These 500 or 700 kw. are not furnished from 500-kw. generators but from large ones of 5000 or 10.000 or more kw.; this means the capital per kilowatt capacity is still further reduced in case of the central station. Moreover, if the isolated plant is to furnish the service, we will have to install a third generator of 500 kw. there as a reserve for breakdowns, that is 50% of the total capacity, whereas in the central station, where we have a larger number of generators, the size of the reserve generator is a smaller percentage of the total capacity. If we have, for instance, five large generators of 10.000 kw. each, installed in the central station for regular service, the sixth unit of the same size, installed as reserve, will be but 20% of the capacity. (Of course, we might also subdivide the 1000 kw. of the isolated plant into five units of 200 kw. each with a sixth one in reserve, but this would not only fail to bring about a reduction of the capital cost but would also increase the operating cost.)

plant can in winter be run with the steam from the heating system of the building and with the attendance provided for the heating plant is of questionable nature, since both heating steam and labor are inferior to that desirable for operating a good electric plant.

The question of the relative cost of providing electric power from the central station or from an isolated plant involves too many factors to permit of a general answer which applies to every individual case. But the scrapping of existing isolating plants in favor of central-station service is a frequent occurrence, whereas the opposite is rarely, if ever, heard of in the territory of supply of a well managed central station. This demonstrates that the central station generally has the economic advantage over the isolated plant. This is the consequence of the progress of the central-station business, chiefly of the accomplishments of recent years in filling in the valleys of the central-station load curve, thereby increasing the load-factor and the diversity-factor, and it is also the consequence of the progress made in central-station rates, chiefly brought about by the clear recognition of the value-of-service principle, particularly in its application to the large consumer.

The relative merits of the central station and the isolated plant to the public are clearly brought out to the engineer if he considers the following hypothetical problem: What would we do if we had to supply most economically a large city, where electricity was hitherto unknown, with light and power on a large scale? It would certainly be very, very unusual conditions which would bring the construction of isolated plants even within consideration.

3. Classification According to the Purpose for Which Electricity Is Being Used.

100. Whereas we have thus to discriminate between the different sizes of the customers in the appl cation of the value-of-service principle, we must also, according to Section 96, pay due regard in this connection to the purpose for which electricity is being used.

The principal uses to which electric central-station service can be put may be roughly classified as follows: (a) lighting service, (b) motor service[4] and (c) heating service.[5]

The value-of-service principle requires that the price of electric service be in general not essentially higher than the price of competitive service (competitive sources of illumination, power and heat) because the customer's valuation of the service is largely determined, or rather limited, by the amount of money for which he can get a similar service from other sources. But considering the convenience, cleanliness, relative fire-safety and other advantages of the electric service its price may in many cases be somewhat higher than that of competitive service.

In the following an investigation is attempted of how the valuations of the three main services (light, power and heating) compare with each other.

101. Electricity for power will be valued slightly lower than electricity for light, at least in case of consumers who are not among the smallest. We can easily understand this from the fact that if these consumers would have to make their own light or power it would cost them almost the same amount to furnish a certain load curve for power or for light. The cost for power load will be slightly smaller than for lighting load because the speed regulation need not be as close, both as regards the regulation between full load

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[4] Industrial motors, railways, elevators, refrigerating, pumping and irrigating, ventilating, motor-generators for electroplating or telephones, etc. Also battery charging for automobiles as this energy is ultimately consumed in motors.

[5] Industrial heating, such as ovens, and domestic heating, such as cooking, flatirons, radiators, etc. "Heating" may also *include electric* welding, also electric steel making and other *electrochemical processes.*

and no load (governor) and the fluctuations of speed during each revolution (flywheel). Also in those instances where for power load a purely mechanical drive is preferable over electric power transmission, that is, where we have only one power-consuming device, or a very few of them close to each other and to the prime mover, a further relative saving may be made in the generation of power vs. generation of light, because then the electric equipment becomes unnecessary and the operator need have no training in electricity. Power rates should therefore generally be calculated with the same or a slightly smaller profit than lighting rates.[6]

For very small customers who have so little demand for power that they cannot think of installing gasoline or gas engines, conditions are different. The mechanical power they require can be furnished by hand or foot and the electric motor is frequently more or less of a luxury for them (household motors, dentists' motors, etc.).

Where the motor is used as a money-earning element, for instance in workshops of small trades people, the valuation of electric power will vary between different trades with its earning power. The price of electric power will not be of such importance to the jeweler who uses it for polishing jewels as to the man who uses electric power for pumping water or crushing stones. Moreover, in one trade electric power will be more of a luxury whereas in another one it will be a necessity.

The valuation of electric light, as far as the influence of the competitive illuminants goes, will be determined by the price of gas and kerosene. The advantages of the electric light, such as convenience, cleanliness, reduction of fire hazard, absence of pollution

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[6] Here again the reader is warned against confusing the question of profit-adding with the question of the price per kilowatt-hour. *Power consumers generally are charged much lower prices per kilowatt-hour* because they are long-hour users.

of the air, etc., will play a greater part in the customer's valuation if he is well-to-do than if he is poorer, because a wealthier man will generally be more willing to lay out a little extra money for these advantages than the man who has to be careful how he spends his pennies.

For all these reasons no general rules can be laid down for the relative valuation of light and power for smaller consumers. But these small power users receive their power under the lighting rates anyway so that this question is not of great importance.

102. A decided difference in the valuation on the part of the customer is seen when we compare heating service with lighting (or power). This will be understood from the following. A steam engine or steam turbine, even of the largest and most economical type, utilizes certainly not more than 20% of the heat energy generated in the boiler furnace. The remaining 80% is not converted into mechanical energy and is lost. About one-fourth or one-third of the mechanical energy at the turbine shaft is then lost in the course of converting it into electrical energy and of transforming and transmitting the same, so that even in the most economic cases certainly not more than about 12 or 15% of the energy contained in the coal arrives at the customer's installation. Further conversion from electricity into useful heat then takes place at a very high efficiency, generally near 100%.

Assuming now that a competitive direct-heating device—stove, furnace, etc.—utilizes only one-seventh of the heat contained in the coal and lets the rest escape into the flue, we can see that we use just as much coal for electric heating as for direct heating by fire. If the electric light company under these conditions would charge the heating customer nothing but the bare cost of coal, the consumer would find that the *cost to him of heating* from both sources is about the same.[7] Now electric heating has not only the ordinary advantages of electric service, such as convenience, etc., mentioned above several times, but also others, such as easy temperature regulation, concentration of the heat at the point where it is needed, or uniform heat distribution, as the requirements of the case may be, absence of contamination by combustion gases, possibility of reaching extremely high temperatures, etc. These advantages make electric heating superior to other methods, for instance in electric steel making. Electric heating can therefore in certain cases compete with direct heating by fire even though the price per heat unit utilized is higher. But even then the above reflections show that we have generally to be satisfied with an extra low profit on heating service if we want to get the business.

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[7] In hydroelectric central stations we would likewise have to charge prices with much lower profits than the average for the same reasons, because the total cost of hydroelectric service is of the same order of magnitude as steam-electric service, though the energy cost frequently is lower. Where we have generation from water power in a region where fuel is expensive, such as in certain parts of the West of the United States or in Sweden, the field for electric heating is better than where fuel is cheaper, especially where the heating load is essentially off-peak, so that the increment demand cost is zero.

PART III

SYSTEMS OF CHARGING

1. General Features.

A. REASONS FOR MAKING DIFFERENT RATES FOR DIFFERENT CLASSES OF CONSUMERS.

NOTE: The examples given in Part III for the various types of rates have been taken from the author's collection of rate schedules and from other sources of information. Pains have been taken to get reliable figures and the given figures are believed to be correct, although some of them may have become obsolete and others may be erroneous in other respects. The author assumes no responsibility for their absolute correctness. His object is to illustrate the type of rate to the reader more quickly and vividly by definite figures than is possible by mere abstract descriptions. It is of very little importance whether the figures are absolutely correct and up-to-date and whether they apply in this one or some other city. This book discusses principles and is *not* intended as a book of reference for the actual rate schedules now used in the different cities.

103. It has been demonstrated in Parts I and II that not only the unit amount of the classified expenses (energy cost, demand cost and customer cost) varies between different classes of customers, but that also the percentage of the profit from different classes should vary between these classes. The rates—that means the prices—to different classes of consumers

will therefore have to be different for these two reasons.[1]

Price is the cost plus profit. If the cost is figured according to the three items of energy cost, demand cost and customer cost, it seems logical that the price should be figured in the same way. But in the case of rates we have to deal with the general public and it is the opinion of most central-station managers that this method is too complicated for the public to understand, especially for the small consumer. The less the average consumer understands about the system of charging, the more prone he will be to assume that the whole rate schedule is nothing but a device made expressly to cheat him out of his money. In case of larger customers we may expect that the electric light company has to deal with somebody who has sufficient arithmetical training to understand a proper explanation of the rates, even if they embody the three-charge system.

The rates to different classes of customers will therefore vary not only in the amounts they charge, but also in the form. We will consequently have generally a number of different rate schedules, each one applying for a different class of service.

B. Classification of Consumers for Rate Purposes in Practice.

104. These classes of service are not standardized among the various central stations, but a selection from the following list is generally found in the rate schedules of most central stations:

1. *General Lighting* or sometimes *General Light and Power.*

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[1] This refers to the unit charges (energy charge, demand charge and customer charge). If we reduce the customer's total payment to the kilowatt-hour consumed, as is frequently done, we get of course much wider fluctuations of the price (average price per kilowatt-hour) between different customers (see Section *13*).

2. *Residence Lighting.*
3. *Commercial Lighting,* which means lighting of business localities.
4. *Display Lighting* (signs, windows, decorative lighting).
5. *Street Lighting.*
6. *General Power.*
7. *Wholesale Power* (incidental lighting sometimes included).
8. *Retail Power.*
9. *Ice Making,* also called *Refrigerating Service.*
10. *Heating and Cooking.*
11. *Primary* or *High-Tension Service.* The current is delivered at the central station's primary voltage and transformed by the customer in his own transformers. This is practicable only for very large customers. This schedule may also be expressed by a deduction under the heading of transformer rentals, if the consumer does not use the company's stepdown transformers.[2]
12. *Off-Peak Service.* Under this schedule the customer agrees not to use his current during certain specified hours, generally the evening hours of the winter months.[3] Sometimes the customer is allowed

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[2] Sioux City Gas & Electric Co.

[3] In Allentown, Pa., for instance, current under the off-peak schedule is not available from 4 to 8 p. m. between Nov. 1 and Feb. 29. In St. Louis the peak hours assumed for the off-peak service are the following:

5:30 to 7:00 p. m. in October;
4:30 to 7:00 p. m. in November and December;
4:40 to 7:00 p. m. in January;
5:20 to 7:00 p. m. in February;
6:00 to 7:00 p. m. in March.

In Chicago an off-peak rate is available where the customer agrees during the peak period not to use more than 10% of the highest demand in the preceding year's peak period. The peak period in this case is counted between 4:00 and 8:30 p. m. from November to January and begins half an hour later in February.

Various methods are used to make the customer meet his obligations for the peak-load period. A recording demand meas*uring instrument* is installed as described in Appendix XVI (for *instance in Chicago) or a recording* ammeter (Superior, Wis.).

to use current under this schedule during the night hours only.[4]

13. *Auxiliary, Emergency* and *Breakdown Service.* For isolated plants in case of an increase of their demand over their capacity or in case of a breakdown of the isolated plant, in which case the installation is connected to the central-station service and its own generators through a double-throw switch.

14. *Battery Charging* for electric vehicles. Sometimes subdivided into "Wholesale" for public garages and "Retail" for private garages.

It is impossible to give a complete enumeration of all the varieties of schedules. Sometimes optional schedules appear under the same heading, so that the customer may choose which one of the two or more he considers preferable (see Section 106). The names of the schedules are, of course, not always the same as given above. Sometimes combinations of the above classes of schedules are found.[5]

105. A classification of the customers for rate purposes into these 14 classes, or into some of them, is carried out in every single large and medium-sized central station, and probably in every small one as well. Although these classes are by no means the only ones possible, other classes of rather isolated

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Where the off-peak schedule prohibits the customer from using any current at all during the peak period an automatic time switch in connection with a clock may be used (Spokane and St. Louis). Portland, Me., uses the expedient of stipulating that not more than 10% of the total power used annually shall be consumed between Nov. 1 and March 1 by consumers under the off-peak schedule. The "Ventilating Service" schedule at St. Louis, which is in effect an off-peak schedule, charges less for the service between April 15 and Oct. 15 than for the rest of the year. See also Section 111 about "Differential Rates."

[4] "Night Service" in St. Louis 10 p. m. to 7 a. m. and Spokane 7 to 7; "Night Power" in Hartford, Conn., 10 to 7, and Altoona, Pa., 11 to 6.

[5] Thus, for instance, in Topeka, Kans., under the "Battery-Charging Rate" customers must agree not to use current for that purpose between 4:30 and 8:30 p. m., so that the rate can be classed *also* as an off-peak rate. Another combination is *for instance*, the "Primary Off-Peak Rate" in Detroit.

occurrence are found only in one or a few electric light companies.

C. Optional Rates.

106. Sometimes we find different types of rates, generally two in number, for the same class of customers in such a manner that the customer is allowed an option. One of the two rate schedules may, for instance, be of greater advantage to the customers with an energy consumption below a certain number of kilowatt-hours or with a demand below a certain amount or a combination of both. The other rate is then more favorable for the other customers (see first footnote of Section 108).

D. Minimum Charges and Guaranteed Minima.

107. Sometimes a rate schedule requires certain minimum guarantees from the customer, for instance a guarantee that his consumption during every month or every year will be large enough to bring his bill to a certain minimum amount either per month or per year.[6]

A minimum charge with the object of insuring a revenue of at least the "customer cost" from every customer is made in the vast majority of schedules which apply to residence lighting.[7] The minimum

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[6] This is not quite the same. For instance, a guarantee of $1200 per year is not entirely equivalent to one of $100 per month. The first guarantee would be fulfilled by a customer who is paying $150 during each one of the six winter months and $50 during each one of the six summer months, whereas under the second named plan that customer would be required to pay an additional $50 for every one of the summer months. The first plan requires a little more work on the part of the central station, since the bills of customers near the limit must be added up at the end of the year, whereas the other plan takes care of itself month by month. These considerations have led in a few cases to a stipulation of a monthly *and* a yearly minimum. The wholesale power schedule of Allentown, Pa., for instance, requires a minimum charge of 50 cents per horsepower connected and the twelve bills in a year must moreover amount to at least $15 per horsepower.

'his refers to central stations in cities of medium and *ize. No information* for statistical purposes is obtain-

charge in that case ranges from 25 cents per month[8] to $1 net.[9]

The opinion of rate experts on the advisability of a minimum charge for residences seems to be strongly in favor of making such a charge, but it is not unanimous. A few of the large central stations in this country (among them New York, Chicago and Cleveland) prefer the gain in simplicity of the rate and the relatively slight risk that some of the customers now and then may pay less than a few dimes to the alternative risk of keeping prospective profitable customers away who are afraid to tie themselves down to a minimum.

These and similar questions in rate making are a matter of local conditions as well as of personal judgment and taste for which no hard and fast rules can be laid down. It is not easy to decide these questions and we have no means of saying, even afterwards, whether the decision was the best one or not. Rate making altogether is a matter of feeling and intuition as well as of exact research. Psychology, business instinct and experience have their place in the design of rates as well as engineering and mathematics. Many of the rate questions have to be decided temperamen-

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able from the innumerable central stations in smaller towns, but we have no reason to assume that their rates should be essentially different from those in the larger cities.

[8] Duluth and New Orleans.

[9] The favorite amounts for the minimum charge in that case are 50 cents and $1; other values are of rare occurrence. In about a dozen cases we find yearly minimum charges in schedules which apply to residences. The most important of these cases are Baltimore, Boston, Brooklyn and Buffalo. In Pueblo, Colo., the residence lighting minimum charge is 3⅛ cents *per day*. St. Joseph, Mo., makes a higher minimum charge for residences in rural districts (75 instead of 50 cents), thus expressing the distance factor by the minimum charge. (A few central stations have entirely different rates for outlying districts.) Salem, Mass., increases the minimum charge from 50 cents to 75 cents for summer residences. The minimum charge of the Display Lighting schedule in Sacramento, Cal., varies with the season and is only about 57% in June of what it is in December. The General Lighting schedule of York, Pa., charges $1 minimum per month in winter and 50 cents in summer.

tally or artistically, if you please, rather than strictly scientifically.

108. An entirely different class from the minimum charges in residence schedules and other small-customer schedules is the minimum charge in schedules destined for large customers. The object here is to keep the small customers away from that schedule by requiring a rather large minimum payment per month or per year. In that case we have optional schedules, one with a high minimum charge and low unit rates per kilowatt-hour (or per kilowatt of demand or both) and the other schedule with a low minimum charge (or none at all) and a comparatively high unit rate per kilowatt-hour (or per kilowatt or both).[10]

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[10] For instance, in Toledo, Ohio, public garages can get battery-charging current at 5 cents per kw-hr. (net) with a minimum charge of $3 per month (this rate to be called Rate A hereinafter for short), but if they guarantee a minimum bill of $50 per month they get their current at 2 cents per kw-hr. (Rate B).

A graphical and analytical investigation of what this means may prove instructive. We step off as abscissae (horizontally) the energy consumption in kilowatt-hours (Fig. 4)

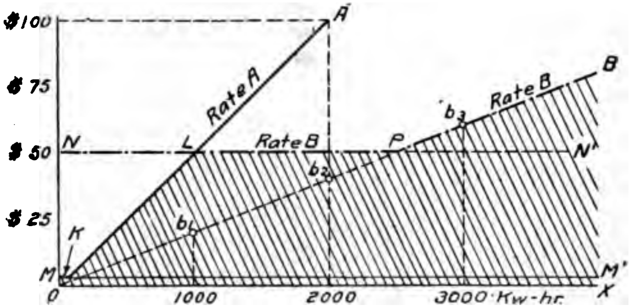

Fig. 4.

and as ordinates (vertically) the amount of the bill in dollars per month. Beginning with Rate B we find that at 2 cents per kw-hr. the customer is to pay $20 for 1000 kw-hr. (point b_1), $40 for 2000 (point b_2), $60 for 3000, etc. The curve representing the bill as a function of the energy consumption will

Whereas the first kind of minimum charge is generally not more than $1 per month, the minimum charge or guarantee of the second type, that is, with the object of reserving a certain schedule for the large consumers, is naturally rather high and we find figures as high as $3000 per year (Primary Service,

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obviously be a straight line *OB* starting from the origin *O*. There is, however, a minimum of $50 for the bill. This minimum is denoted by the horizontal line *NN'*. The portion of the inclined line *OB* which is situated below horizontal *NN'* does not apply and *NN'* applies instead. Rate B is thus represented by the heavy dash-and-dot duct from *N* over *P* to *B*. On the other hand, we have the optional rate A of 5 cents per kw-hr. with a $3 minimum. We get the straight line *OA* and here again a minimum charge applies. This is now $3 and is represented by the horizontal line *MM'*. Rate A is represented by the heavy outlined duct from *M* over *K* to *A*. As the customer has the desire to get the lowest bill possible he will pick out that one of the two rates which shows the lower bill for his particular energy consumption; in other words, the rate represented by the duct which is lower for that respective energy consumption. The bill of the customers who choose their schedule wisely is then given by the ordinates of the shaded area. Small customers will be better off on Rate A, large ones under Rate B.

Where is the limit between those two sizes of customers, in other words, up to what energy consumption is it more advantageous for a customers to use Rate A? This limit is evidently given by the point *L* where the line duct ("curve") of Rate A intersects that of Rate B. The intersection means that the bill is the same for the respective energy consumption, whether the customer is being charged under Rate A or Rate B. This intersection is given by point *L* in Fig. 4 and the energy consumption belonging to that point will give the limiting value. We can either find that value in kilowatt-hours by measuring it in the drawing with the kilowatt-hour scale or we can figure it in the following way:

The ordinate of *L* is $50 because *L* lies on the minimum-charge line *NN'*. On the other hand, the unknown amount of energy multiplied by 5 cents per kw-hr. must give $50. It is easy to figure out that 1000 kw-hr. are necessary to make up a charge of $50 at the rate of 5 cents per kw-hr. All customers with less than 1000 kw-hr. energy consumption should choose Rate A, the others Rate B.

The graph Fig. 4 also shows that all customers using Rate B between the points *L* and *P* are charged the identical amount of $50. Point *P* can easily be found as corresponding to

$$\frac{\$50}{2 \text{ cents per kw-hr.}} = \frac{5000 \text{ cents}}{2 \text{ cents per kw-hr.}} = 2500 \text{ kw-hr.}$$

Therefore, all customers using between 1000 and 2500 kw-hr. per month and choosing the schedule which is of best advantage for them will have to pay uniformly $50 no matter how large their energy consumption was within these limits.

We find further that the minimum charge of $3 (Rate A) affects the 5-cent-per-kw-hr. rate for such customers only as have a kilowatt-hour consumption of less than 300/5 = 60 kw-hr. per month (point *K*).

Indianapolis) or even $340 per month (Primary Large Lighting and Power, Chicago).

So far in this discussion the minimum charge has always been understood as a guarantee on the part of the customer to pay a certain amount in dollars and cents every month or year. This is the most frequent case. Not infrequently, however, we find a stipulation in a rate schedule requiring the customer to pay for a certain minimum number of kilowatt-hours or for a certain minimum of demand in kilowatts, watts, horsepower, or whatever the unit for the demand may be. This is not quite the same as guaranteeing a minimum bill (except in case of the simplest rates), as will be shown later (Section 168), but the object is the same.

E. Maximum Unit Charges.

109. In some cases we find a clause in the rate schedules to the effect that where the average charge per kilowatt-hour figures out higher than a certain specified maximum the bill shall not be higher than what corresponds to that maximum average amount in cents per kilowatt-hour, regardless of what the schedule says elsewhere. This is in a certain measure the reverse of the minimum charge discussed in the preceding sections. Here, as well as with the minimum charges (see first footnote to Section 107) this restriction may apply either to the individual monthly bill or to the sum of the 12 monthly bills rendered during the year. For examples and further discussion of this principle of the maximum unit charge (which is not frequently applied) see Section 124, first and third footnotes.

F. Prompt-Payment Discounts and Delayed-Payment Penalties.

110. A prompt-payment discount, to induce the customers towards early settlement of their bills can be found in about three of every four rate schedules

of the more important central-station companies in this country. The time within which this discount is allowed is 10 days in the great majority of cases, and the percentage taken off is usually either 10% or 5%. Sometimes the discount is expressed in terms of cents per kilowatt-hour, generally 1 cent or 0.5 cent per kw-hr. In a few cases the discount is limited, for instance the discount is given only on the first 200 kw-hr.[11] In some cases the gross unit charge is an odd figure which becomes a round figure after the deduction of the discount.[12]

Of course, most of the customers avail themselves of the prompt-payment discount and this makes the rate look higher on paper than what is actually being paid. If, for instance, we have a 10-cent-per-kw-hr. rate with a prompt-payment discount of 1 cent per kw-hr., the rate which almost everybody is actually paying will be 9 cents per kw-hr. and yet almost everybody will say: "We have a 10-cent rate." The company will get the blame for a higher rate than it actually charges. For this reason a few companies have reversed the statement by introducing a delayed-payment penalty. This would mean in the example quoted above: 9 cents per kw-hr. with a delayed payment penalty of 1 cent per kw-hr. It amounts to the same, only it sounds better and the impression given is more correct.[13] In one case[14] an aditional stipulation is made that the minimum amount of the de-

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[11] Lighting and Power schedule, Indianapolis, Ind.

[12] For instance, the cooking rate in Toledo, Ohio, is a straight-line meter rate of 5.56 cents per kw-hr. with 10% discount if the bill is paid within 10 days, thus making the net rate practically 5 cents per kw-hr.

[13] Delayed-payment penalties can be found in the rates of Philadelphia, Cincinnati, Washington, D. C., and a few other cities. In New Orleans, where two electric light companies are operating, one of the two charges a 7-cent rate with a 10-day delayed penalty of 1 cent for general lighting, whereas the other one charges an 8-cent rate with a 10-day prompt-payment discount of 1 cent.

[14] Topeka, Kans.

layed-payment penalty is 25 cents. The idea of this is evidently that as soon as a customer gets on the delinquent list he causes the company a certain amount of clerical work and other expenses, no matter how large or how small the amount of his bill is. We see here again the principle of the minimum charge.

II. The Various Types of Rates.

A. INTRODUCTION.

111. The various types of rates base the amount of the bill of a certain consumer on his energy consumption or on his maximum demand or on both.

There are a very few exceptions to this rule, but under certain unconstrained assumptions we can bring even these under the above rule.

In the first place we find some isolated cases of rates which make the amount payable dependent also on the amount which the customer has been willing to guarantee in advance as a minimum payment per month.[1]

Evidently there will exist a certain guarantee for every customer which makes his bill smaller than any other guarantee would. If he guarantees less than that amount he will have to pay an unnecessarily high unit price and if he guarantees more his guarantee

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[1] An example is the Retail Lighting and Power schedule of Pittsburgh, which charges the following average prices per kw-hr.:

Cents per kw-hr.	If maximum demand is	And guarantee is
10	1	$ 1.00
9	1	3.00
8	1	5.00
6.5	1	10.00
5.5	1	15.00
5	1	20.00
4.25	1	25.00
3.75	2	36.00
3.25	3	52.00
2.75	5	79.00
2.25	8	107.00

Add $1.00 to guarantee for each additional kw. of demand.

Another example is the Toledo Battery-Charging rate, *which is quoted and* discussed in footnote 10 of Section 108.

is higher than what he actually consumes. It is clear that, provided the consumer chooses his guarantee wisely, he will find his bill dependent only on the energy consumption and the demand, in other words, under that assumption there will be only one amount of the bill possible for every conceivable combination of kilowatt-hours consumption and kilowatts demand.

Another exception to the above rule, that the price charged to the customer under a certain schedule depends only on the amount of the kilowatts used and kilowatt-hours consumed by him, is the so-called time-differential rate which, however, though extensively in use in European countries, is hardly used in this country. Almost the only instance the author could find of this rate in this country is the "Time-Differential Service" rate in Detroit for auxiliary and emergency service for private plants. Ten cents per kw-hr. are charged for all energy consumed between 1:00 p. m. and 6:30 p. m. and 4 cents per kw-hr. at all other hours. This really is nothing but a combination with an off-peak schedule (cf. Section 104, footnote). If we consider the high-rate hours and the low-rate hours separately, we have again reduced the amount payable to the original elements of kilowatts and kilowatt-hours in each one of the two daily periods.

If, therefore, in this manner all rates can be assumed to be based upon the two elements of kilowatt-hour consumption and kilowatt demand only, we can distinguish the following three classes of rates:

(1) Rates based on the energy consumption only.

(2) Rates based on the maximum demand only.

(3) Rates based on both energy consumption and maximum demand.[2]

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[2] We might add to this for the sake of completeness a fourth class, that is, rates which are independent of either energy consumption and maximum demand. This class has, however, only academic and slight historic interest. Every customer of the class, for instance every residence customer, would have to pay under that rate the same amount, no matter how large his installation and the size of the lamps in hi

B. RATES BASED ON THE ENERGY CONSUMPTION ONLY.

1. *The Straight Meter Rate.*

112. The straight meter rate is one of the simplest and one of the oldest systems of charging, but by no means one of the most satisfactory ones, at least not for general use. The customer's bill is proportional to the number of kilowatt-hours used, as measured by a watt-hour meter.[3] Frequently the straight meter rate is combined with a minimum charge[4] or sometimes with a customer charge.[5] The advantages of the straight meter rate are its simplicity and the readiness with which it is understood by the public.

113. A variety of the straight meter rate is the *prepayment-meter rate* for very small customers. A prepayment meter is a watt-hour meter constructed in combination with a vending machine ("penny-in-the-slot" machine). The customer inserts a coin of certain specified value, for instance a quarter, into the slot of the prepayment meter and then is furnished with current until he has consumed the amount of energy for which he has paid by that sum. After that time his circuit is automatically opened by the

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sockets and no matter how long he is burning them. The rate is reduced to the customer charge, and all other costs, but the customer cost, are averaged into the customer charge, the assumption being that the difference in cost between residence customers is not very large and that we can simply charge the average cost for every customer plus a percentage of profit. Such rates have been in use in a very few cases during the earliest days of central-station history. If the author's information is correct, one instance was in Monticello, N. Y. This type of rate has been abandoned long ago. (See later Part V—"Accuracy of Rates").

[3] An example of a straight meter rate is the Residence Lighting rate of the Bronx Gas & Electric Co., of New York, where the customer is simply charged 12 cents per kilowatt-hour.

[4] For instance, in the General Lighting and Power schedule of Boston, which charges 8.5 cents for every kilowatt-hour consumed, but with a minimum charge of $9 per year, payable monthly.

[5] Battery-Charging schedule of Springfield, Ohio, which makes a customer charge of $1.50 and an energy charge of 8 cents per kw-hr.

mechanism of the prepayment meter and his lights go out, usually after a warning of some kind has been given to him.

Opinions on the merits of the prepayment meter, as on so many other rate questions, are divided. The idea of paying in small amounts in preference to paying comparatively larger sums at the end of the month certainly appeals to some classes of the poorer population and is based on sound psychology. Meter readings are unnecessary and bad debts are avoided. On the other hand, the meter is more expensive and complicated than an ordinary watt-hour meter and gives more occasion for complaints. The extinguishing of the lamps at inopportune times is a great annoyance, especially if there is no coin of the required denomination at hand.*

2. *Application of Lower Average Kilowatt-Hour Charges to Larger Energy Consumers.*

a. Reasons for the Desirability of a Graduation of the Kilowatt-Hour Charges.

114. The defect of the straight meter rate—apart from the defects which it shares with all other rates based on the energy consumption only—is that it makes the large consumer pay the same unit price as the small one. The adequate prices per kilowatt-hour under a pure meter rate are far from being the same for all consumers. They become steadily lower as the energy consumption of the customer becomes larger and larger. There are a number of reasons for this which will be explained in the following.[1]

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*About another rate especially adopted for the smallest customers, the limited flat demand rate, see Sections 128-130.

[1] Graphical methods convey the best and most accurate conception of these several reasons and their effects. Drawings have been termed "the engineer's language" and it has been the author's experience that many commercial men get shy when it comes to representation of facts by curves. According to the leading principle of this book to make their contents accessible as far as this can be done, to those who do not take kindly to mathe*matics and* graphical representation, an attempt will b

115. Wherever business transactions are made, it is an established principle that the wholesale buyer gets lower prices than the small consumer. Very few exceptions can be found from this sound business principle. As far as the electricity supply business is concerned, it is clear (from what has been said in Part I about the cost of electric service) that large consumers cause smaller unit cost and inasmuch as the cost is at least one of the factors that make up the price, we are justified in expecting to find lower unit prices for larger consumers in the electricity supply business.

The "customer cost" which is the same for all customers, as we have seen, makes the average cost per kilowatt-hour higher if a small consumer is being served than if the consumer is a large one. These differences in cost can be very considerable. Let us assume, for instance, that the customer cost to the central station be 60 cents per customer per month and that all the rest of the total cost to the central station (demand cost and energy cost together) be 3 cents per kilowatt-hour.[2] These values are in no way extremes and they can be said to be near average conditions. (The figures of the different central stations vary within wide limits.)

Now with these assumptions we can figure out the

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made in the following to explain purely by words the question why and how the unit price varies with the amount of the energy consumed, but the same question is treated graphically and much more thoroughly in Appendix X. Readers who entertain no objections against curves are strongly advised to substitute Appendix X for Sections 115-117. Later in the articles graphical and elementary mathematical methods will be found indispensable.

[2] From what has been explained before (Part I) it is not quite logical to divide the demand cost by the number of kilowatt-hours. The result of the operation is an average value which applies only under the given load-factor conditions. But as long as we deal with rates which do not take any cognizance of the maximum demand of the customer, we must of necessity be satisfied with this rough average, inaccurate as it may be. This is a disadvantage intrinsic to all rates which are based on *the energy consumption* only.

following table for the total average cost of customers with various energy consumptions:

Monthly Energy Consumption in kw-hr.	Average Cost to the Central Station in cents per kw-hr.
5	$\frac{60 + (3 \times 5)}{5} = 15$ cents per kw-hr.
10	$\frac{60 + (3 \times 10)}{10} = 9$ cents per kw-hr.
20	$\frac{60 + (3 \times 20)}{20} = 6$ cents per kw-hr.
100	$\frac{60 + (3 \times 100)}{100} = 3.6$ cents per kw-hr
1000	$\frac{60 + (3 \times 1000)}{1000} = 3.06$ cents per kw-hr
10000	$\frac{60 + (3 \times 10000)}{10000} = 3.006$ cents per kw-hr.

We see from this that the average cost per kilowatt-hour falls, first rapidly, with increasing energy consumption, and then more slowly, until for very large energy consumptions it becomes practically constant and converges towards the limit of 3 cents per kilowatt-hour.

Another reason why the average cost of the kilowatt-hour becomes lower as the energy consumption of the consumer increases is that the large consumer generally has a better load-factor, or a better diversity-factor, or both. A consumer, for instance, who is using only 10 to 20 kw-hr. per month is generally a residential customer or a small store owner, and this type of user is, as a rule, a short-hour user. If the kilowatt-hour consumption is a little larger, that is if it approaches the first 50 or 100 kw-hr. per month, we will in some cases still have a residential customer or a store owner, but a larger one. People living in larger residences are more liable to use current-consuming appliances other than lamps, such as flatirons toasters, heating and cooking apparatus, and the like. Now these appliances generally have a much greater capacity than the ordinary lamp; a flatiron, for instance, is using 300 to 500 watts, or roughly as much as ten lamps. These appliances are largely used during the daylight hours and consequently they bring about an improvement of the customer's load-factor and with that of the central station's load-factor. Medium-sized stores are liable to use all-day lights in the back part of the establishment, also ceiling fans, desk fans, etc., which has a tendency to improve the customer's load-factor. If we now proceed to customers using several hundreds or thousands of kilowatt-hours per month, this increase in energy consumption is due generally not only to an increase in the size of the installation but also to the increase in the number of hours of daily use. If we go into a hotel, or a restaurant, or a large store, a theater, an office building, etc., we will always find a number of lamps burning in daytime, not a few of them may be even burning 24 hours a day; for instance, in the lobby and other portions of a hotel. Such large establishments will frequently also have some motors connected to the lines, for elevators, ventilators, refrigerators, and the like. All this means that large lighting consumers will have a better effect on the central-station load-factor than small ones. The same applies to power consumers, only for different reasons. A little workshop with not more than one or two machines consuming electric power will at some times during the day run at full load and at others at no load. The variation between the maximum and the minimum load will be 100%. If we have 100 motor-driven machines there will be a certain diversity between their power demands, just as we have seen a diversity between the power demands of different consumers. Every single one of the hundred machines *y vary between full* load and no load, but the more machines we have, the larger is the probability that at the time when one machine is running at full load another will have very little or no load so that the peaks and valleys of the various machines' loads compensate each other and the load curve of the total establishment will be flattened out and become the smoother the larger the number of the power-consuming machines is; that is, in general the larger the energy consumption of the consumer is.

116. The above refers to the cost. If we were to add the same percentage for profit in every case (cost-of-service principle, see Part II) obviously the same relations would subsist between the prices charged to various customers as have been found between the cost of these customers. The value-of-service principle requires, however (see Part II), different percentages of profit for different customers and we shall now investigate in what manner the relations between prices for various sizes of customers will differ from the relations between costs of the same customers.

It has been shown in the discussion of the value-of-service principle (Sections 97-99) that the percentage of profit from the largest and from the smallest customers should be smaller than that from the medium-sized ones. This furnishes another reason why the consumers with a large energy consumption should receive lower unit prices per kilowatt-hour. But at the same time the value-of-service principle seems to point towards giving lower prices to the small consumers also and this in opposition to what we have found so far about what the cost of the small customer is. The tendency of the value-of-service principle to lower the price per kilowatt-hour to the smallest consumers doubtless exists, but in case of the straight meter rate it is more than offset by the influence *of the fixed* customer cost which has just bee *explained.* We see, for instance, in the table

Section 115 that the change of the average cost per kilowatt-hour which is due to the influence of the customer cost is much more marked in the region of the smallest consumers than in that of the medium and large consumers. It vanishes the more the larger the consumers become. If we assume, for the sake of an example, the typical small consumer to be one of 20 kw-hr. and the typical medium-sized consumer to be one of 200 kw-hr. monthly consumption, the average cost per kilowatt-hour of the small customer would be 6 cents per kw-hr. and that of the medium one 3.3 cents per kw-hr. The cost to the central station of the typical small consumer is then nearly twice as high, if referred to the kilowatt-hour, as that of the medium-sized one and even if we add a very much lower percentage of profit for the small consumer in obedience to the value-of-service principle, the price charged to him per kilowatt-hour will still be higher than for the medium-sized consumer.

117. Summarizing, we can say: Where the customers are classified by their kilowatt-hour consumption only, the unit kilowatt-hour prices should decrease with the size of the consumer for the following three reasons:

1. The fixed customer cost is distributed over a larger number of kilowatt-hours.
2. The load-factor is liable to be better for large energy consumers than for small ones.
3. The percentage of profit is to be reduced for both the largest and the smallest consumers according to the value-of-service principle. This works towards further lowering of the prices of very large consumers. As regards the smallest consumers this effect is more than offset by item 1.

Several attempts have been made to follow this principle in the meter rates, that is to correct the straight meter rate in such a manner that the larger energy consumer gets lower average prices per kilowatt-hour. These methods consist either of a restriction of a given straight meter rate to certain classes or sizes of customers or of a modification of the meter rate which results in the so-called *sliding-scale rate* or of the addition of an explicit customer charge. The term "sliding-scale rate" means meter rates which specify decreasing charges per kilowatt-hour as the energy consumption of the customer increases.

These methods will be discussed hereinafter.

b. The Methods for Applying Lower Average Kilowatt-hour Prices for Larger Energy Consumers.

1. Limited Application of the Straight Meter Rate.

118. Those central stations as are using a straight meter rate at all restrict it to certain classes of consumers, at least certainly all the large and medium-sized central stations. Some central stations have restricted the straight meter rate to small customers only, chiefly residential customers or "general lighting" customers. Others have several straight meter rate schedules, those with higher charges applying to such classes as are liable to consist of small energy users only, such as residence lighting, whereas the lower charges apply to such uses where the amount of energy consumed is greater or the load-factor is better, or both.[8]

2. The Step Meter Rate.

119. Another one of these methods is given by the step meter rate. The range of kilowatt-hours, beginning from zero, is divided into a certain number of "steps" and the unit price charged per kilowatt-hour depends upon the "step" which the customer's

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[8] For example, Reading, Pa., charges 10 cents per kw-hr. for residence lighting and 3 cents per kw-hr. for cooking and heating. This is also based on the "value of service" principle (see Part II.) The Toledo rate discussed before (Section 99) that is, a straight meter rate varying with the guaranteed minimum in such a manner that the higher the guarantee the lower the kilowatt-hour *charge*, belongs also in this class.

energy consumption has reached in that respective month. The higher the step which has been reached, the lower is the charge made per kilowatt-hour. In contradistinction to the "block rate" to be discussed hereafter, all kilowatt-hours consumed by a certain customer in a certain month are charged at the same price.[4]

A step meter rate can also be expressed in the form of a straight meter rate with "quantity discounts," that is, discounts which increase as the quantity consumed increases.[5]. The quantity discounts need not be given according to the number of kilowatt-hours. They can also depend on the amount of the bill.[6] In case of pure meter rates this is only

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[4] The example of the General Lighting schedule at Allentown, Pa., will illustrate this rate. Under this schedule the charge per kilowatt-hour is as follows:

10 cents, if the customer's monthly energy consumption is 100 kw-hr. or less.
9 cents, if the customer's monthly energy consumption is between 101 and 200 kw-hr.
8 cents, if the customer's monthly energy consumption is between 201 and 400 kw-hr.
7 cents, if the customer's monthly energy consumption is between 401 and 800 kw-hr.
6 cents, if the customer's monthly energy consumption is 801 kw-hr. and over.

[5] For instance, the Retail Power rate in Birmingham, Ala., is a straight-line meter rate of 7 cents per kw-hr. with the following quantity discounts applying on the total bill:

10% if the energy consumption is 450 kw-hr. or less.
15% if the energy consumption is more than 450 kw-hr.

This means that the rate is 6.3 cents per kw-hr. if 450 kw-hr. or less are consumed and 5.95 cents per kw-hr. if more than 450 kw-hr. are consumed.

In the General Lighting rate of Mobile, Ala., which is a step meter rate, the prompt-payment discount is stepped off according to the energy consumption. The gross rates are:

10 cents per kw-hr. up to 50 kw-hr.
9 cents per kw-hr. from 51 to 150 kw-hr.
8 cents per kw-hr. from 151 to 300 kw-hr.
7 cents per kw-hr. from 301 to 500 kw-hr.
6.5 cents per kw-hr. over 500 kw-hr.

The prompt-payment discount is 3 cents per kw-hr. if the consumption is 1 to 50 kw-hr. and 2 cents per kw-hr. if the consumption is over 50 kw-hr.

The net amount per kilowatt-hour for the first step is therefore $10 - 3 = 7$ cents, and for the second step $9 - 2 = 7$ cents. The prompt-payment discount is therefore in this case graded in such a way as to entirely wipe out the difference between the first and the second step.

[6] General Lighting rate of Jacksonville, Fla. The following

another form of stating the same thing, but we will later see some of the rates where it does make a difference whether we apply the quantity discounts according to the number of kilowatt-hours or to the amount of the bill (see Section 168).

120. The plain step meter rate has a serious drawback and that is the possibility of a reduction of the bill by an increase of the consumption.[7] The man who is careful about his bill and understands the rate may under certain circumstances reduce his bill by wisely wasting energy shortly before the meter reader comes around. He can thus not only make the company furnish gratuitously the wasted energy, but he can reduce the company's revenue from him besides.[8]

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discounts are given on a straight meter rate of 7 cents per kw-hr.:

10% if the monthly bill is at least $50, 15% if the monthly bill is at least $100, 20% if the monthly bill is at least $150; 25% if the monthly bill is at least $200; 40% if the monthly bill is at least $300.

This is, of course, the same as if we were to say: The rate is a step rate charging (net)
7 cents per kw-hr. for 1 to 5000/7 or 1 to 714 kw-hr.
6.3 cents per kw-hr. for 715 to 10,000/7 or 715 to 1428 kw-hr.
5.95 cents per kw-hr. for 1429 to 15,000/7 or 1429 to 2142 kw-hr.
etc.

[7] A customer, for instance with a consumption of 200 kw-hr., in the above Allentown schedule, will have to pay 200 × 9 = 1800 cents or $18. If now he increases his energy consumption by one kilowatt-hour, that is to 201 kw-hr., he will have to pay 201 × 8 = 1608 cents or $16.08.

[8] This is clearly shown in a graphic representation of the step rate. We again step off the energy consumption horizontally

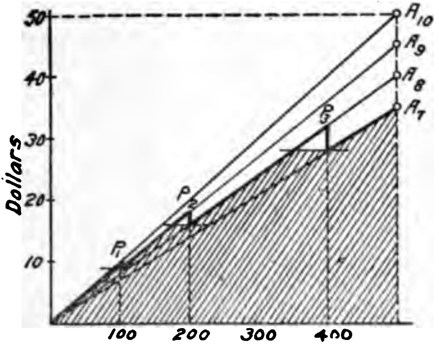

Fig. 5.—Step Meter Rate.

This sudden reduction in the income of the company from the consumer is smaller and less important if the difference of the prices per kilowatt-hour between neighboring steps is smaller. If, therefore, we wish to design a schedule with a given unit price in the first step (maximum price) and another given unit price in the last step (minimum price) it follows that the difference between these two prices should be split up and divided between a large number of intermediate steps if the effect of the above-mentioned undesirable feature shall be kept in narrow limits. We find for this reason that step rates in practice frequently have a large number of steps.[9]

Another method for getting around the drawback mentioned is to make a statement in the step-rate schedule to the effect that the total charge is never to be greater than what would be due (at the next lower rate, that is at the next higher step) for a greater consumption.[10]

This may also change the form in which the step rate is expressed, inasmuch as the upper end of a step must no longer coincide with the beginning of the

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in kw-hr. as abscissae and plot the amount of the corresponding monthly bills vertically as ordinates (Fig. 5). The line representing the 10-cent-per-kw-hr. rate (for the above Allentown schedule, footnote 4, Section 119) is found by joining the origin O with a point given, for instance, by the abscissa 500 kw-hr. and the corresponding ordinate $500 \times 10 = 5000$ cents or $50 (point A_{10}). This inclined straight line represents a straight 10-cent-per-kw-hr. rate and applies as far as the 10-cent rate applies, that is from 0 to 100 kw-hrs. From there on (point P_1) the 9-cent rate applies, which is represented by another straight line drawn from the origin under a correspondingly smaller angle to point A_9 (500 kw-hr., $45). The figure shows the resulting peak at P_1 which indicates how the bill suddenly drops as soon as we exceed the 100-kw-hr. mark; 100 kw-hr. are billed at $10 and 101 kw-hr. at $9.09. The same condition is repeated with the transcending of every one of the other steps (points P_2, P_3, etc., heavy outline).

[9] The Optional Power rate schedule of Newark, N. J., has as many as 42 steps. The Retail Power rate of Wilmington, Del., has 24 steps, and many schedules have a dozen steps or so.

[10] The Retail Power schedule of Albany, N. Y., is an example of a step meter rate with this stipulation.

The effect of this stipulation represented graphically is the g off of the sawtooth-like peaks of the charge curve by ntal lines. (Compare the shaded area in Fig. 5.)

next higher step (or remain within 1 kw-hr. below it). The whole range of energy consumption within which the charge is constant is then exempted from the steps, or counted as separate steps.[11]

121. A step meter rate with this stipulation can to a certain degree also be expressed as a system of optional straight meter rates, each one involving a minimum guarantee of its own in such a way that the rates with the higher kw-hr. charges require lower guarantees and vice versa. (Compare the example in footnote 10 of Section 108.) This represents a step meter rate of the described variety, *provided* the customer chooses that one of the optional rates which results in the lowest total payment for his energy consumption. The customer may therefore have to pay more than under the equivalent step meter rate, but he can never be charged less.[12]

If a step rate contains the stipulation that the charges are never to be higher for a smaller energy consumption than for a larger one—no matter in which one of the above forms the stipulation is made—the customer can no longer decrease his bill by

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[11] For instance, the General Lighting schedule of Lynn, Mass., charges:

10 cents per kw-hr. (net) for 1 to 228 kw-hr.
8 cents per kw-hr. (net) for 285 to 1800 kw-hr.
6 cents per kw-hr. (net) for 2400 and over.

No bill is made out larger than what would be obtained for a greater consumption at a lower rate.

It will be seen that 228 kw-hr. at 10 cents per kw-hr. cost the same as 285 kw-hr. at 8 cents per kw-hr., etc.

The step meter part of the General Lighting and Power schedule of Washington, D. C., is worded after the following fashion:

6 cents per kw-hr. for the first 3200 kw-hr.
$192 for 3200 to 3500 kw-hr.
5.5 cents per kw-hr. from 3500 to 4545 kw-hr.
$250 for 4545 to 5000 kw-hr.
5 cents per kw-hr. from 5000 to 7500 kw-hr.
$375 for 7500 to 8333 kw-hr.
etc. . . . .

This is obviously only another way of expressing the above stipulation.

[12] The Off-Peak Emergency Lighting and Power schedule of Des Moines, Iowa, for instance, provides the following optional rates:

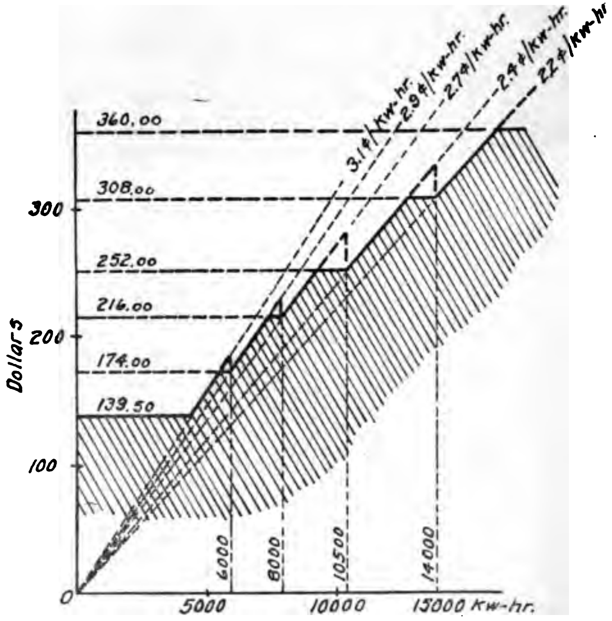
3.1 cents per kw-hr. with guarantee 4,500 kw-hr. (that is $139.50) judiciously increasing his energy consumption. But he can increase his consumption in a considerable range without being charged for the increase. The drawback of the rate is mitigated, but not removed.

2.9 cents per kw-hr. with guarantee 6,000 kw-hr. (that is $174.00)
2.7 cents per kw-hr. with guarantee 8,000 kw-hr. (that is $216.00)
2.4 cents per kw-hr. with guarantee 10,500 kw-hr. (that is $252.00)
2.2 cents per kw-hr. with guarantee 14,000 kw-hr. (that is $308.00)
2.0 cents per kw-hr. with guarantee 18,000 kw-hr. (that is $360.00)
etc.

This is evidently the same as the following step meter rate with a minimum charge of $139.50 (as above):

3.1 cents per kw-hr. for less than 6000 kw-hr.
2.9 cents per kw-hr. for 6000 to 8000 kw-hr.
2.7 cents per kw-hr. for 8000 to 10,500 kw-hr.
etc.

with the provision that no charge shall be higher for a lower number of kilowatt-hours than for a higher one. (See Fig 6.)



6.—*Identity of Step* Rate and Variable Meter Rate in Combination *with* Variable Minimum Charge.

For some remarks on the theory of the step rate see also Appendix XI.

3. The Block Meter Rate.

122. The block meter rate is the third one of the methods we can use to make the average kilowatt-hour charges lower for the larger energy consumer. The range of kilowatt-hours, beginning from zero, is here also divided into a number of ranges corresponding to the steps of the step meter rates. These ranges are in this case called "blocks." A certain unit charge per kilowatt-hour is made for all energy used within the first block, and that part of the energy which reaches into the second block is charged at a certain lower rate per kilowatt-hour. After the number of kilowatt-hours which are contained in the second block and are charged at that lower rate, the excess of energy over the second block is charged at a still lower rate until the third block is filled, etc. The difference from the step rate is that the charge connected with a certain block applies only to the kilowatt-hours within that block and not to those of the preceding blocks, whereas with the step rate the total bill is always calculated at one single rate.

Let us assume, for instance, that the first 100 kw-hr. be charged at 10 cents per kw-hr. and all excess energy over 100 kw-hr. at 5 cents per kw-hr. We have then in the range of 0 to 100 kw-hr. a straight meter rate of 10 cents per kw-hr.; for the consumers of 100 kw-hr. and more the charges (total bill) will be as given in the second column of the following table:

1.	2	3
Kw-hr. consumed.	Charges. Block rate.	Charges. Straight meter rate, 5 cents per kw-hr.
100	$10.00	$5.00
101	10.05	5.05
102	10.10	5.10
103	10.15	5.15
.......etc.......		

Supposing now for a minute we had a straight meter rate of 5 cents per kw-hr. in the range both below and above 100 kw-hr., then the amounts will be those of column 3 of the above table. The block rate (column 2) is just $5 higher in every instance than the 5-cents-per-kw-hr. straight meter rate (column 3), regardless of the amount of energy consumed (provided, of course, that we remain within the second block). We can say, therefore: This block rate corresponds in the range of its second block to a combination of a straight 5-cent meter rate with a customer charge of $5. Where we have more than two blocks in the rate schedule the same general rule applies for all blocks: The block rate is equivalent to a combination of a straight meter rate with a customer charge. The customer charge is zero in the first block and increases with every successive block, whereas the energy charge decreases. This is more fully and more generally discussed in Appendix XII, which also describes the resulting simplification of the method of figuring the bills of the block meter rate.

The block meter rate furnishes us with the first examples of what might be called a "concealed charge" or a "disguised charge." This means that one of the three "fundamental charges" (energy, demand, customer) is expressed by one or both of the other two charges so that it is not visible on the surface.

The block meter rate is the most frequently found type of the pure meter rates. The number of the blocks does not go up as high as that of the steps in case of step rates; it ranges generally between two and eight, a larger number is a rare exception and no greater number of blocks than eleven[1] has been found in a very large number of schedules examined.

The block meter rate is also found combined with a minimum charge or a customer charge.

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[1] *General Power* **Schedule, Salem, Mass.**

A block meter rate is sometimes briefly described by simply stating the successive kilowatt-hour charges and omitting for the sake of brevity any mention of the size of the blocks. Thus the Sacramento rate described and analyzed in Appendix XII would be called "a 6-5-4-3-cent block meter rate." The same applies to step meter rates.

4. Combinations of Block Meter Rates With Step Meter Rates and Straight Meter Rates.

123. A block meter rate may be combined with a step meter rate in such a way that the lower energy consumptions are charged under the block meter system and anything above a certain amount of energy consumption is charged under the step meter system. The step rate may also contain one single step only, which means that it reduces to a straight meter rate. The order can also be reversed, which means that the step rate applies to the lower range of kilowatt-hours and the block rate to the higher one. (If we again consider the special case of the step rate comprising one step only, that is a straight meter rate, every plain block meter rate can be considered as a combination of a straight meter rate with a block meter rate, where the unit charge of the straight meter rate equals the unit charge of the first block of the block meter rate and where the straight meter rate extends over a portion, but not the whole, of the first block.)

These combinations do not occur frequently. Examples and their full discussion are given in Appendix XIII.

5. Explicit Customer Charge.

124. A further method of doing away with or reducing the inequity which lies in the use of equal unit charges per kilowatt-hour to large and small consumers *alike by* the straight meter rate is to make an explicit customer charge in addition to either a straight meter rate[2] or a step rate[3] or a block rate[4]. This combination with a customer charge (especially if it is applied to any system of block meter rates) can make

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[3] Battery Charging rate of Springfield, Ohio; customer charge 60 cents, energy charge 8 cents per kw-hr.

The General Lighting Rate of the New York and Queens Electric Light & Power Co. (customer charge 60 cents, energy charge 9 cents per kw-hr.) is only an apparent example of this combination because this rate contains a stipulation that the average rate per kilowatt-hour shall not exceed 11 cents per kw-hr. This makes the rate in effect a block rate. This example will be treated more fully here as it is typical for those schedules which make a stipulation that, no matter what is said elsewhere in the schedule, the average rate shall never be more than a certain specified amount per kilowatt-hour (compare Section 109).

Fig. 7 illustrates this rate. *OA'* is the customer charge of 60 cents and *A'x* is the 9-cent-per-kw-hr. rate added to this customer charge. This combination will be called Rate A hereafter. *Oy* is the straight 11-cent-per-kw-hr. meter rate (Rate B) which limits the amounts of the bill under Rate A. The drawing shows that Rate B is lower than A for all points to the

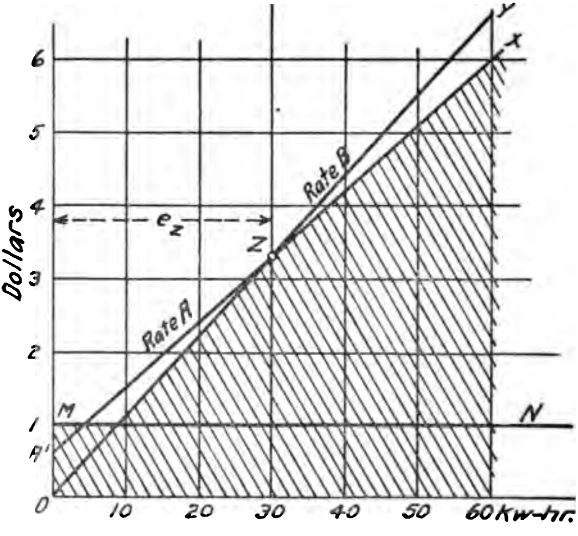

Fig. 7.—Stipulation of a Maximum Charge per Unit Results in the Equivalent of a Block Rate.

the curve of actual charges follow very closely the curve of the theoretical charges, as is demonstrated in Section 4 of Appendix XII, and is the most accurate pure meter rate, especially for small energy consumers.

Notwithstanding this, such a combination rate system is not a very popular one, probably because the public has not yet been educated to understand the equity of a fixed customer charge.

c. The Average Charges per Kilowatt-hour for Various Customers.

125. It has been shown (Section 117) that it is desirable to get lower average charges per kilowatt-hour for larger energy consumptions than for smaller ones. It will, therefore, be of interest to see how with the different systems discussed above (Sections 118-124) the average charges per kilowatt-hour vary as the amount of the energy consumption increases. This is fully discussed in Appendix XIV, to which the reader is herewith referred.

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left of the intersection point *Z* between the two rates. This means Rate B will apply for all energy consumptions smaller than the energy consumption e_x belonging to that point *Z* and A will apply to all energy consumptions exceeding that amount; e_x can either be read off from the drawing Fig. 7 directly, or with greater accuracy and as a check it may be figured out arithmetically as follows: The amount of the bill for e_x kw-hr. under the Rate A is $60 + 9e_x$ and under Rate B it is $11e_x$. These two amounts are equal to each other because the point *Z* is situated on both rate curves and we have $60 + 9e_x = 11e_x$, from which $60 = 2e_x$ and $e_x = 30$ kw-hr. The combination of Rates A and B is therefore identical with a block rate as follows: 11 cents per kw-hr. for the first 30 kw-hr.; 9 cents per kw-hr. for the excess over 30 kw-hr.

In addition to this the rate states that a minimum charge will be made of $1 per month, which is represented by the straight line *MN* and applies to the equivalent block rate as well. The resulting curve is shaded in Fig. 7.

[8] General Lighting, York, Pa.: Customer charge 10 cents per month (this is the smallest customer charge the author could find in any rate) plus an energy charge of 10 cents per kw-hr. The bill is reduced by stepped quantity discounts as follows:

5% on bills from $1.01 to $ 2.00
20% on bills from $2.01 to $ 5.00
25% on bills from $5.01 to $10.00
etc.

These discounts change the apparent straight meter rate into a *step meter rate* as explained at the discussion of the step rat *(Section 120).*

C. Rates Based on Demand Only.

1. *Nomenclature.*

126. Rates which are based on the demand

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[4] General Service Schedule of Lincoln, Neb.: **Customer harge** 40 cents per month plus an energy charge of

5 cents per kw-hr. for the first 10 kw-hr.
4 cents per kw-hr. for the next 30 kw-hr.
3.5 cents per kw-hr. for the next 460 kw-hr.
3 cents per kw-hr. for the excess over 500 kw-hr.

The General Lighting rate of Memphis, Tenn., is interesting in this connection: Customer Charge 30 cents per month plus an energy charge as follows:

6 cents per kw-hr. for the first 80 kw-hr
5 cents per kw-hr. for the next 120 kw-hr
4 cents per kw-hr. for the next 300 kw-hr
3 cents per kw-hr. for the excess
} To be called Rate I.

Now the rate further provides that no customer's bill for service through one meter shall exceed 7.5 cents per kw-hr. on the average in any calendar year. If this clause could be read "on the average per month" we would have a case parallel to that of the New York & Queens Co. treated in the preceding footnote and the above rate would be simply reduced by that clause to a block rate as follows:

7.5 cents per kw-hr. for the first 20 kw-hr
6 cents per kw-hr. for the next 60 kw-hr
5 cents per kw-hr. for the next 120 kw-hr
etc., as above.
} Rate II (monthly average)

As the clause applies, however, to the *yearly* average charge it is possible that the monthly bill of a certain customer exceeds this average charge of 7.5 cents per kw-hr., without reduction, for instance during the summer months, if his winter consumption is large enough to bring the yearly average charge below 7.5 cents per kw-hr. (It can easily be shown that this is the case if the average monthly consumption of the consumer over the whole year is greater than 20 kw-hr. See second preceding footnote, third paragraph.) This means that the reduction of the bills by the maximum of the average energy charge takes place in more cases if the maximum energy charge applies to the monthly average than if it applies to the yearly average. The monthly average for the maximum energy charge is on the whole more favorable to the customers than the yearly average.

The maximum-average-charge clause is in a certain measure the opposite of the minimum-charge clause (see Sections 107 and 108). The former is a guarantee on the part of the *electric service company* that the average charge per kilowatt-hour over a certain period shall never be *greater* than a specified amount in cents per kilowatt-hour and the latter is a guarantee on the part of the *customer* that the payment for a certain period shall never be *smaller* than a specified amount in cents or dollars. That period may be a month or a year, and the choice of the larger period (the year) will have the opposite effect in the case of the maximum-average-charge clause than in case of the minimum-charge clause. Whereas, for instance, a yearly minimum charge of $12 has the tendency to allow customers' bills, as a whole, lower than a $1 monthly charge (see Section 107, first footnote), a maximum-average charge of, for instance, 10 cents per kw-hr. will tend to make the payments of the customers, as a whole, higher, if applied to the yearly average than if applied to the monthly average.

only of the customer (in watts, kilowatts or some other unit or substitute) are called flat demand rates or simply flat rates.[1] The customer is charged simply in accordance with his maximum demand in watts, kilowatts, etc., and no explicit charge is made for the energy consumption in kilowatt-hours. The latter may be more or less strictly defined by the maximum demand, as in the case of display lighting (see later), or it may be only loosely connected with the maximum demand, as in some cases of residence lighting.

2. *Types of Service for Which Flat Demand Rates Are Used.*

a. Flat Demand Rates for Display Lighting, Etc.

127. The flat demand rate adapts itself most naturally to display lighting service, that is, advertising signs, outline lighting, store windows, etc. This kind of lighting is in use during a certain definite number of hours every day or month, either with all the lamps burning all the time or a certain prearranged percentage of the total capacity burning on the average (flashing signs). The number of kilowatt-hours per year (or per average month) is therefore, with a given kind of sign and use of the sign, proportional to the capacity of the sign, that is, to the maximum demand, and the energy charges can be most naturally averaged into the demand charges, thus doing away with the necessity of employing a meter.

Not infrequently the electric light company furnishes a patrol service for lighting and extinguishing the sign lights at the correct prearranged time. The lamps are, of course, lighted at dusk and they are extinguished usually at 11 o'clock, at midnight or at dawn, as local conditions warrant or the customer desires. The charges for this patrol service are then included in the flat rate charges.

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[1] In England the term "flat rate" is used to designate straight meter rates.

The ways in which the payments are made to depend on, and vary with, the maximum demand are very diversified. One group of rate schedules makes the charges proportional to the capacity of the sign in watts. We may have then what might be called a straight demand flat rate, in analogy with the straight meter rate[2], usually with a minimum charge. Or the rate may be framed as a block demand rate, in analogy with the block meter rate[3]. An explicit customer charge may also occur[4].

Another group of display lighting schedules make the charges depend not directly on the number of watts connected, but they charge a certain specified amount for every standard size of lamp. This is not quite the same, if the charges per lamp are not exactly proportional to the wattage of the lamp. The charges per watt of lamp capacity are generally smaller for the larger lamps than for the smaller ones.[5] This devi-

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[2] Erie Pa.: 1.5 cents per watt connected, with minimum charge of 75 cents. East St. Louis: 0.85 cents per watt connected, with minimum charge of $3. Both rates are charges per month for lights burning from dusk to midnight.

[3] St. Joseph, Mo.:
8 cents per 10-watt lamp, first 100 lamps
7 cents per 10-watt lamp, next 100 lamps } from dusk to 11 p. m.
6 cents per 10-watt lamp, excess lamps
For another example see footnote 6 (Macon, Ga.).

[4] Rockford, Ill. (optional): Customer charge $12 per year added to the demand charge of

$ 5 per year per 25-watt Mazda lamp
$ 8 per year per 40-watt Mazda lamp
$12 per year per 60-watt Mazda lamp
$20 per year per 100-watt Mazda lamp
$30 per year per 150-watt Mazda lamp
$50 per year per 250-watt Mazda lamp

As the demand charges are strictly proportional to the wattages of the lamps, this rate is the equivalent of a straight demand rate with a customer charge superimposed.

[5] Peoria, Ill.:

	Cents per Lamp per Month.	
Size of Lamp, Watts.	Dusk to 11 p. m.	Dusk to midnight.
5	5.1	6
10	8.5	10
20	17	20
25	21.25	25
30	25.5	30
40	34	40
60	51	60

ation from strict proportionality is occasioned by the fact that the charges in these cases include the renewals of burnt-out lamps. The price of a large lamp per watt is smaller than that of a small one, whereas the cost of labor for renewing the lamp is independent of the wattage of the lamp and this labor cost is by no means small on roof signs, etc. Moreover, the signs with the larger lamps are on the average the signs of greater total capacity and their users are therefore the larger consumers.

Some central stations diversify their flat rate schedules for display lighting according to the time during which the sign is being used, or, since all signs are lighted at dusk, according to the time when the sign is extinguished.[6]

The charges are generally figured per month, but

This is a straight demand rate with exception of the 5-watt lamps where the charge per watt is greater than with the rest of the lamps.

South Bend, Ind.:

Size of Lamp, Watts.	Cents per Week.	Equals Cents per Watt per Week.
2.5	2.5	1
5	3	0.6
10	5	0.5
15	7.5	0.5
20	10	0.5
25	12.5	0.5
40	18	0.45
60	25	0.42
100	40	0.40
150	50	0.33
250	80	0.32
500	150	0.30

Minimum charge 25 cents per week. Here, as the last column shows, the charges per watt decrease pretty regularly from 1 cent per watt per week down to 0.3 cent per watt per week.

[6] Macon, Ga.:

Watts Connected.	Cents per Watt Connected.	
	Burning till midnight.	Burning all night.
First 500	1.8	3.3
Next 500	1.4	2.5
Next 1000	1.1	2.0
Next 2000	1.0	1.8
Excess	0.9	1.6

See also footnote 5 (Peoria, Ill.).

occasionally display lighting schedules can be found which charge per year[7] or per week.[8]

In a few instances flashers on the signs are quoted in the schedules as bringing about a reduction of the charges.[9] From the theoretical point of view this is correct as it reduces not only the energy consumption of the sign but also the kilowatt demand on the central station and its lines on account of the diversity between a number of flashing signs. This means that the period of lighting of the sign will coincide with the period of darkness of some other sign, so that the total demand of all the flashing signs is smaller than the sum of the maximum demands of the individual signs in the ratio of the periods of darkness to the total time the flasher is connected, that is in the ratio of the energy consumption saved by flashing.

The great majority of electric light companies which have special schedules for display and sign lighting employ flat demand rates for that purpose.[10]

There are some other kinds of service which share the characteristic of the display and sign lighting service inasmuch as the number of burning hours per month can be predicted with a fair degree of accuracy from the size of the installation, so that for a given demand the monthly or yearly energy consumption is fairly well known without employing a meter. These

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[7] See for instance footnote 4 (Rockford, Ill.).

[8] See for instance footnote 5 (South Bend, Ind.).

[9] Topeka, Kans.: 5.5 cents per 5-watt Mazda lamp instead of 6 cents; Birmingham, Ala., states that a flasher discount is made in proportion to the current saved, usually about 40%.

[10] Among the few exceptions to this rule are the following: New York City (three of the companies operating in the territory); Rockford, Ill., (one of the optional display lighting rates); Sacramento, Cal., (Pacific Gas & Electric Co.). All of these employ straight meter rates for that purpose. Birmingham, Ala., has a step meter rate for display lighting, in which the lower kilowatt-hour charge takes place not after a certain energy consumption has been reached but after the connected exceeds 3000 watts. Muskogee, Okla., has a block meter *for display* lighting.

services are chiefly commercial lighting[11] (stores) and all-night porch lighting.[12] The employment of flat demand rates based on the rated capacity of the installation is, however, by no means as general for these purposes as it is for display lighting; on the contrary, it is rather the exception than the rule.

b. Flat Demand Rates for Small Customers.

128. Flat demand rate schedules are used also for the service of residences, small stores, etc. Whereas with the display lighting customers, treated in the preceding Section 127, the advisability of the flat-rate lighting schedule is based on the fact that for every consumer the energy consumption is practi-

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[11] Commercial Lighting, Pueblo, Colo.:

Hours per Night.	Price per 100 Watts per Month.
6	$1.00
8	1.15
10	1.25
12	1.35
14	1.45
16	1.55

The company reserves the right to set check meters and to bill the excess at 5 cents per kw-hr.

Flat Rate Commercial, Canton, Ohio:

	Daily Use in Hours.		
Tungsten Lamps.	6	10	15
250-watt	$2.50	$3.30	$4.00
150-watt	1.75	2.00	2.50
100-watt	1.10	1.30	1.60
60-watt	.75	.85	1.00
40-watt	.50	.60	.75

[12] Flat Rate for Porch and Hall Lighting, Waterbury, Mass.: $1 per 10-watt lamp per month.

Porch and hall lights, being intended as all-night lights for protection against burglars, etc., should be charged at lower rates—reduced per kilowatt-hour—than the ordinary residence lamps because otherwise no customer would avail himself of that rate and that kind of lamps would be burned under the ordinary residence rate, if at all. The electric light company can afford to give cheaper rates—reduced per kilowatt-hour—for that kind of service not only on the basis of the value-of-service principle but also because the cost per kilowatt-hour is lower on account of the good load-factor. A kilowatt-hour consumed in the regular residential lamps will be largely used during the peak hours of the central station, whereas if consumed in an all-night porch light the largest part of the energy will be demanded at off-peak hours. On the other hand, the cost will be enhanced by the cost of inspection, or in the absence of this, by the use—fraudulent or otherwise—of larger lamps than contracted for by some of the customers.

cally proportional to the maximum demand, no such statement can be made for the small customers, such as residences, etc. The energy consumption of various residence consumers with the same maximum demand may indeed vary within rather wide limits. The reasons which recommend the employment of a flat demand rate for small residences and stores are entirely different from what we saw in the case of display lighting.

The chief reason is the large reduction of the "customer cost" which can be effected by the flat demand rate through elimination of the customer's individual meter. From what has been previously explained (Sections 54-55) about the "customer cost," it is obvious that a large part of the "customer cost" consists of the expenses incidental to the meter, that is, capital charges on the meter, meter reading, meter maintenance, computing of the bills, etc., and this is eliminated by using a flat demand rate. Now the customer cost, as stated in Section 56, has the order of magnitude of 50 cents or one dollar per month per customer and therefore constitutes a large part of the total cost of serving a small customer. Consequently, if we are able to reduce the "customer cost" of the small customer, we will reduce the total cost of that customer's service in a proportion which is not much smaller than the percentage reduction of the customer cost. Therefore the charges to the small customer for a certain service can be made lower under a flat demand rate than under a meter rate, even though the profit to the company is not reduced or possibly even enhanced as compared to the profit from the same service if furnished under a meter rate. It has been shown before (Section 97) that cheapness of the service to the small customer is essential since money is of greater value to those customers as a class.

In addition to the cheapening of the service for the reasons just discussed, the flat demand rate permits of simplification in the office work of the company for the small flat-rate customers, especially if certain wattages are standardized for the capacity of flat-demand-rate installations to the exclusion of all other wattages. Thus, for instance, it might be stipulated that the maximum demand of every flat-rate installation must be rated at a multiple of 25 watts between the limits of 75 and 300 watts. This would mean that the customer has ten different standard wattages to choose from if he wants to be served under the flat demand rate. Then we might have ten different sets of bills, receipts, etc., printed, each with the correct amount imprinted, thus saving clerical work.

The simplification can be carried to such an extent that neither bills nor collectors are sent out to the flat-rate customer. The flat-rate customer is required to take care himself that his monthly or quarterly payments reach the company before a certain day of the month (or quarter) without waiting for a bill as the other customers.[13] Payments can be made at the company's offices or at the agencies appointed by the company all over the city.

Finally the company's dealing with individual consumers on the company's books in the monthly routine work can be eliminated altogether, as far as the small flat-rate customers are concerned. This is done by assigning to each flat-rate customer an identification number within a group. Each group comprises all customers whose demand amounts to a certain one of the standardized numbers of watts (see above). Each customer of a certain group has then

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[13] It is even feasible to require that these payments be made in advance, since their amounts are definitely known in advance. Losses from bad debts and expenses for collecting back payments can thus be restricted. The principle of prompt-payment discounts or delayed-payment penalties, respectively (Section 110) can as readily be applied to the flat rate the *meter rate*.

to pay the same amount per month as every other customer of the same group. The electric light company then deals only with the entire groups of customers on the company's books, at least in the regular course of operations, that is, if the customers pay regularly, or do not change their contracts, etc. Those customers who do not pay in time or who change their contract, etc., must of course always be singled out and dealt with individually, which causes extra expense; but even there the schedule could be framed in such a way that the customers who require individual dealing are charged a little additional fee in that month to cover the extra expenses.

These simplifications with their incidental cheapening of the service are not used as much in this country as they might be and as they are in Europe.[14]

Another advantage of the flat demand rate for small customers is the possibility of exactly and definitely foretelling to the prospective customer what amount he will have to pay every month for the lighting of his home with a given installation. This makes it easier to induce these small customers to try electric service than where they are to be served on a meter basis and where therefore no definite promise can be made as to the cost of the service.[15]

129. Nothing has been said so far about the method of determination of the customer's maximum demand for the residential flat demand rate. Simply using the number of the sockets connected for a basis and leaving it to the discretion of the customer what size of lamps he wishes to use would be too

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[14] See also S. E. Doane: "The Successful Handling of the Small Consumer in Europe," Proceedings of the National Electric Light Association, 1914, also reprinted in Electrical World, May 23, 1914.

[15] A booklet by the present author explaining fully the advantages of the flat demand rate if applied to small customers and containing a detailed description of the bookkeeping methods, with samples of the printed forms required can be obtained n the author, care of the publishers. As to the good results ined by flat demand rates for soliciting the small consumer, Earl A. Whitmore: "The Value of the Poor Man's Busi-" Electrical World, Jan. 22, 1916.

inaccurate for a flat rate; this method is at present nowhere employed for a flat rate in this country as far as the writer is aware. Using the connected load in watts as a basis for charging the residential customers, as in case of display lighting has also certain disadvantages. Whereas in case of display lighting the illicit exchange of lamps by the customer for larger ones or the unauthorized installation of additional lamps would be easily detected by an observer on the street, frequent surprise inspections would be necessary in case of residential consumers to be safe against unauthorized increases of the customer's wattage, whether fraudulent or unintentionally. These inspections would be, to say the least, cumbersome, costly, and inefficient, and would frustrate the object of the flat demand rate, viz., to make the cost of service low.

Special sockets into which the ordinary lamps would not fit have been tried, the lamps for these sockets which are not obtainable on the market being furnished by the electric light company. The disadvantages of this system are so great that to the writer's knowledge this method is no longer in use anywhere. In the first place dealers cannot easily be prevented from eventually selling lamps with bases to fit into these special sockets and in the second place lamp sockets have been standardized during recent years with an expense of work and money much greater than the outsider would suspect, so that every attempt to introduce nonstandard sockets nowadays means a step backwards.

Similar considerations apply to the use of low-voltage lamps (for instance 55 volts) for residential flat-rate customers to prevent the customers from buying higher wattage lamps and inserting them into the sockets. Strenuous and successful co-operative efforts are being made at present by the electrical industry to exclude all but a very few standard voltages

(110, 115, and 120 volts) in order to reduce the cost of production.[16]

Monthly measurements of the actual maximum demand would bring accurate results, but they would necessitate just what we want to avoid with the introduction of the flat rate—the installation of expensive measuring instruments for every consumer which necessitate periodic reading, etc.

130. The best method is to prevent the small flat-rate consumer by automatic devices from using a larger wattage than he has contracted for. This is done by the use of an inexpensive instrument called "demand limiter" or "current limiter" or sometimes, though not quite as accurately, "demand indicator." As soon as the customer at any time switches on a larger wattage than he is allowed under his contract, that is, as soon as he uses a current in excess of what he has paid for, this instrument, which is connected in series with his entire installation, will rythmically break and make the circuit until the excess current is turned off again. The current is broken about 20 to 300 times a minute according to the make of the instrument. This causes a very disagreeable flicker of all the lights in the entire installation which warns the customer that he has—intentionally or otherwise—exceeded his allowance and which most effectively induces him to shut off the excess wattage. This kind of flat demand rate is sometimes called the "controlled flat rate."

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[16] Only very few examples of flat residence rates on the basis of the connected load could be found by the author.

Dallas, Tex., charges per month:

$0.60 per 16-cp. lamp for 3 to 5 lights.
$0.50 per 16-cp. lamp for 6 to 8 lights.
$0.40 per 16-cp. lamp for 9 to 12 lights.
.... etc. ....

All the other instances found (three in number) are close together, in Connecticut, and apply to the use of low-voltage (55-volt) lamps, for instance: Hartford, Conn.: $1 per month for ten 10-cp. low-voltage tungsten lamps, 6 cents for each additional lamp; $1.50 for ten 20-cp. tungsten lamps, 12 cents for each additional lamp.

This instrument acts on the principle of an ordinary electric bell. It is so adjusted that it does not work unless the current passing through it exceeds a certain amount, namely the equivalent of the wattage for which the customer has contracted. These instruments require no periodic inspection as the meter and they are much simpler in construction and therefore cheaper than the meter.

The customer may now have installed as many and as large lamps as he chooses, but he never can burn a larger wattage at any time than what the current limiter allows to pass, that is, what the customer's contract permits. The greater, however, the ratio is of the installed capacity to the contracted maximum demand the oftener the current limiter will come into action and this, of course, means annoyance and dissatisfaction to the customer. If, for instance, a customer sits downstairs in his home reading and his daughter turns on the light in her own room at a time when the entire contracted demand is being used, all the lights in the entire house will begin to flicker and thereby cause a certain amount of inconvenience. It is but human that the customer does then not blame himself for not having contracted enough wattage to cover his needs but that he curses the electric light in general and the electric light company in particular. Such a thing never had happened to him before he had electric light. To avoid this it seems good policy on the part of the electric light company to make a rule in the flat-rate schedule that a customer must contract for the entire wattage of his installation or at least for a certain stipulated minimum percentage of that wattage.[17] If the lights then begin to flicker it is a warning to the consumer that he has violated his agreement and it is brought home to him forcibly

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[17] The controlled flat rate of Scranton, Pa., specifies that the demand shall not be less than 75% of the total sockets installed at the rate of at least 25 watts per socket (with a few secondary exceptions).

that he is doing something wrong so that he is the one to blame and not the electric light company.

The "controlled flat rate" is especially suited for the man of small means and by turning a large number of individuals into satisfied customers it contributes towards strengthening the good-will of the public towards the electric light company.[18]

D. RATES BASED ON BOTH ENERGY CONSUMPTION AND DEMAND.

1. *Introduction.*

131. Since the cost of service to a certain customer depends, as we have seen, about as much on the energy consumption as on the maximum demand, a rate which contains these two elements will in general be more accurate than any rate which contains only one of these two elements; that is, the unintentional deviations of the charges from the cost (plus a fixed percentage of profit) can be avoided more effectively. (This will be explained in Part V, "Ac-

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[18] Examples of the controlled flat rate: Worcester, Mass.:

Watts Demand.	Net Charges Per Month.
100	$1.00
125	1.25
150	1.45
175	1.65
200	1.85
225	2.05
250	2.25
275	2.45
300	2.65
325	2.85
350	3.05

It is easily seen that this means a (net) charge of 0.8 cent per watt plus a customer charge of 25 cents (with exception of the 100 watts demand, which is 5 cents lower).

In Allentown, Pa., the charges of the controlled flat rate are diversified between residences and stores and in the latter class they are again diversified according to the time of closing in the following manner:

1 cent (net) per watt per month for residences.
1.5 cents (net) per watt per month for stores, etc., closing at 9 p. m. or earlier.
2.0 cents (net) per watt per month for drug stores, saloons etc.
2.5 cents (net) per watt per month for establishments *open* all night.

curacy of Rates," Section 171). The charges from the two elements of energy consumption and demand may or may not be combined with a customer charge and in this manner we get either three-charge or two-charge rate systems under this heading.

2. *Determination of the Demand.*

a. Theory and General Remarks.

132. As shown in the theoretical part of this book (Sections 27-29 and Appendix VI), the exact evaluation of that portion of the central station's capacity which is determinative for the demand cost of a certain customer is so complicated an undertaking that an attempt to introduce it into general practice without far-reaching approximations would be hopeless.

The first step of approximation towards practical determination of the portion of the central station's capacity just mentioned and with that towards determination of the customer's demand cost and demand charge is the assumption that the demand cost is proportioned to the peak responsibility, that is, to the kilowatt load which the customer draws from the central station at the time of the peak load of the latter, assuming as peak-load time of the central station not an instant, but a certain short interval of one or several minutes or hours duration. This latter assumption is necessary not only for practical reasons, inasmuch as the top of the central station's load curve is too flat to permit of pointing out one definite second of the year as bringing a heavier load on the central station than any other second of the year, but theoretical considerations prove it to be correct so that we would use it even if our measuring instruments were accurate enough to determine just at what second the absolute maximum of the central station has occurred. It has been shown in Appendix VI that, whereas the customer's load at every single moment of time has a certain influence on the customer's demand cost, the influence of the customer's load is practically imperceptible during those moments of time when the central station's load, expressed in percentages of the central station's peak load, is not large. On the other hand, during those elements of time when the central station's load is near 100% of the central station's peak load the influence of the customer's load is paramount, so that these latter moments of time can be exclusively considered. But the decrease in the importance is a steady though rapid one as we go from the 100% limit downwards; in other words, the influence of the moment with the central station's real peak load (100%) is not of a greater *order of magnitude* than that of a moment with 99.9% or 99% load,[19] etc., and there are no *abrupt* changes in the degree of influence as we go down the scale of percentages on a continuous load curve.

We have therefore to take the customer's demand during a certain interval of time when the loads are near 100% and, as the customer's demand will in general vary during that interval, we will have to take the average of all the customer's instantaneous demands during that interval.

We have then to define the "peak responsibility" as the average of the customer's instantaneous demands taken over a given or chosen interval of time. This term "average demand taken over a certain interval of time" is not as definite as it seems at first sight. This question, which is of secondary practical importance, has been dealt with in Section 34-37.

The question now suggests itself how large the interval should be over which the customer's instantaneous demands are to be averaged in order to furnish

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[19] The former (100% load influence) may even be smaller than the latter (99.9% or 99%) although as a rule it will be larger.

the most exact result. The solution of this question depends entirely on the shapes of the load curves. In order to solve that problem for a particular case we would first have to find out how large the correct unapproximated value of the demand cost is with the given shape of the customer's and central station's load curve (Appendix VI). But the object of the whole approximation is to avoid just that complicated calculation of the true value. We will therefore have to select more or less by guesswork some standard length of interval over which the customer's demand is to be averaged.

Fortunately it does not make much difference generally whether we use those moments of time when the central station's load is above 95%, or when it is above 98%, or 99%, etc. It does make a difference with an individual consumer whose load curve has a freak shape containing large and abrupt rises and falls during the assumed interval of the central station's peak which are symmetrical to the latter, but with the ordinary shape of load curve the duration of this interval is not a matter of great importance.

What would be the effect of using too large or too small values for that interval? If it is chosen too short, this would mean penalizing a customer who once in a while, as an exception, or even as a consequence of an accident has a heavy but short demand. This short demand would as a rule not necessitate the electric light company keeping an additional equipment of the same capacity in readiness as the chances are that these exceptional demands do not occur during the central station's peak-load time. Even if they do, they will affect the central station's size in a much smaller measure than the amount of kilowatts of the maximum demand indicates, because these extremely short demands of a number of customers would not come all at the same moment but generally one after the other; in other words, the diversity-factor betwee these peaks would be very great on account of their short duration. This penalizing of the very short demands would result in a corresponding unjustified favoring of the customers with demands which are steady during a longer interval of time and thereby do not give the possibility of a large diversity so that they affect the central station's size in a much greater measure than the same number of kilowatts would for a short period. If we go to extremes, the customer might have to pay during the whole month, or even year, for the current occasioned by a short-circuit.

On the other hand, if the interval is chosen too great, the element called "peak responsibility" loses more and more the character of a real peak responsibility by approaching more and more the element of energy consumption. This is best illustrated by again assuming the extreme case, which now means that the interval within which the "peak responsibility" is to be measured (averaged), is no longer so small as to be measured in minutes or even hours, but extends over the entire 8760 hours of the year. In that case the rated "peak responsibility" would be the average demand of the customer over the whole year and would be proportional to his yearly energy consumption.

133. The next step towards enabling practical application is the replacement of the customer's peak responsibility as basis for the demand cost (or for the demand charge respectively) by the customer's maximum demand, regardless of the time when the latter occurs. With this approximation we arrive at a method of general practical applicability.

The maximum demand of the customer is usually not defined as the instantaneous maximum demand, but the demand as averaged over a certain given interval of time, just as this has been done in the first approximation with the customer's peak responsibility and for the same reasons. That interval of the *given length* within which the demand averages to a larger amount than during any other interval of the same length is then selected and the average demand during that particular interval is the rated "maximum demand." The maximum instantaneous demand must not necessarily occur during the same interval.

b. Details of the Determination of the Measured Demand in Practice.

1. Interval of Time Over Which the Customer's Demand Is to Be Averaged.

134. As has just been shown, it is impossible to determine by exact theory how long the interval should be over which the customer's demand is to be averaged for the determination of his rated maximum demand. Consequently this interval in practice varies within very wide limits[1] and it is impossible to say that the time interval chosen by one company is more correct than that of another company as they all remain within reasonable limits. Here again is an instance where a wide field is left to the judgment of the designer of the rate. Where we know or have good reason to assume that the customers of a certain class all have a peak of about the same duration, it may be well to choose the duration of that peak as the interval over which the maximum demand is to be averaged.

Sometimes several intervals are quoted in the schedule as alternatives, so that smaller percentages are to be taken of the short-interval demands than of

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[1] 1 minute: Duluth. General Power, see footnote 2 below.
2 minutes: Buffalo, Wholesale Lighting and Power (optional schedule).
3 minutes: Binghamton, N. Y., Wholesale Power.
5 minutes: East St. Louis, Ill., Retail Power.
10 minutes: Waterbury, Conn., General Power.
15 minutes: City Electric Co., San Francisco. Wholesale Lighting and Power.
30 minutes: Chicago, General Lighting and Power.
1 hour: Detroit, Standard Power, where lighting amounts to more than 10% of total load.
3 hours: Detroit, Standard Power, where lighting amounts to *less than* 10% of total load.

the long-interval ones.[2] Or shorter intervals are chosen for such loads as are subject to violent fluctuations, such as hoists, elevators, welding machines, etc.[3]

The instantaneous demand may also be made the basis for the rated demand, but then it is generally reduced by a certain large percentage to arrive at the rated maximum demand and special provisions may be made in that case to exclude the effects of any possible short-circuits and accidents on the rated demand.[4]

2. Period Over Which a Certain Amount of Demand, Once Determined, Remains the Basis of the Charges.

135. As the capacity of the central station is determined by the yearly peak and the monthly peaks have nothing to do with it, it seems logical to take the customer's yearly maximum demand as the basis for all his demand during the whole year and not to change it until his peak load for the following year has been determined.[5]

In most cases it is felt, however, that a single high demand would penalize a customer for the whole year

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[2] One of the two optional General Power schedules of the Great Northern Power Co., Duluth, rates the demand at the option of the company as the highest of the following: 40% of the instantaneous peak, or half the maximum 1-minute peak, or two-thirds of the maximum 3-minute peak, or the maximum 5-minute peak.

[3] Primary Power, Cincinnati: 5 minutes instead of 20 minutes.

[4] The Duluth schedule mentioned in footnote 2 provides that peaks due to short-circuits and accidents shall not be counted.

[5] This would mean that the highest recorded demand of the previous 12 months is to be chosen, as in the Wholesale Light and Power schedule of Washington, D. C. A little different interpretation is made in the Standard Lighting schedule of Detroit, where the demand indicator is reset June 30 each year so that the customer has to pay for the maximum demand since the preceding June 30 only. In the first case the rated demand will never be smaller than the maximum of a full 12-month period, whereas in the second case the demand to be paid for will generally steadily increase from June until December and n remain stationary until June, when a sudden decrease to *June monthly* maximum takes place.

and thus might work hardship and injustice. The maxima are, therefore, taken usually for shorter periods than one year; most commonly the monthly maximum is chosen. As the bills are almost universally rendered monthly, the demand-meter reading can be easily taken together with the watt-hour-meter reading every month. We are trying in this manner to get a value which is proportional to the *probable* value of the customer's maximum demand as given by the character of his load and to eliminate the accidentals of a single maximum demand. Since the maximum demand is nothing but a substitute for the peak responsibility, it seems that the average of the 12 months' maximum demands is a good substitute for, or even a better one than, the amount which the demand happens to reach only once a year. If customer *A* reaches 50 kw. in one month and 25 kw. only during each one of the remaining 11 months, and if customer *B* reaches 40 kw. of maximum demand month after month, the 50-kw. maximum of *A* may be the result of some casual contingency (neglecting in this example, for the sake of simplicity, the seasonal regular variations of load) and *B* is then liable to have a larger peak responsibility than *A*; yet under a system which takes into account the yearly maximum demand *A* would have to pay more for demand charges than *B*.

The demand charges are mostly based simply on the monthly readings, but other plans for calculating the rated maximum demand are also found.[6]

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[6] Standard Power schedule, Detroit, Mich.: The maximum demand is determined by load tests every 60 days.

Long-Hour Lighting schedule of Boston: The highest reading between Nov. 1 and Feb. 1 is taken to be in force for 12 months. That means a yearly maximum, but only that part of the yearly maximum demand is counted for which there is a possibility of its coinciding with the central station's peak.

The Retail Power schedule of the same city bases the demand charge on the average of the November-December-January-February peaks, whereas the demand charges of the Wholesale Power schedule are based on the average of the

3. *Sundry Details of the Determination of the Measured Demand.*

a. Influence of the Demands of Previous Billing Periods.

136. Various rate schedules provide that the rated demand depends on the previous demands of the same customer in such a manner that during the term of contract it may not decrease substantially below the highest demand established at a previous period[7], or that it cannot decrease below a certain percentage of the highest previous demand[8], or that the previous demand retains such a restricting influence

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highest half-hour peaks that have occurred in the 12 months previous to the bill.

The Primary Industrial Power and Railway Generating Power schedule of Baltimore requires that the demand be based on the highest readings (not less than ½ hour) between 4 and 8 p. m. any week-day during the months of November, December and January and not less than two-thirds of the actual demand at any other time.

The Wholesale Power schedule of the Sioux City Gas & Electric Co. rates the demand as the measured average over 15 consecutive minutes during the month, provided that the instantaneous demand is not more than twice the annual maximum demand. In the latter case the demand for the month shall be rated as 50% of the instantaneous demand.

The Large Lighting and Power schedule of Chicago provides that the demand be determined either monthly or yearly. When on the monthly basis, the average number of kilowatts in the highest 30-minute interval in the month is to be taken. When on the yearly basis the average number of kilowatts in the three highest 30-minute intervals in the month is selected from three different days, the first month's maximum to prevail for 12 months, unless a higher maximum takes its place, in which case a new 12-month period begins.

[7] Industrial Lighting and Power schedule of Baltimore, where the demand is not substantially to be decreased during the term of the contract. The minimum term of contract under this schedule is three years.

[8] Wholesale Power schedule of Macon, Ga., where the demand shall in no case be less than 75% of the highest previous recorded demand. Under the Primary Power schedule of Minneapolis the demand must never be under 50% of the highest previous peak occurring throughout the 10-year term.

[9] General Lighting and Power, Portland, Me.: Once determined, the demand may not be decreased during the year. Primary Lighting and Power schedule of Des Moines, Iowa: The demand shall not be less than 75% of the demand occurring in any of the preceding 12 months. Wholesale Power schedule wtucket, R. I.: The demand shall never be under 50% of ghest in the preceding 12 months.

at least for a certain length of time[9] (usually one year). In the latter case we get either the equivalent of a yearly demand, or if the maximum monthly demand may not exceed a certain percentage of the highest reading of the last 12 months, we arrive at a hybrid between the monthly and the yearly establishment of the demand.

b. Influence of the Power-Factor.[10]

137. A small number of electric supply companies make provision for a modification of the rated demand in case of the power-factor being below certain limits. Thus in some cases the demand is increased over the measured demand if the power-factor is smaller than a certain minimum percentage.[11] In other cases the regular demand charge of the schedule is not based on a 100% power-factor but on a smaller one; consequently the demand charge is not only increased for smaller power-factors but also decreased if the power-factor is large.[12]

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[10] The nontechnical central-station man will find an explanation of the term "power-factor" and allied terms in Appendix XV.

[11] Primary Large Lighting and Power schedule, Chicago: If the power-factor is less than 70%, the maximum demand is increased for billing purposes. In other cities the limit of the power-factor is 80%, thus in Philadelphia (Wholesale Power Lighting), Harrisburg, Pa. (Primary Power), and Little Rock, Ark. (Wholesale Lighting and Power). In Pittsburgh (Off-Peak Service) the limit is 90%.

Another way of expressing the same thought is that of the Wholesale Power schedules of Boston, which specify that either the kilowatts, or 80% of the kilovolt-amperes, are used for the demand, whichever is higher.

The Wholesale Power schedule of the Sioux City Gas & Electric Co. specifies that the monthly demand charge will be increased by 1% for each 1% the lagging power-factor on the consumer's load is below the following:

75% at the time of the demand.
70% at 75% of demand.
65% at all other loads.

In Cambridge, Mass., the average demand during the peak-load period is determined from the kilowatt-hour consumption as indicated by a watt-hour meter in case of noninductive load and from the kilovolt-ampere-hours as determined by a voltmeter and and amperemeter in case of inductive load. (See Section 139, "Determining the Demand by the Service Watt-hour Meter.")

Naturally provisions for an influence of the power-factor on the charges are found only in power rates for large consumers (wholesale power rates and general power as distinguished from retail power, etc.).

c. Methods Used for Measuring the Maximum Demand.

138. There are two methods in practical use for finding the maximum which the average demand over a certain interval (for instance 30 minutes) reaches within a given period (for instance, one year).

1. Determining the Demand by the Service Watt-Hour Meter.

139. The first method is the calculation of the average maximum demand from the number of kilowatt-hours consumed during the interval. The service watt-hour meter of the installation can be very conveniently used for that purpose in combination with a stop watch. If, for instance, the readings on the watt-hour meter show that during one certain hour 500 kw-hr. have been consumed, the average hourly demand during that particular hour is evidently 500 kw. If we now obtain a number of such readings at different hours when the maximum demand may be expected to take place, the maximum of these readings will be the nearer to the maximum average hourly demand the larger the number of readings has been. If we desire to know the average demand over a shorter (or longer) interval than an hour, we have to divide the kilowatt-hour reading of that interval by the length of that interval in hours.[13]

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[12] The General Power schedules in the following New England cities: Waterbury, Conn., New Britain, Conn., and Salem, Mass., provide that the maximum demand be adjusted accordingly if the power-factor is below 75% or above 80%.

An extra discount of 10 cents per kilowatt per month is granted on the demand charge of the Sioux City rate (referred to in the preceding footnote) if the power-factor is 95% or better.

[13] For instance, if in the 2 hours between 3 and 5 o'clock

This method can, of course, be applied for large customers only and is not suitable for general use. It is not as reliable as the other method, to be described hereafter (special demand-indicating instruments) because it is left to chance whether the watt-hour-meter readings are made actually at that interval which shows the highest average demand. The test method is, therefore, not frequently used and where it is used it is generally either as an alternative to some other method[14] or for secondary purposes.[15]

This method may also be carried out in a more accurate way by means of the so-called "printometer," that is an attachment to the customer's watt-hour meter which prints on a strip of paper in regular intervals—for instance, 5 minutes or half hours, etc.—the number of kilowatt-hours used.

2. Demand Metering Instruments.

140. The second method of determining the maximum demand makes use of special instruments for that purpose. This method is much more frequently employed than the calculation from watt-hour-meter readings and in all cases where the actual amount of the maximum demand in kilowatts or watts

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100 kw-hr. have been used, the average demand during that interval has been 100/2 = 50 kw. Or if in some 10-minute interval 8 kw-hr. have been used, the average demand during that interval has been 8 ÷ 1/6 = 48 kw. (because 10 minutes equals 1/6 hour).

[14] Wholesale Power and Lighting schedule, Birmingham, Ala.: If no demand indicator is used, the demand in kilowatts is assumed to be four times the highest recorded consumption in kilowatt-hours for any 15-minute period.

Primary Power Service, Buffalo: The demand is determined by the kilowatt-hour meter for 2 minutes, or by an indicating or graphic recording wattmeter.

[15] For instance, Cambridge, Mass., has pure block meter rates, but the knowledge of the customer's demand is necessary for writing out the bill in one of the schedules, because the minimum monthly charge is based on it and another one of the schedules applies only for the customers whose load-factor exceeds a specified minimum. The demand in these cases is determined as the average of three 5-minute readings of the watt-hour meter (or, in case of low power-factor load, of a voltmeter and ammeter), the readings to be made with the use of a stop watch.

must be known the use of these instruments is the rule.

A description of the principles on which these instruments act is given in Appendix XVI.

In a few cases the readings of the demand meter are modified by a stipulation in the schedule that the demand must never be taken lower than a certain percentage of the connected load.[16]

In some isolated cases the demand charge is based not on the measured demand, but on a certain fixed percentage of the demand.[17] This is, of course, merely a matter of changing the form; the total demand charge in dollars is the same as if the total measured demand had been chosen with a correspondingly reduced unit charge per kilowatt. But certain conditions, for instance the historical development of the rates of a company, may make this way of expressing the charges preferable.

The demand limiters (see Section 130) and similar devices, like fuses, automatic cutouts, etc.,[18] do not strictly belong in this class of instruments as they give no lasting record of the amount of the maximum demand, but they simply disturb the customer's supply as long as he tries to draw a larger demand than he has subscribed for. They have a very small time interval, which means that they act practically on the instantaneous demand.

d. Substitutes to Approximate the Measured Maximum Demand.

1. General Remarks.

141. Measurements of the maximum demand as described in the preceding Sections require either

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[16] General Power schedule in South Bend, Ind.: 50%; Optional Commercial Lighting schedule of the Universal Electric & Gas Co., San Francisco: 60%.

[17] General Power schedule of Fitchburg, Mass., and Haverhill, Mass.; 75% of the measured demand.

[18] Auxiliary Lighting and Power schedule, Buffalo: Demand

special labor, or special instruments, or both. Moreover, as has been pointed out in several places in this book, the element of the customer's demand as a basis for the demand charges has been derived from the theoretically correct basis by a series of approximations and therefore is not the accurate basis anyway. For these two reasons many electric utility companies avoid in all or in a part of their schedules the complication occasioned by the measuring of the maximum demand and use substitutes for the measured maximum demand which are more easily determined, even though by the additional approximation a further element of inaccuracy is introduced.

The possibility of substituting other values for the measured maximum has already been proviously mentioned, in Section 129. It has been stated there that these substitutes, such as connected load and number of sockets, are not popular among central-station managers for the flat rates other than display lighting. It is different in case of the rates which are based on both the maximum demand and the energy consumption. Here the inaccuracy introduced by the employment of a substitute for the real maximum demand of the customer affects a portion only of the total charges, so that the inaccuracy is, so to speak, diluted. A customer who increases the wattage of his lamps or the number of his sockets will in general also increase his energy consumption and therefore he will not get for nothing the additional, and possibly fraudulently obtained, service, as he would in case of a flat rate.

2. The Various Substitutes for the Measured Demand.

a. Size of Transformer Required.

142. To begin with a rather isolated type of substitutes for the measured demand, the size of the

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limited by fuses or other device. Auxiliary Power, Cleveland: The customer shall install an automatic circuit-opening device which the company will set and seal for an amount slightly above the *demand* contracted for.

transformer actually required for the customer's service may be used as the measure of his maximum demand.[1]

b. The Connected Load.

143. A very frequent substitute for the measured maximum demand is the connected load of the customer in some form or other.

Sometimes the measured demand and connected load are used together in such a manner that the measured demand is limited by the connected load so that the demand is never counted as less than a certain specified percentage of the connected load.[2]

1. The Full Connected Load.—The full connected load as the determining factor of the demand charge is used mostly in power rates,[3] although lighting rates on the same basis can also be found.[4]

The unit of the connected load in these instances is, in case of power rates, frequently the horsepower connected, for instance, the rating of the motor or motors.[5] Otherwise it may be the load in watts or kilowatts as determined by inspection of the installa-

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[1] The Electric Welder Service schedule of the Standard Electric Light Co. of Kansas City: Transformer capacity connected for each service.

All schedules of Lansing, Mich., where the demand enters into the rates: Transformers actually required.

Wholesale Power schedule, Erie, Pa.: 75% of the transformer capacity, where installation is greater than 15 kw. (otherwise demand is measured).

[2] Wholesale Lighting and Power schedule, Norfolk, Va.: The demand is never to be counted as less than one-half the connected load.

[3] For instance, in the wholesale power rate of Denver, Colo., a three-charge rate system where the monthly demand charge is $2 per horsepower connected.

The Wholesale Power schedule of Omaha (which applies also to lighting where this is not more than 20% of the total load) determines the demand as the motor rating of the connected load, not counting the portion formed by the lighting load.

[4] For instance, in the Retail Lighting Schedule of Washington, D. C., or in the Residence Lighting schedule of Pueblo, Colo., (Wright demand rates).

ee footnote 3 above.

tion.[6] The use of the number of 50-watt units installed,[7] or the rather obsolete method of using the 16-cp. lamp or equivalent,[8] as unit for the connected load are varieties of this rating.

2. Percentages of the Full Connected Load.—Instead of the full connected load a certain percentage of the load is very frequently made the basis of the charges and that percentage may be either a fixed one[9] or it may vary—within the same schedule[10] or from one schedule to the other[11]—with the character of the business served.

144. The percentage of the connected load which is to constitute the rated demand is frequently also varied with the *size of the connected load.* This is justified by the following considerations.

With a very small load, consisting, for instance, of one lamp or a very few lamps, the maximum demand will be equal to the connected load. If we have a larger installation the chances are in many installations that not all lamps will burn at the same time. The various lamps will have a diversity among themselves which in the average will be the greater, the larger the installation's capacity is in kilowatts. The same applies to power loads, especially where we have

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[6] See footnote 4 above.

[7] One of the two optional Commercial Lighting rates in Rockford, Ill., is a three-charge rate with a monthly demand charge of 16.6 cents per 50 watts connected.

[8] Denver, Colo.: Combination Lighting schedule, which is a three-charge rate system with a monthly charge of 15 cents per 16-cp. lamp.

[9] General Power, Haverhill, Mass.: For smaller installations 75% of the connected load or by test, but never under 60% of the connected load.

[10] The General Lighting schedule in Brooklyn rates the demand as 50% of the connected load in residences and 70% elsewhere, with exception of sign lighting where the full connected load is used.

[11] Buffalo: The Residence Lighting schedule and the Commercial Lighting schedules are identical except that the demand in the former schedule is determined as 25% of the total installation and in the latter schedule as 50%. The minimum rated demand for residences of 250 watts is accordingly raised for commercial lighting to 500 watts.

more than one motor installed. The chance that these motors will never be all running at the same time at full load is the greater the larger the number of motors is. Where we have one motor only, even if it be driving a number of power-consuming devices which have a diversity amongst one another, we must assume that this diversity had been anticipated when the selection was made of the size of the motor. At the same time the choice of a smaller percentage for the rated demand in larger installations expresses the principle of granting lower unit prices to the larger consumer.

The percentage which determines the rated capacity is therefore reduced as the *connected load* increases or as the *number of motors* in the installation increases, or both.

145. Where the connected load is the determining factor of the percentage—which is quite frequent practice—we can use methods entirely analogous either to the "step" or the "block" method, explained in the description of the step meter rate and the block meter rate, respectively (Sections 119 and 122). The same laws which govern the decrease of the energy charge per kilowatt-hour with increasing energy consumption in the step and block meter rates apply here to the decrease of the percentage with increase of the connected load.[12]

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[12] Step method: Power rates, Cincinnati:

Connected Load.	Rated demand in % of connected load.
Under 5 hp.	90%
5 to 10 hp.	75%
10 to 20 hp.	70%
20 to 50 hp.	65%
50 to 100 hp.	60%
100 hp. and over.	55%

Block method: Optional Commercial Lighting schedule, Bay City, Mich.:

Connected Load.	Rated demand in % of connected load.
First kw.	95%
Next kw.	90%
Next 2 kw.	85%

As regards those rather infrequent cases where the number of motors installed is used to determine what percentage of the connected load is to make up the rated demand, the simplest conceivable method, and the only one used in practice, is to state how large the percentage of the connected load is which corresponds to every number of motors. Usually only two or three different percentages are used in this manner; for instance: one for one motor, another one, say, for two to five motors, and the third one for more than five motors.

The use of the number of motors alone, without any reference to the size of the connected load, does not occur. It is either combined with the step or block method of the connected load as just described in such a manner that the percentages for the various numbers of motors are different for the different steps or blocks of the connected load[18] or the use of the number of motors applies to a certain range of the connected load only and outside of that range the per-

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Next 6 kw	80%
Over 10 kw	Varies with the class of business.

[18] A. Combined with steps of connected load:

1. Optional Commercial Lighting and Power schedule, Cleveland:

Connected load.	One motor.	Two or more motors.
Up to 5 kw	80%	75%
5 to 10 kw	75%	70%
Over 10 kw	70%	65%

2. Wholesale Power, Newark:

Connected load.	One motor.	Two or more motors.
Up to and incl. 50 hp	70%	60%
Over 50 hp	70%	50%

B. Combined with blocks of connected load:

General Power schedule of the Portland Railway, Light & Power Co., Portland, Ore.:

Connected load.	One motor.	2 to 5 motors.	Over 5 motors.
First 5 kw	95%	90%	85%
Next 5 kw	75%	70%	65%
Excess	65%	60%	55%
Minimum	75%	70%	65%

centage is entirely independent of the number of motors.[14]

Another measure to take into account the effect of the diversity of the motor loads can be made by assigning a lower percentage of the connected load to the rated demand if the motors are driving a group of power-consuming machines than if we have individual drive of the power-consuming devices. In both cases we will have the same diversity between the power demands of the various machinery,[15], but in case of the group drive—if the size of the motors has been chosen intelligently—the diversity of the loads on an individual motor has been anticipated and the capacity of the motor, that is the "connected load," is selected correspondingly smaller than in case of the individual drive, although the character of the mechanical load on the motors is exactly the same in both cases. Consequently the actual maximum demand is a larger percentage of the motor capacity in case of group drive than in case of individual drives.[16]

In a few cases the size of the largest motor installed is chosen to determine the percentage of the connected load which is to form the rated load. In these cases the block method is used: The rated demand is a certain percentage of the capacity of the

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[14] Retail Power schedule, Philadelphia:

Rated demand:

One motor connected—85% of connected load .... Two or more motors connected—75% of connected load	If less than 10 hp. are connected.

Optional Power schedule, Lancaster, Pa.:

Rated demand:

One motor connected—85% of connected load .... Two or more motors connected—80% of connected load	If 25 to 50 kw. are connected.

When the connected load is different from that specified (under 10 hp. or between 25 and 50 kw., respectively) another method of determining the rated demand applies.

[15] This means that their joint maximum demand will be smaller than the sum of their individual maximum demands *because* their maxima do not occur at the same time.

[16] *This discrimination* of the demand charges between group

largest motor plus another percentage of the rest of the connected load.[17]

146. Instead of reducing the percentage of the connected load which is to be used as the basis for the determination of the rated demand, we can get the same numerical results by rating the demand as the full connected load and reducing the demand charge accordingly. Whichever law—step or block—we use for the reduction of the percentage of the connected capacity, the same law will apply to the equivalent reduction of the unit demand charge and the latter will be reduced in the same proportion as the percentages of the connected load have been reduced.[18]

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drive and individual drive is very rare; only one example could be found, that is the Retail Power schedule of Wilmington, Del.:

		Two motors.		3 to 5 motors.		6 to 10 motors.		11 to 19 motors.		Over 19 motors.	
Connected load. Kw.	One motor.	Group.	Indv.	Group.	Indv.	Group.	Indv.	Group.	Indv.	Group.	Indv.
Under 3		98%	95%								
3 to 4.9	99%	97%	93%	90%	82%						
5 to 9.9	97%	95%	90%	88%	80%	85%	78%				
10 to 19.9	95%	92%	86%	85%	77%	82%	73%	75%	68%		
20 to 49.9	93%	90%	83%	82%	75%	78%	70%	73%	65%	70%	62%
50 to 99.9	92%	85%	78%	80%	72%	75%	68%	70%	62%	68%	60%

[17] Wholesale Lighting and Power schedule of San Francisco: For installations under 50 hp. the demand may be estimated at the option of the company as 100% of the rated capacity of the largest motor installed, plus 60% of the rated capacity of the additional motors or other devices.

Retail Power, Louisville:

Up to 10 hp. connected: 30% of largest motor	+40% of the total connected load.
10 to 50 hp. connected: 25% of largest motor	
Over 50 hp. connected: 20% of largest motor	

This might also be written to conform with the form of expression chosen above, as:

Up to 10 hp. connected: 70% of largest motor	+40% of the capacity of the additional motors.
10 to 50 hp. connected: 65% of largest motor	
Over 50 hp. connected: 60% of largest motor	

[18] *To quote* a simple example, the demand charge of the

Both principles of reduction—step and block—are sometimes found combined with each other as block-block, step-block or step-step systems. For instance, the percentage of the connected load decreases with increasing connected load according to the block law and the unit demand charge also decreases with the increasing connected load according to the block law. This "block-block" or "double-block" system can be reduced to one single-block system[19] either

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(optional) Commercial Lighting rate of Bay City, Mich., is $2 (net) per kilowatt of active load and the active load is—

95% for the first kw. connected.
90% for the next kw. connected.
85% for the next 2 kw. connected.
80% for the next 6 kw. connected, etc.

This plainly amounts to the same numerical effect as if we would change the demand charge to

95% of $2 = $1.90 for the first kw. connected.
90% of $2 = $1.80 for the next kw. connected.
85% of $2 = $1.70 for the next 2 kw. connected.
80% of $2 = $1.60 for the next 6 kw. connected.

[19] The following example will illustrate this:

The Lighting and Power rate of Grand Rapids, Mich., charges a yearly demand charge of—

$24 per kw. of the first 50 kw. of active load (rated demand).
$18 per kw. of the next 50 kw. of active load.
$15 per kw. of the excess.

Plus an energy charge which is of no interest in this connection.

The active load or rated demand is determined in the following way from the connected load:

100% of the first 20 kw. connected.
90% of the next 20 kw. connected.
80% of the next 20 kw. connected.
60% of the next 20 kw. connected.
50% of the next 20 kw. connected.
30% of the excess.

To reduce this rate to one based on the full connected load we proceed as follows:

The first 20 kw. connected are evidently charged at $24 per kilowatt connected.

The demand charge for the next 20 kw. connected is obviously 0.90 × $24 = $21.60.

The demand charge is changed from $24 per kilowatt of active load to $18 as soon as the active load reaches 50 kw. We have first to determine what the corresponding connected load is.

40 k.w connected corresponds to 20+(0.90 × 20) = 38 kw. rated load.
60 kw. connected corresponds to 20 +(0.90 × 20) +(0.80 × 20)= 54 kw. rated load.

Consequently 50 kw. of rated load lies in the third block *of the* original load system, that is, between 40 kw. and 60 kw. *connected load. The* first two blocks of connected load together based on full connected load instead of percentages of the same and employing a varying demand charge or, if preferred, with a constant demand charge and varying percentages of connected load. (This does not mean that either of these single-block systems is always preferable to the double-block system or vice versa; the fact is mentioned here only as an instance of the interchangeability of various forms of rates with the same numerical effect, to introduce the reader to the methods of analyzing the meanings of the various rate schedules). Similarly a "double-step" system is equivalent to a "single-step" system with the number of steps generally equal to the sum of the numbers of steps in the two systems. Finally a "step-block" system is conceivable which could be resolved into a combined step and block system, again either based on full connected load, or with a constant demand charge.

147. The block and the step method and their combinations are the simplest methods of varying the demand charges with the connected load. There

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comprise 38 kw. rated load; this leaves 12 kw. rated load reaching into the third block or, as 80% is the prevailing percentage in that block, $12/.80 = 15$ kw. connected load. The total connected load corresponding to 50 kw. rated load is therefore $40 + 15 = 55$ kw.

After 50 kw. of active load (55 kw. connected) has been reached, the demand charge changes from $24 per rated kw. to $18 per kw., that is, from 0.8 × $24 = $19.20 per connected kw. to 0.8 × $18 = $14.40 per connected kw.

In this manner we arrive by successive steps at the following table:

First 20 kw. connected	$24 per kw. connected
Next 20 kw. connected	21.60 per kw. connected
Next 15 kw. connected	19.20 per kw. connected
Next 5 kw. connected	14.40 per kw. connected
Next 20 kw. connected	10.80 per kw. connected
Next 20 kw. connected	9.00 per kw. connected
Next 80 kw. connected	5.40 per kw. connected
Excess over 180 kw. connected	4.50 per kw. connected

In the same manner we might also express this rate as a rate with one single kilowatt charge applying to different percentages of the connected load.

The reader who is familiar with elementary mathematics will find preferable the algebraic methods as described in Part IV (Rate Analysis), page 169 et seq.

are other methods which are very frequently used. In these the demand charges depend on the energy consumption as well. The demand charge is then generally altered not when the *demand* but when the *load-factor* exceeds a certain limit. These less simple methods will be discussed later (Wright system, etc.).

c. The Number of Sockets or Outlets.

148. Just as the connected load is an approximation of the maximum demand so the number of sockets or outlets is an approximation of the connected load, and just as the introduction of the connected load in lieu of the maximum demand brings about certain advantages and disadvantages, namely an increase in simplicity and a decrease in accuracy, so this same advantage and disadvantage is correspondingly further enhanced by the introduction of the number of sockets as basis for the demand charge.

The use of the number of sockets is an exception in case of rates which are based on both demand and energy consumption. The basis for the demand may be in these cases either the full number of sockets connected[20] or a certain percentage of that number,[21] in accordance with the parallel proceedings with the connected load (Section 143).

Sockets for lights which are liable to burn a few minutes only at a time (so-called convenience lights) and for lights which are liable to burn only occasionally or not at peak-load time are sometimes exempted

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[20] Residence Lighting schedule, Indianapolis, Ind.: All lighting sockets; sealed sockets, baseboard and floor receptacles for small power appliances not counted.

[21] Residence Lighting schedule, Youngstown, Ohio: Of the sockets in the principal rooms as specified in the schedule (that is, omitting closets, bath room, attic, etc.) seven-tenths are considered as 50 watts each and three-tenths as 25 watts each. Four-tenths of the wattage so obtained is then counted as the customer's demand. An easy calculation reveals that this simply means 17 watts are counted for every one of the sockets in the respective rooms. The demand charge per kilowatt may thus be reduced to a demand charge per socket.

from the rating for the demand,[22] so as to avoid a restricting influence on the installation of the customer and to let him get the benefit of the convenience of electric lighting without unduly high charges.

d. Encouragement of the Use of Domestic Appliances. Number-of-Rooms and Floor-Area Basis.

149. Whereas in the earliest times of central-station history electricity was used in residences purely as a lighting agent, it is now being employed more and more for heating and power purposes around the house, in radiators, stoves, toasters, percolators, milk-warmers, flatirons, ranges, washing machines, vacuum cleaners, fans, battery chargers for automobiles and in similar domestic appliances.

These appliances are used as a rule at a different time of the day than the electric lights and their use increases the peak of the central station in a far smaller measure, if at all, than the use of the same capacity in lamps would. Therefore, their operation costs the central station much less than their capacity implies. Their use should be cheapened also from the point of view of the value-of-service principle, since nobody would, for instance, care to use an electric range if he had to pay the same price for current as he willingly pays for electric light. Moreover, the use of these appliances improves the central station's load-factor and thereby it cheapens the service eventually to all customers, because a central-station equipment which is being used 24 hours a day is able to produce the unit kilowatt-hour of energy as well as kilowatt of demand more cheaply than one used during a short period of the day only and then resting idly.

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[22] **Residence Lighting schedule, Youngstown, Ohio:** Lamps in the following rooms are not counted: Vestibule, porch, closets, bath room, lavatories, dressing rooms, attic, basement, pantries, summer kitchen, servant's room, back halls, back stairways, billiard room, barn and yard.

The demand and energy consumption of these appliances is as a rule quite considerable and is a multiple of that of an ordinary household lamp. The capacity of almost any of these household appliances runs into the hundreds of watts and sometimes (ranges, etc.) into the thousands. The capacity of one or two of these devices may exceed that of the whole lighting installation. Therefore the leveling effect of these appliances on the total residential load curve can be a very considerable one.

For these reasons it is generally recognized by central-station managers that the use of domestic appliances should be encouraged in the mutual interest of the central station and of the customers.

150. The ordinary step and block meter methods, either as applied to the energy (step and block meter rate) or as applied to the rating of the demand are a step in this direction. However, with these two methods the lowering of the charges does not clearly apply to the use of appliances, but to the larger user in general.

The reduction of the charges to the user of the appliances, regardless of the amount he is using is more outspoken in the following method used by a number of central stations which base their rated demand on the connected load (either full load or a certain percentage) or on the number of sockets or outlets. The demand of the above mentioned appliances is then exempted from the rated demand [28] or it is counted at a smaller percentage than the light-

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[28] Residence Lighting schedule of Pueblo, Colo.: The rated demand is the full connected load of lamps only.

Retail Lighting schedule of Washington, D. C. (includes all residence service): The rated demand is the full load, not counting small fans, heating and cooking appliances.

Residence Lighting schedule of Cincinnati: The demand is determined by inspection as 70% of the connected load. Small motors, heating devices and appliances for domestic use consuming less than 1 kw. are not figured in the demand.

Residence Lighting schedule of Cleveland: 17 watts for each outlet. Fans, heating and cooking appliances, charging sets and motors for domestic purposes not counted.

ing load.[24] Where the demand is rated by sockets or outlets the first receptacle in each room may be exempted.[25]

The same object of automatically allowing the residential customers a lower rate for "appliances" than for lights is at the bottom of the most modern system of residential rates, the "number-of-rooms" and the "floor-area" rates. The idea of these rates is that the actual lighting demand of a residence depends on the number of rooms or on the floor area and that the deviations from the average in individual cases are generally not large.

To illustrate this by a simple case, a demand charge might be made of a certain amount for every room or for every 100 sq. ft. of floor area, and an energy charge of, let us say, 5 cents would then be added for every kilowatt-hour consumed.

This rate, like every other one with a demand charge apparently penalizes those customers who are using their installation for a shorter time than the average customers of that class, that is, the customers with a small load-factor, inasmuch as the average price paid by them *per kilowatt-hour* is higher. Conversely, it apparently favors the customers with a large load-factor by reducing their average charge per kilowatt-hour. A special feature of the rates under discussion is the fact that the customer who has not installed lamps of sufficient size and number to give adequate illumination in the whole house or who is using electricity as a supple-

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[24] General Lighting schedule of Saginaw, Mich.: Residence customers 80% of first 500 watts connected plus 60% excess. Appliances, such as electric ranges, ovens, etc., will be included at two-thirds of their maximum capacity, while other miscellaneous appliances are not included in determining the active load.

General Lighting schedule, Flint, Mich.: Residence customers full connected load. Electric ranges, ovens, etc., will be included at 75% of their maximum capacity. Other miscellaneous appliances not included.

[25] *Cleveland, Ohio.* See "Rate Research," Vol. VIII, page 179.

ment only to other illuminants (gas), is also penalized by being charged a relatively higher price, because he has to pay the same amount for the demand—though not for the energy—as if he were using lamps of sufficient or ample total wattage and number. On the other hand, he gets additional service for "appliances" against payment of not more than the energy charges for the additional service.

151. The question is hard to decide whether the number of rooms or the floor area is a better basis theoretically. It depends to a certain degree on the size of the rooms, as we can see from the following. If we imagine a very large hall, an auditorium, a skating rink, or the like, the wattage required for a given illumination (with a given type of lamp, etc.) depends obviously on the floor area. If we subdivide the hall by partitions into several smaller rooms, these partitions will cast shadows and shut off the light of certain lamps from certain parts of the floor, thereby reducing the illumination. Every room will get light only from the lamps immediately above it and the rest of the light will be lost (neglecting the fraction which is reflected from the partition walls). To maintain the illumination the wattage per floor area must therefore be increased as the number of rooms in the same total floor area increases, that is as the size of the rooms decreases.

In practice the great majority of central-station managers who have chosen this system of rates have decided for the number-of-rooms in preference to the floor-area[26] basis, probably because the number of rooms is easier to determine than the floor area and the method of determination is easier understood by the average customer.

It is possible to combine the number-of-rooms principle to a certain degree with the floor-area prin-

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[26] For statistical figures see the end of Section 152.

ciple by stipulating in a number-of-rooms rate that rooms with an area over a certain size count as two rooms[27] or that they count as many times as they exceed that maximum area.[28]

In at least one instance the floor-area principle is combined with the connected-load principle for determining the demand, in such a manner that the demand is rated as the connected load with the restriction that lighting equipment in excess of one watt per square foot of floor area is not considered.[29]

152. It has been shown above (Section 148) that, where the number of sockets is the basis of the rated demand the sockets in certain localities are not counted. In the same manner and for the same reason certain rooms—or their floor area, respectively—are exempted in the number-of-rooms and floor-area rates. We thus get "active" and "nonactive" rooms. The definition of an active and a nonactive room is as a rule made by simply enumerating the different kinds of rooms which fall under one or the other of both classes. The rates of different companies differ from each other to a slight degree in that respect. Nonactive rooms are in all or almost all schedules: halls, bathrooms, basements, attics, closets, and porches. In several of the schedules we find further specified as inactive: The first three bedrooms, laundry, pantry, alcoves, garage, unfinished rooms, coal shed, storage rooms, etc. The real estate rating[30] or the architect's rating simply is made the criterion to distinguish the active from the nonactive rooms in isolated cases.

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[27] El Paso, Texas, 300 sq. ft.

[28] Superior Water, Light & Power Co., Superior, Wis., 300 sq. ft.

[29] Portland Railway, Light & Power Co., Portland, Ore.: Nor are heating, cooking and power appliances considered in this residence rate.

[30] Minneapolis. (See "Rate Research," Vol. XI, page 164.)

[31] Pittsburgh. (In apartment houses bedrooms and bathrooms are counted as second-floor rooms, all other rooms and halls *as first-floor* rooms).

The rooms of the first floor are rated differently in one case.[31]

The rates on the number-of-rooms principle and the floor-area principle have not yet found as wide an application in this country as they deserve. An investigation (which does not claim completeness) showed 16 companies in the United States employing the number-of-rooms principle in 17 cities, and two companies only using the floor-area principle in three cities.

The number-of-rooms rates and the floor-area rates are almost exclusively designed on the principle of the so-called Wright demand rate or its modifications. Further discussion will therefore be put off until that principle has been explained (see Section 155 *et seq.*).

3. Description of the Various Rate Systems Based on Both Energy and Demand.

a. The Hopkinson Rate.

153. The simplest system of rates which embodies both an energy charge and a demand charge is the rate system which was devised by Dr. John Hopkinson in 1892. The customer's bill under the Hopkinson system consists of two separate portions, a demand charge and an energy charge. The demand charge is figured as the product of a certain fixed unit demand charge times the number of kilowatts (or horsepower or whatever unit has been chosen for the demand), and likewise the energy charge is the product of the unit charge times the number of kilowatt-hours consumed.[1] (It goes without saying that the energy charge is lower than it would be under a straight meter rate for the same service.)

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[1] Refrigerating rates of the Universal Electric & Gas Co., San Francisco: Demand charge $2 per hp. connected, plus an energy charge of 1 cent per kw-hr. Auxiliary service of the same mpany: Demand charge of $2.25 per kilowatt connected plus energy charge of 2 cents per kw-hr.

The unit demand charge as well as the unit energy charge can be varied with increasing demand or energy, respectively, according to the block system ("demand block," "energy block" and "double block" systems), or according to the step system as shown for the pure meter rates in Sections 122 and 119.[2] "Demand block" and "demand step" systems with unchanged energy charges do not occur in practice as far as the author's knowledge goes.

b. The Doherty Rate.

154. This rate is an amplification of the Hopkinson rate by the addition of a customer charge.[3] It

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[2] Energy Block System (demand charge unchanged):
Wholesale Lighting and Power, South Bend, Ind.:
Demand charge $1 per hp. connected, plus an energy charge of—
6 cents per kw-hr. for the first 50 kw-hr.
4 cents per kw-hr. for the next 50 kw-hr.
3 cents per kw-hr. for the next 100 kw-hr.
2.3 cents per kw-hr. for the next 300 kw-hr.
2 cents per kw-hr. for the next 500 kw-hr., etc.
Double Block System:
Wholesale Lighting and Power, High Tension, Boston:
Demand charge—
$60 per year per kw. for the first 15 kw. of demand.
$36 per year per kw. for the next 40 kw. of demand.
$30 per year per kw. for the next 100 kw. of demand.
Energy charge:
5 cents per kw-hr. for the first 1500 kw-hr.
3 cents per kw-hr. for the next 4000 kw-hr.
1.5 cents per kw-hr. for the next 50,000 kw-hr.
1.25 cents per kw-hr. for the next 50,000 kw-hr., etc.
Step System of Demand Charges and of Energy Charges:
Primary Power, Atlanta, Ga.:

Kw. demand	Demand charge.	Plus energy charge of
10 to 20	$1.70 net	0.9 cents per kw-hr. net
21 to 50	1.70 net	0.7 cents per kw-hr. net
51 to 100	1.60 net	0.6 cents per kw-hr. net
101 to 200	1.40 net	0.5 cents per kw-hr. net

Etc.

[3] Optional Lighting rate, Denver, Colo.:
Customer charge $9 per year, payable monthly
plus a demand charge of $1.80 per year per 16-cp. lamp of demand payable monthly (one-third of connected load)
plus an energy charge of 5 cents per kw-hr.
Wholesale Power, St. Joseph, Mo.:
Customer charge $100 per month.
plus a demand charge of 25 cents per hp. connected
plus an energy charge of 1 cent per kw-hr.
In this latter case the object of the customer charge is obviously to restrict the use of these cheap unit charges for demand and energy to the wholesale customer.

is named after Henry L. Doherty, the well-known New York engineer and financier who, to the author's best knowledge, introduced this system for the first time in Denver in the first decade of this century.

The Doherty rate can also be found in practice with a graduation of the energy charges.[4] Graduations of the demand charges by blocks or any graduations by steps are not found in practice with Doherty rates, as far as the author's knowledge goes.

c. The Wright Demand Rate (Multiple Rate).

155. This rate, which is one of the most generally used systems, has been designed by Arthur Wright, of Brighton, England, after whom it is generally called. Sometimes, especially if it embodies more than two steps, it is also called the "multiple rate."

This rate mentions only kilowatt-hour charges and yet, as will be demonstrated later (Sections 160, 161, 166 and 167) it embodies the equivalent of a demand charge and sometimes of a customer charge as well. A certain charge per kilowatt-hour ("primary charge") is made for the first block of kilowatt-hours of every customer; another one ("secondary charge") is made in the next block of kilowatt-hours, and so on, just like in the block meter rate, with this difference that the blocks are not determined by a certain fixed absolute number of kilowatt-hours but by some relation of the customer's kilowatt-hours to the customer's demand, in most cases as a certain fixed number per every kilowatt (or other unit) of demand. In other words, the blocks are not energy blocks, but generally load-factor blocks.[5]

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[4] Optional Commercial Lighting schedule of the Universal Electric & Gas Co., San Francisco:

Customer charge $10 per month
plus a demand charge of $1 per kw. maximum demand.
plus an energy charge of 1.5 cents per kw-hr. for the first 4000 kw-hr. and 1 cent per kw-hr. for the excess.

[5] *General* Lighting rates of Sioux City Service Co.:

Practically all the rates on the number-of-rooms or floor-area basis are Wright demand rates, either in the simple form just discussed that the number of kilowatt-hours in each block is proportional[6] to the "demand" (number of rooms or number of square feet of floor area, respectively), or in some other more complicated form.[7] This latter form of rates will be discussed and analyzed later (Sections 161 and 167) and here will be mentioned only, anticipating this analysis, that it embodies the equivalent of a customer charge for certain customers.

A variety of the Wright system is based not on the block system of gradation, but on the step system;[8] such systems are exceptions, however.

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12 cents per kw-hr. for the first 40 hours' use of the connected load
6 cents per kw-hr. for the excess.

General Lighting schedule, Portland Railway, Light & Power Co.:
9 cents per kw-hr. for the first 6% of the "monthly maximum consumption"
7 cents per kw-hr. for the next 6%
4 cents per kw-hr. for the excess.

"Monthly maximum consumption" means the number of kilowatt-hours which would result from continuous use of the demand.

[6] Optional Residence Lighting schedule, Oklahoma City, Okla.:
10 cents per kw-hr. for the first 5 kw-hr. per active room,
7 cents per kw-hr. for the next 5 kw-hr. per active room,
3 cents per kw-hr. for the excess.

Residence Lighting schedule of Tacoma, Wash.:
5 cents per kw-hr. for the first 40-watt-hours per sq. ft. active floor area,
1 cent per kw-hr. for the excess.

[7] Residence schedule of St. Louis, Mo.:
8 cents per kw-hr. for the first 4 kw-hr. for each of the first 4 active rooms, plus 2½ kw-hr. for each excess room,
6 cents per kw-hr. for excess up to 7 kw-hr. per room for all active rooms,
3 cents per kw-hr. for the excess.

This latter kind of Wright rate is not easy to understand for the beginner; the analysis of Sections 161 and 167 will explain it fully.

[8] Retail Power, Akron, O.:
5 cents per kw-hr. for 30 hours' use of the demand
4.4 cents per kw-hr. for 60 hours' use of the demand
3.9 cents per kw-hr. for 90 hours' use of the demand } 21 steps
.... etc. ....
1.5 cents per kw-hr. for 630 hours' use or over

What has been said before (Sections 119-121) about step meter rates applies also to step Wright rates, for instance that the steps can be formed by discounts, except that in this case the discounts are not quantity discounts but load-factor discounts.[9]

The Wright rate principle can also be applied to block or step meter rates.[10]

d. Combination Rates.

156. In this and similar ways a large number of combinations between the various systems of rates and various principles of rate-making[11] can be put into effect and a great variety of "combination rates" of this kind are found in practice. It is almost impossible and is of little purpose to enumerate or classify all these combination rate systems. Just to show the fertility of this field a few examples from practice will be quoted here at random:

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[9] **Wholesale Combined Light and Power rate of Dallas, Tex., (see next footnote).**

[10] As, for instance, in the example of Dallas, Tex., where the rate is a step meter rate with a load-factor discount. This discount is given only, however, if the energy consumption is in excess of 50,000 kw-hr., which brings a further element into the rate.

Or, General Lighting rate, Northern Electric Co., Portland, Ore.: Primary rate covering the first 100 hours' use of the demand:

8 cents per kw-hr. for the first 10 kw-hr.
7 cents per kw-hr. for the next 70 kw-hr.
6 cents per kw-hr. for the next 100 kw-hr.
5 cents per kw-hr. for the next 720 kw-hr.
4 cents per kw-hr. for the next 1100 kw-hr.
3 cents per kw-hr. for the next 1500 kw-hr.
2.5 cents per kw-hr. for the excess.

Secondary rate covering the excess over 100 hours' use of the demand:

Demand 5 kw. or less. 3 cents per kw-hr. for next 1000 kw-hr.
2 cents per kw-hr. for excess.
Demand 5 to 30 kw... 2 cents per kw-hr. for next 1500 kw-hr.
1.5 cents per kw-hr. for excess.
.... etc. .... etc. ....

[11] Such as, for instance, changes of the unit prices by blocks or by steps, guarantees for a minimum bill or minimum demand or minimum consumption, discounts of various descriptions such as quantity discounts, discounts for guaranteed bill, for guaranteed consumption, for guaranteed demand, for guar-*nteed* term of contract (for instance, Lowell, Mass.), seasonal *scounts, etc.*

1. The Retail Power schedule of Spokane is a combination of a Hopkinson and a Wright demand rate as follows:

Demand charge: $1.50 per kv-a. connected
plus an energy charge of

Nothing for first 20 hours' use of connected load
3 cents per kw-hr. for next 30 hours' use of connected load
1.5 cents per kw-hr. for next 50 hours' use of connected load
1 cent per kw-hr. for next 300 hours' use of connected load
0.5 cent per kw-hr. for excess.

We see, moreover, that in this particular Wright rate the unit energy charge is not steadily decreasing with increasing load-factors but that it first increases from zero to 3 cents per kw-hr. and then decreases by blocks. We might also look at it in this way that the Wright rate begins only at the load-factor or of 20 hours' use instead at the load-factor zero, as in all other Wright rates.

2. The General Power rate of the Great Western Power Co., of Sacramento, Cal., is a straight meter rate stepped according to the size of the demand (in connected load) as follows:

3.75 cents per kw-hr. under 10 hp. connected.
3 cents per kw-hr. from 10 to 50 hp.
2.5 cents per kw-hr. over 50 hp.

3. The Commercial Lighting schedule of Atlanta, Ga., is a combination of a step, block and Wright demand system as follows:

A. Demand 25 kw. or less:

Primary kw-hr. charge 7.77 cents per kw-hr.
Secondary kw-hr. charge, 6 cents per kw-hr.

Primary charge applies according to the following table:

Demand in 50-watt equivalents.	Primary charge applies to the following number of kw-hr.
1 to 50	150
51 to 60	175
61 to 70	200
71 to 75	225
76 to 85	250
....................etc.	....

B. Demand 25 kw. to 50 kw.:

Primary kw-hr. charge, 7.77 cents per kw-hr.
Secondary kw-hr. charge, 6 cents per kw-hr.
Tertiary kw-hr. charge 4 cents per kw-hr.

The primary and the secondary charges apply according to the following table:

Demand in 50-watt equivalents.	Primary charge.	Secondary charge.
501-600...............	First 925 kw-hr.	Next 925 kw-hr.
601-700...............	First 945 kw-hr.	Next 945 kw-hr.
701-800...............	First 965 kw-hr.	Next 965 kw-hr.
801-900...............	First 985 kw-hr.	Next 985 kw-hr.
901-1000..............	First 1000 kw-hr.	Next 1000 kw-hr.

C. Demand above 50 kw.:

Primary kw-hr. charge 7.77 cents per kw-hr.
Secondary kw-hr. charge, 5 cents per kw-hr.
Tertiary kw-hr. charge 3 cents per kw-hr.

The primary charge applies for the first 20 hours' use of the connected load, the secondary for the next 20 hours' use and the tertiary, of course, for the excess. We see, therefore, that beginning from demands of 50 kw. we have a pure Wright demand rate, but not for demands below that amount.

Some of these rates require more or less study in order to arrive at a thorough understanding of their meaning. This study is very much simplified and the understanding of the various rate systems in general is made much easier by the systems of analysis explained in the following (Part IV), especially by the graphic system.

PART IV

RATE ANALYSIS

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157. In case of the more complex rate systems it is not easy to get a clear general conception of what the rate means to the various customers. It is, of course, very easy to figure out what the amount is which a certain given customer has to pay under the rate system, but that is not enough for a critical examination of the character of the rate. A little arithmetic, algebra or geometry can be applied with astonishing results to get an insight into the nature of a certain rate or rate system.

I. ARITHMETICAL AND ALGEBRAIC RATE ANALYSIS.

158. The arithmetical system of rate analysis has been demonstrated incidentally in a few of the simplest cases in the previous sections and will now be discussed more fully.

Let us call

a = the *amount* of the customer's total bill in cents for the period to which the bill applies.

d = the customer's maximum *demand* in kilowatts, horsepower, number of rooms, or whatever else the unit is.

e = the customer's *energy* consumption in kilowatt-hours.

Let us now take first the example of the straight meter rate, for instance, charging 10 cents per kw-hr. Then we obviously get $a = 10e$, or in the general case if z cents[1] are charged per kilowatt-hour we get $a = ez$.

Likewise, we get in a straight demand rate $a = dy$, where y is the demand charge in cents per kilowatt, horsepower, or per room, etc.

In a Hopkinson rate we get $a = dy + ez$
and in a Doherty rate: $a = x + dy + ez$,
where x is the specified customer charge.

Conversely, if we can express a rate by a formula composed of three members, one of them being constant (x), the second (dy) being proportional to the demand d, and the third (ez) being proportional to e, we have the equivalent of a three-charge system as follows: The member (x) which is free from demand and energy consumption is the equivalent customer charge, the member containing the demand d indicates the equivalent unit demand charge times the multiplier d, and the member containing the energy consumption e indicates the equivalent unit energy charge times the multiplier e. Since x, y or z, or any two of these values may be zero, we may accordingly get a two-charge or a single-charge system.

If we include those cases where one or two of the values x, y and z are zero, we can state that practically every rate system can be reduced to an equation of the type described and we have in every rate, whatever its apparent form, the equivalent of a three-charge, a two-charge or a single-charge rate with the unit charges expressed in the way described above.

The quickest and easiest way to show the working and the principles of this arithmetical and algebraic analysis is to work out a few examples. The gen-

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[1] Although the symbols x, y and z are usually employed to designate variable quantities, they are used in the following equations for the quantities: customer charge, unit demand charge and unit energy charge, respectively, although these charges are constant in the first rate systems to be discussed *and the variables are* d and e. The reason for this choice of *will become* apparent later.

eralizing deductions will then offer themselves automatically.

159. Supposing we have a block meter rate which charges 10 cents per kw-hr. for the first 100 kw-hr. per month (first block), 9 cents per kw-hr. for the next 50 kw-hr. (second block), and 8 cents per kw-hr. for the balance (third block). The amount of the bill is then in the first block,

$$a = 10e$$

In the second block ($100 < e < 150$) it is

$$a = 10 \times 100 + 9(e - 100) = 100 + 9e$$

This means that all customers in the second block are charged just the same amount as they would have to pay under a rate having a customer charge of 100 cents ($1), plus an energy charge of 9 cents per kw-hr. We can thus say, although we have no customer charge *specified* in the rate system, yet we have the equivalent of a customer charge of $1. We have no equivalent demand charge, or, more accurately expressed, the equivalent demand charge is equal to zero.

Likewise we get in the third block

$$\begin{aligned} a &= (10 \times 100) + (9 \times 50) + 8(e - 150) \\ &= 250 + 8e. \end{aligned}$$

The equivalent customer charge has increased to $2.50, the demand charge is still zero, and the energy charge has decreased to 8 cents per kw-hr.

A customer, for instance, who has used 200 kw-hr. will have to pay, being in the third block, 250 cents + (8 × 200) = $18.50, which means a simplification of the computation even in this simple case.[2]

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[2] Using general symbols where z_1, z_2, z_3...are the energy charges in the first, second, third, etc., block and e_1, e_2, e_3... are the sizes of the successive blocks in kilowatt-hours we get in the first block $a = ez_1$
in the second block, that is for energies between e_1 and e_2 kw-hr.,

$$a = e_1z_1 + (e - e_1)z_2 = e_1(z_1 - z_2) + ez_2$$

The customer charge is $e_1(z_1 - z_2)$, the energy charge is z_2.

In the third block:

$$\begin{aligned} a &= e_1z_1 + e_2z_2 + (e - e_1 - e_2)z_3 \\ &= \underbrace{e_1(z_1 - z_3) + e_2(z_2 - z_3)}_{\text{Customer Charge}} + ez_3 \end{aligned}$$

Energy Charge $= z_3$. Likewise we get analogous terms for the *customer and* energy charge in the following blocks.

160. Next let us take the example of a plain Wright demand rate, charging, for instance, 10.5 cents per kw-hr. for the first 35 hours' use of the demand, 8.5 cents per kw-hr. for the next 25 hours, 5 cents per kw-hr. for all the balance. In the first zone, block or "range," that is from 0 to 35 hours' use, we have

$a = 10.50e$; this means $x = 0$, $y = 0$, $z = 10.5$ cents per kw-hr.

In the second block (35 to 60 hours' use) the customer is made to pay for all kilowatt-hours which correspond to the first 35 hours' use of the demand at the rate of 10.5 cents per kw-hr. and the balance at 8.5 cents per kw-hr. The number of kilowatt-hours which corresponds to the first 35 hours of use of the demand d is $= 35d$ and the unit price to be paid for these $35d$ kw-hr. is 10.5 cents, so that the total price for them is $10.5 \times 35d$. The balance of $e - 35d$ kilowatt-hours is to be paid at 8.5 cents per kw-hr. and the total amount paid for that balance is therefore $8.5\ (e - 35d)$. The total bill is therefore

$$a = (10.5 \times 35d) + 8.5\ (e - 35d) = 70d + 8.5e.$$

This means a customer charge $x = 0$, a demand charge of 70 cents per kilowatt per month and an energy charge of 8.5 cents per kw-hr. (equivalent of a Hopkinson rate).

In the third block we get likewise $70d + 8.5e$ to be paid for the kilowatt-hours corresponding to the first 60 hours' use, that is for $e = 60d$, and 5 cents per kw-hr. for the balance. Therefore

$$a = 70d + (8.5 \times 60d) + 5(e - 60d) = 280d + 5e.$$

A customer having, for instance, 1.5 kw. demand and 100 kw-hr. energy consumption would have to pay $(280 \times 1.5) + (5 \times 100) = 420 + 500 = \$9.20$. Compare this simple calculation with the explicit method of figuring the price for 100 kw-hr. at 1.5-kw. demand under this rate:

I. (35×1.5) kw-hr. $\times$ 10.5c/kw-hr. $= 551.25$c
II. (25×1.5) kw-hr. $\times$ 8.5c/kw-hr. $= 318.75$c

Brought forward 870.00c

	Carried over	870.00c
III.	$(100 - 60 \times 1.5)$ kw-hr. $\times$ 5c/kw-hr. =	50.00c
		920.00c

161. As the next example, consider the St. Louis Residence Lighting rate, which is a Wright demand rate on the number-of-rooms basis, but no longer a simple load-factor Wright rate but of somewhat more complicated specifications as given below. The charges are as follows:

8 cents per kw-hr. for the first 4 kw-hr. for each one of the first 4 active rooms, plus 2.5 kw-hr. for the excess rooms;
6 cents per kw-hr. for the excess up to 7 kw-hr per room for all active rooms;
3 cents per kw-hr. for the excess.

The unit for the demand *d* is here not the kilowatt but the active room. To determine the different "ranges" or "zones" in which the three unit kilowatt-hour charges of 8, 6 and 3 cents per kw-hr., respectively, apply, it is convenient to draw a diagram (Fig. 8), plotting the number of rooms in horizontal direction (as abscissæ) and the number of kilowatt-hours in vertical direction (ordinates). The 8-cent-per-kilowatt-hour zone reaches from

0 to 4 kw-hr. for 1 room
0 to 8 kw-hr. for 2 rooms
0 to 12 kw-hr. for 3 rooms
0 to 16 kw-hr. for 4 rooms.

This means, anything below the straight line *OA* in Fig. 8 is to be paid at 8 cents per kw-hr. For 5 rooms and upwards we have to add only 2.5 kw-hr. for each room, which means the 8-cent-per-kw-hr. zone reaches

from 0 to 18.5 kw-hr. for 5 rooms
0 to 21 kw-hr. for 6 rooms
0 to 23.5 kw-hr. for 7 rooms
..........etc.............., see line AM

The range below the line O*AM* is therefore straight meter rate at 8 cents per kw-hr. or $a = 8e$.

The 6-cent-per-kw-hr. zone (zone 2 in Fig. 8, consisting of 2*a* and 2*b*) reaches from this line up to the line *ON*, which represents 7 kw-hr. per room (7 kw-hr. for one room, 14 kw-hr. for 2 rooms, etc.). What is the equivalent three-charge system in this zone?

Dealing first with that part where the number of rooms is not greater than 4 (zone 2*a*), we find in the same manner as before (using the symbol *d* for the demand in rooms):

$$\begin{aligned} a &= (8\text{c/kw-hr.} \times 4d) + 6\text{c/kw-hr.}(e - 4d) \\ &= 8d + 6e \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1) \end{aligned}$$

The customer charge $x = 0$, the demand charge $y = 8$ cents per room per month, and the energy charge $z = 6$ cents per kw-hr.

Where the number of rooms is greater than 4

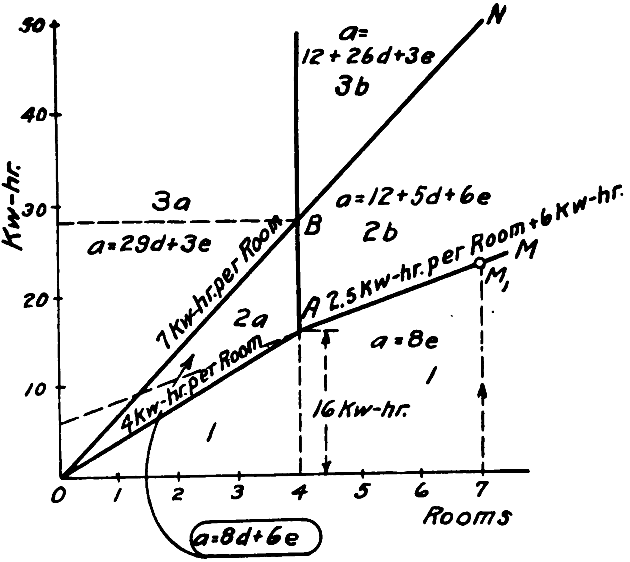

Fig. 8.—Analysis of Number-of-Rooms Rate.

(zone $2b$) we have to pay 8 cents per kw-hr. for a certain number f kw-hr. and 6 cents per kw-hr. for the balance, so that

$$a = 8f + 6(e - f) = 2f + 6e \quad \ldots\ldots\ldots\ldots (2)$$

where f is composed of two parts: 4 kw-hr. for every one of the first four rooms and 2.5 kw-hr. for every room in excess of 4, so that $f = (4 \times 4) + 2.5(d - 4) = 6 + 2.5d$. Substituting this into the equation (2) we get

$$a = 12 + 5d + 6e \quad \ldots\ldots\ldots\ldots\ldots (3) \text{ zone } 2b$$

We see, therefore, that for all customers in this range ($d > 4$ rooms) the equivalent of a three-charge system applies with

a customer charge $x =$ 12c per customer per month,
a demand charge $y =$ 5c per room per month, and
an energy charge $z =$ 6c per kw-hr.

Thus this is practically a Doherty rate, although the rate schedule mentions nothing but kilowatt-hour charges.

On the dividing line between the two portions $2a$ and $2b$ of the 6-cent-per-kw-hr. zone, that is, for $d = 4$ rooms, we can, of course, use either of the formulæ (1) or (3) as each one furnishes $a = 32 + 6e$ for $d = 4$. Likewise, we can, of course, use the formula for either zone on the dividing line between any two zones in any rate system, because the formulæ must of necessity furnish identical numerical results for the dividing line that lies between them.

The bill in the third zone (3-cent zone) where $e > 7d$ (zone 3, consisting of $3a$ and $3b$) is figured for $d < 4$ (zone $3a$ in Fig. 8) in analogy with what has been explained previously [using equation (1)] $a = 8d + (6 \times 7d) + 3(e - 7d) = 29d + 3e$ (equivalent of Hopkinson rate).

Or, if we wanted to work this out without reference to formula (1), applying to the second zone, we get $a = (8 \times 4d) + 6(7-4)d + 3(e-7d)$ with the same result as above.

For the third zone with $d > 4$ (zone 3*b* in Fig. 8) we get, using the formula (3) for zone 2*b*, as far as that zone applies, that is up to $e = 7d$, $a = 12 + 5d + (6 \times 7d) + 3(e - 7d) = 12 + 26d + 3e$; this means $x = 12$ cents per customer per month, $y = 26$ cents per room per month, and $z = 3$ cents per kw-hr. (equivalent of Doherty rate).

162. Another example for the working of this method will be given in the analysis of the High-Tension Wholesale Lighting and Power rate of Boston. This rate is a double-block Hopkinson demand rate as follows:

Demand charge
$60 per year per kw. for first 15 kw. of demand
36 per year per kw. for next 40 kw. of demand
30 per year per kw. for next 100 kw. of demand
15 per year per kw. for next 100 kw. of demand
12 per year per kw. for excess demand
plus an energy charge of
5 cents per kw-hr. for the first 1,500 kw-hr.
3 cents per kw-hr. for the next 4,000 kw-hr.
1.5 cents per kw-hr. for the next 50,000 kw-hr.
1.25 cents per kw-hr. for the next 50,000 kw-hr.
.... etc. ....

We will restrict ourselves to the investigation of the rate for such customers who come into the highest demand block ($d > 255$ kw., or the $12 demand block) and into the energy block of more than 55,500 kw-hr. per month (1.25-cents-per-kw-hr. block).

$$a = (6000 \times 15) + (3600 \times 40) + (3000 \times 100) + (1500 \times 100) + 1200(d - 255) + (5 \times 1500) + (3 \times 4000) + (1.5 \times 50{,}000) + 1.25(e - 55{,}500)$$
$$= 403{,}125 + 1200d + 1.25e \text{ (in cents).}$$

We see that we have in this rate a very considerable customer charge amounting, for instance, to *$4031.25 per month* per customer for all customers with a demand over 255 kw. and an energy consumption between 55,500 and 105,500 kw-hr. Yet the rate schedule mentions only demand and energy charges.

163. The great importance and extreme usefulness of a proper rate analysis is clearly demonstrated in the following example of a rate of a large company in the West which the present author analyzed in the *Journal of Electricity, Power and Gas* of July 27, 1912. Quoting from there: Under the rate the consumer had to pay a certain monthly fixed charge f cents per kilowatt maximum demand; let us say[8] \$4 per kw. ($f = 400$ cents). This entitles him to use current up to the load-factor L (for instance, $L = 300$ hours' monthly use of the maximum demand), the excess to be paid at z cents per kw-hr. (for instance, 2.5 cents per kw-hr). Without mathematical analysis, this rate looks perfectly correct and harmless, and it was not until the customers themselves happened to find out the defects and took advantage thereof that the rate was revoked. With a little mathematical analysis, as follows, the defects of the rate would have shown up glaringly at once and the rate would never have been put into effect.

In the range of load-factors smaller than L hours (300 hours) we have $a = fd$, where a is again the customer's bill and d his maximum demand; and if L is larger than 300, the customer with a total energy consumption of e kw-hr. has to pay, on top of this, for the excess of e over Ld kw-hr. at the rate of z cents per kw-hr., so that his total bill will be:

$$\begin{aligned} a &= fd + (e - Ld)z \\ &= (f - Lz)d + ez \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4) \end{aligned}$$

The first term on the right side is proportional to d and independent of e; the second term is proportional to e and independent of d. Consequently, $f - Lz$ is the equivalent demand charge and z is the equivalent

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[8] The figures are changed for obvious reasons, but in suc a way that the characteristic features of the rate are preserv

energy charge in the range of load-factors larger than L hours.

Substituting now the above values from the schedule for f, L and z, we get

$$a = (400 - 300 \times 2.5)d + 2.5e = -350d + 2.5e.$$

We thus have a *negative* demand charge of $3.50 per kw.!

In other words, all the customer has to do in order to reduce his bill—provided he has a load-factor of more than 300 hours' use per month—is to *increase* his maximum demand before the meter reader comes, with as little increase of the energy consumption as is feasible, until he has reduced the load-factor to 300 hours per month. For instance, let us assume his demand meter reads 50 kw. near the end of the month and his kilowatt-hour meter 30,000 kw-hr.; his bill therefore would be

$$\begin{array}{r} 50 \times 400 = 20{,}000 \text{ cents} \\ (30{,}000 - 50 \times 300) \times 2.5 = \underline{37{,}500} \text{ cents} \\ 57{,}500 \text{ cents} = \$575. \end{array}$$

If now, before the meter reader comes, the consumer for a short time purposely puts on a heavy load on his motors, or for instance lets them all work simultaneously (instead of with the diversity they show under ordinary operation) and thus runs his maximum demand meter up to 80 kw. with a practically unchanged kilowatt-hour consumption, he will have to pay

$$\begin{array}{r} 80 \times 400 = 32{,}000 \text{ cents} \\ (30{,}000 - 80 \times 300) \times 2.5 = \underline{15{,}000} \text{ cents} \\ 47{,}000 \text{ cents} = \$470. \end{array}$$

He has increased his demand by 30 kw.; therefore, his bill must be smaller by 30 × $3.50 = $105. This reduction of the bill with increasing maximum demand
se goes on until the load-factor is reduced to
rs.

To avoid this drawback with a rate of the type quoted we must choose *f*, *L* and *z* so that $f \lesseqgtr Lz$, then the demand charge will be $\lesseqgtr 0$, as equation (4) demonstrates. If, for instance, we maintain *f* at $4 per kilowatt and *z* at 2.5 cents per kw-hr., *L* must not be chosen larger than 400/2.5 = 160 hours per month. If *f* is just equal to *Lz* (in our case, if *L* is chosen at just 160 hours per month) the demand charge is zero and the rate for all customers with a load-factor above 300 hours is a straight kilowatt-hour rate; if $f > Lz$ ($L < 300$ hours' use) the rate for all load-factors above *L* is a Hopkinson rate. In both cases the rate for all load-factors $< L$ is a straight demand rate

II. Geometrical Rate Analysis.*

164. A rate system is nothing but a statement of the way in which the amount to be paid by the customer varies with his maximum demand and with his energy consumption.[1] We have

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*The principles of geometrical rate analysis were first brought out by the author in the 1911 report of the National Electric Light Association Rate Research Committee.

[1] Engineers and other readers who are more familiar with mathematics may skip Section 164, and substitute the following briefer statement in lieu of the same: Mathematically expressed a rate system is nothing but the function $a = f(d, e)$. This function contains three variables *a*, *d*, and *e* so that three dimensions are necessary for a graphical representation. Demand *d* and energy consumption *e* are used for the horizontal axes of an orthogonal system of axes in space and the amount *a* is stepped off in the vertical direction. We thus get a surface in space representing the function $a = f(d, e)$. The surface and the rate determine each other.

Every customer is characterized by his "characteristic point" *P* on the bottom plane. The co-ordinates of this point are the customer's demand and energy consumption. The locus of the characteristic points of all customers with the same load-factor is a straight line drawn from the origin at an angle λ with the axis of *d*, where tan λ is proportional to the load-factor *l*. Consequently where $l = 0$ this line is identical with the *d*-axis, whereas for $l = 100\%$ it is not identical with the *e*-axis, but includes a certain angle λ_{max} with the *d*-axis. The size of λ_{max} depends on the relation of the scales for *d* and *e*. Any portion of the bottom plane as well as of the surface in space for which $\lambda > \lambda_{max}$ has no meaning in practice.

Now all rate systems, with practically no exception, are of such a nature that the function $a = f(d, e)$ is of the first order so that the surface representing the rate is a plane, at least *for all* customers within a certain range of demand an therefore three variable amounts: the maximum demand *d*, the energy consumption *e* and the amount of the bill *a*. Some rate systems do not take cognizance of the demand or of the energy consumption (pure meter rates and flat demand rates, respectively). In those cases we have therefore only two variables and we can demonstrate the way in which one of the two variables depends on the other one, by a graphic representation in a plane (see Figs. 4 and 5). If we want to extend the graphic representation to rates that are based on both demand and energy consumption we must add a third dimension. The rate will be represented by a surface in space or in other words by a solid model.

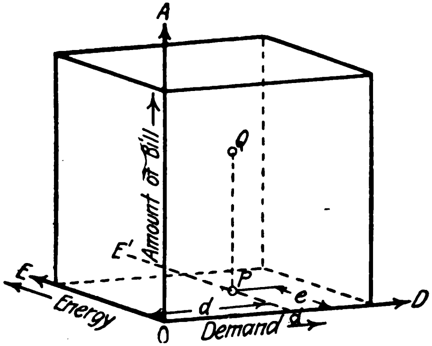

Fig. 9.—Principle of Three-Dimensional Rate Representation.

To explain this more fully: In Figs. 4 and 5 we stepped off the energy consumption to a certain scale in horizontal direction and the amount of the bill in vertical direction. We had, as it is termed, two *axes* standing normal upon one another. Now we

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energy consumption. For other ranges we get another function for $f(d, e)$ which also, however, is of the first order so that the surface for all customers is a combination of planes.

will have to add the demand in another horizontal direction (assuming this page to be held vertically in front of the reader) normal to the direction in which the energy consumption has been stepped off, that is normal to the plane of the paper. We thus get three axes *OA, OD* and *OE,* of which each one stands normal to the two others, like three edges of a cube which meet in one point (Fig. 9). Given now a customer with a certain demand *d* and a certain energy consumption *e*, we choose a scale for the demand, for instance we say that every inch shall represent 10 kw. We then measure off the number of inches which corresponds to the given demand *d* from *o* in the direction towards *D*. Let this length be *Od* and we thus arrive at the point *d*. From this point we draw a parallel to *OE,* shown as *dE'* in Fig. 9. In the same manner as above we choose a scale for the energy consumption and step off on the parallel *dE'* the length *dP* corresponding to the energy consumption of the customer. We thus reach the point *P*.

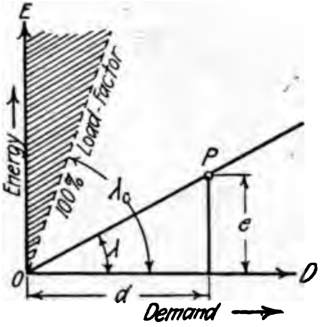

Fig. 10.—The Bottom Plane of a Rate Model.

Point *P* is called the characteristic point of the customer. As soon as the customer changes either his demand or his energy consumption, or both, his characteristic point will shift to another position. Fig. 10 shows the bottom plane *ODE* viewed vertically downwards from above. If we connect *P* with the origin *O* by a straight line all customers whose characteristic points are situated anywhere on this line (or its production beyond *P*) evidently have the same load-factor *e/d* as the customer with the point *P*. The smaller the angle λ is the smaller is the load-factor of all the customers whose characteristic points lie on that line. For the load-factor $l = e/d = 0$ the angle $\lambda = 0$, but for $l = 100\%$ the angle λ is not 90°

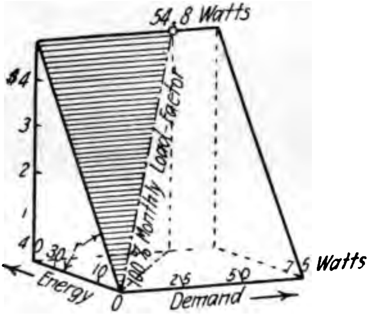

Fig. 11.—Straight Meter Rate.

as might be expected on superficial observation, but it reaches a certain maximum value λ_{max} which is smaller than 90°. Just how large it is depends on the scales which have been chosen from *d* and *e*, but it must of necessity always be smaller than 90°.

If we step off now in the vertical direction from the characteristic point of a certain customer the amount *a* (to a certain scale which we are free to choose) we get a point in space above the plane (*Q* in Fig. 9). The position of this point in space determines the customer's demand, his energy consumption, and the amount he has to pay for both.

If we now do the same thing as just described for every point on the bottom plane the upper ends of all the vertical lines will form a surface in space which determines and represents the demand-energy rate in the same manner as the plane curves of Fig. 4 or 5, for instance, represent pure meter rates. Instead of the surface we can also speak of the total space between the surface and the bottom plane, that is of a solid or of a solid model, always meaning, of course, in this case the upper surface of the model.

A few examples will make this clearer.

165. One of the simplest cases is the straight meter rate. The charge is proportional to the energy consumption and independent of the demand. This rate is obviously represented by a plane passing through the demand axis and sloping at a certain angle r from the bottom plane upwards. Fig. 11 is, for instance, the model of a 12-cent per kw-hr. rate.

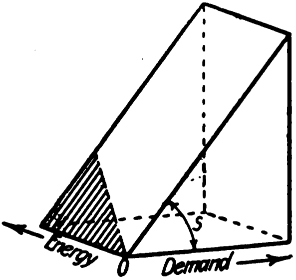

Fig. 12.—Flat Demand Rate.

In designing a model of a rate we have first to decide over what portion of the bottom plane the model is to extend. Fig. 11, for instance, extends over the range from 0 to 75 watts demand and from 0 to 40 kw-hr. energy consumption. At $e = 40$ kw-hr., that is, at the rear end of the model the amount to be paid is $12 \times 40 = 480$ cents ($4.80) for any demand d.[2] The part of the solid which corresponds to load-factors of more than 100%, and which therefore has no practical meaning, is shaded in the drawing of this and the following models. It is found in the following manner: For 40 kw-hr., for instance (rear plane of the model) we have 100% load-factor (730 hours' use per month) at $d = 40{,}000$ watt-hours/730 hours = 54.8 watts. Place a vertical plane through this point and the origin O; the line of intersection of this plane with the surface of the model is the 100% load-factor line.

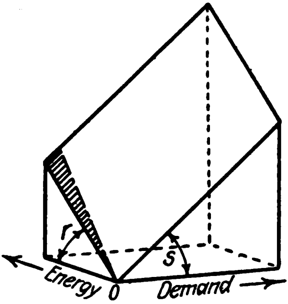

Fig. 13.—Hopkinson Rate.

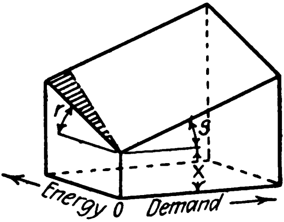

Fig. 14.—Doherty Rate.

A similar surface or solid would represent a flat demand rate except that this model would have to be turned around by 90° to the former, see Fig. 12.[3]

Supposing now that we have a Hopkinson rate, then it becomes clear that we will get a combination of the planes shown in Figs. 11 and 12. The plane will be in such a position in space that the angle r corresponds to the energy charge and the angle s

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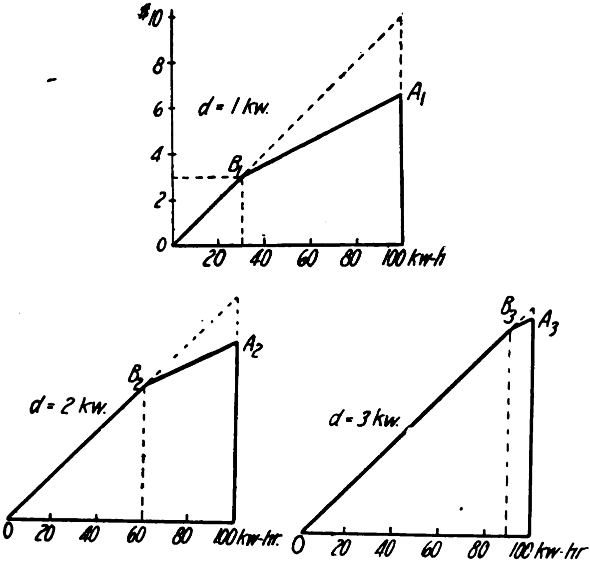
[2] By mathematics: $a = f(d, e) = ze$, that is the equation of a plane passing through the axis of d and sloping at an angle r from the bottom plane, where $\tan r = z$.

[3] Mathematics: $a = f(d, e) = yd$, that is the equation of a plane passing through the axis of e and sloping at an angle s from the bottom plane, where $\tan s = y$

to the demand charge (Fig. 13). A large energy charge will result in a large angle *r* and vice versa. The same applies to the relation between the demand charge and the angle *s*.[4]

The next example is logically the Doherty three-charge rate. The difference from the Hopkinson system is that a customer charge of constant size (to be



Figs. 15a, 15b and 15c.—Wright Demand Rate—Monthly Bills for Various Customers With Various Maximum Demands.

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[4] Mathematics: The equation of the plane in space is $a = yd + ze$. (It should be remembered that the co-ordinates are here not called x, y and z, as usually, but d, e and a, respectively, whereas x, y and z are co-efficients). The trace on the a-d plane of co-ordinates is found by setting $e = 0$ in this equation, resulting in $a = yd$. Likewise the trace on the a-e plane is found as $a = ze$. These two traces rise therefore at angles r and s from the horizontal which are determined by $\tan r = z$ and $\tan s = y$. The traces determine the position of the plane in space.

called x) is added to the charges of the Hopkinson system.[5] This means that the plane of the Hopkinson system will simply have to be lifted up parallel to itself over a distance corresponding to the amount x (to the scale previously chosen) and if the demand and the energy charges are to be lowered the angles r and s will have to be made smaller. The plane or model will assume the general shape of Fig. 14.

166. Without going into theoretical details[6] we will investigate now how the plain Wright demand rate looks in this space representation, for instance a rate charging 10 cents per kw-hr. for the first 30 hours of use and 5 cents per kw-hr. for the balance. Assuming first a customer with a fixed demand of 1 kw. it is evident that the quoted Wright rate will be a block meter rate for all such customers (demand 1 kw.) charging 10 cents per kw-hr. for the first 30 kw-hr. and 5 cents per kw-hr. for the excess (See Fig. 15a, also Appendix XII). For every customer with

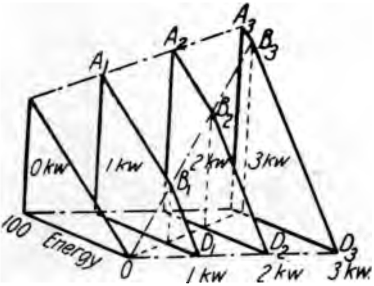

Fig. 16.—Evolution of Model of Wright Demand Rate.

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[5] Of course, if this shall not simply mean making the rates simply so much higher to every customer, the kilowatt-hour charges or the kilowatt charges, or both, will have to be lowered, just as the kilowatt-hour charges of a straight meter rate have to be lowered if we change over to a Hopkinson rate by adding *a demand* charge.

[6] *For these see* the author's article on "Space Representation *tral-Station* Rates," *Electrical* World, Nov. 4, 1911.

2 kw. demand the Wright rate is the equivalent of another block meter rate, viz., 10 cents per kw-hr. for the first 60 kw-hr. and 5 cents per kw-hr. for the excess (Fig. 15b). For 3 kw. demand we get Fig. 15c, etc.

Supposing now that we cut these diagrams, Figs. 15a, 15b, 15c, etc., out of stiff cardboard and arrange them behind one another in proper order and at distances from each other corresponding to 1 kw., as shown in Fig. 16. The interstices between these cardboards are then filled in by more cardboard diagrams representing the rate for customers with fractional-kilowatt demands, for instance 1½ kw., then 1¼ and 1¾ kw., etc., at the proper spacing, until we finally get a solid block of cardboards (Fig. 17). Then this block evidently represents the space model of the rate. Every point on the bottom plane is determined by the customer's demand and energy consumption and the vertical distance from there to the upper surface is the amount paid by the customer.

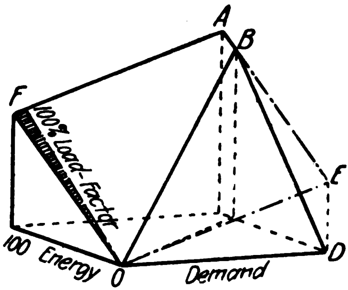

Fig. 17.—Model of Wright Demand Rate.

This surface consists of two planes *ODB* and *OBAF* which intersect in the straight line *OB*. Plane *ODB* represents a straight meter rate. OBAF is a

Hopkinson plane (compare Fig. 13), as can be easily seen if we produce that plane until it intersects the vertical plane passing through axis OD (point E and dot-and-dash lines in Fig. 17). The line of intersection OB between the two planes where the 5-cent charge begins, corresponds of course to the 30-hour load-factor, this means that if we project it down on the bottom plane we get the 30-hours load-factor line.[7]

To construct this model the simplest way will be to proceed in the manner indicated above by the cardboard diagrams. We would place the last cardboard $D_3B_3A_3$ at the proper distance from the origin O (Fig. 17) and pass the plane ODB through DB and

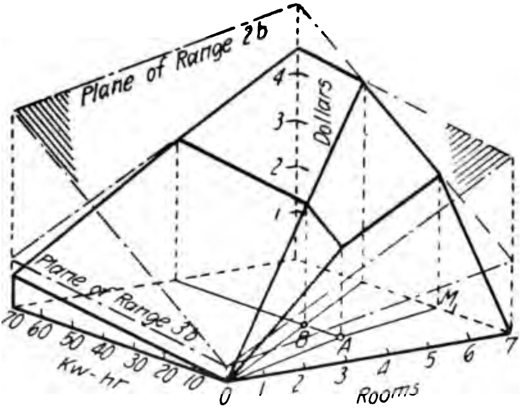

Fig. 18.—Model of Number-of-Rooms Rate (12-6-1c/kw-hr.).

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[7] The mathematics of the case is as follows: Call p the primary charge in cents per kw-hr. and s the secondary charge and let l be the load-factor up to which the primary charge applies. Then we have in the primary range $a = pe$ and in the secondary range $a = p\,l\,d + s(e - ld) = (p - s)ld + es$. This means, in the secondary range we have a demand charge of $(p - s)l$ and the energy charge is s cents per kw-hr. We will therefore have in the primary range a straight meter plane and in the secondary range a Hopkinson plane with tan $r = s$ (secondary energy charge) and tan $s = (p - s)l$.

the point *O* (primary plane) and then pass *OBAF* (secondary plane) through *O* and *AB*, or through *O* and *AF*. A check of the correctness will hardly be necessary in this simple case, but we could check whether the secondary plane *OBAF* actually is identical with the Hopkinson plane as computed.

167. Turning now to a somewhat more complicated example, that of the St. Louis Residence Lighting rate, described and partly discussed in Section 161, we have in range 1 (see Fig. 8) a straight meter rate, of course. Ranges *2a* and *3a* have been found by the algebraical analysis to be represented by Hopkinson planes. The rate schedule is therefore equivalent to a simple three-block Wright demand system for all customers whose demand element does not exceed 4 rooms. If the demand is greater than 4 rooms, we have in range *2b* a Doherty plane (see the inscription in range 2b of Fig. 8). This plane may be constructed in several different ways, one of which is this: The point of the rate surface corresponding to point *A* in Fig. 8 must be a point of the plane under construction. It belongs to range 1 as well as to range *2b* (and incidentally to *2a*). The number of rooms corresponding to this point is 4, consequently the number of kilowatt-hours is 4 × 4 because the point lies on the dividing line. The amount to be paid is $4 \times 4 \times 12 = 192$ cents,[8] computed from range 1. We have thus located one point of the plane for the range *2b* and in an entirely analogous way we can determine two more points of the plane, for instance those corresponding to *B* and M_1 of Fig. 8. Points on the dividing lines are of course preferable because they can be used for the determination of more than one plane and thus the work is simplified.

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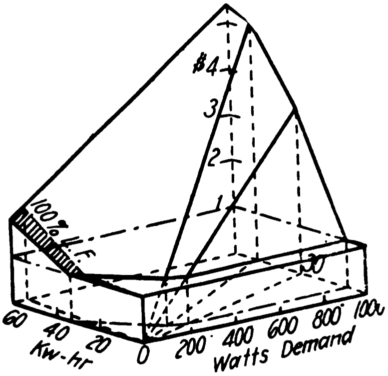
[8] To exhibit more plainly the characteristic qualities of this type of rate, the St. Louis rate has been arbitrarily modified for the construction of the model Fig. 18 from a 8-6-3-cent rate to a 12-6-1-cent rate which makes the angles between the various planes greater. The ground plane is unchanged.

The whole model of the St. Louis residence rate (modified as per footnote 8) looks like Fig. 18.

168. The stipulation of a minimum bill in a rate schedule means that, whatever the rate surface may be, no parts of it shall be considered which are nearer to the bottom plane than the distance corresponding to the amount of the minimum bill. We therefore draw a horizontal plane at that distance above the *bottom plane* and wherever that plane is higher than



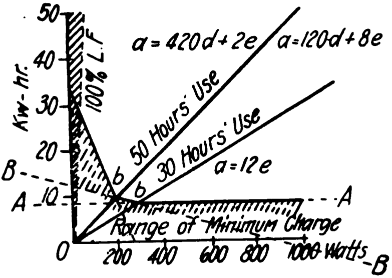


Fig. 19.—Minimum Charge.
Fig. 19a.—Plan of Fig. 19.

the surface of the model it replaces the latter. To demonstrate how this works out, Fig. 19 shows a model of a Wright demand rate[9] with a minimum charge (of \$1), indicating also in the ground plane Fig. 19a in which range the minimum bill applies.[10]

Fig. 20 is the same Wright rate with the stipulation of a certain minimum number of kilowatt-hours instead of a minimum bill. Suppose, for instance, the rate would say that, whenever the actual energy consumption is less than 20 kw-hr. it shall be figured as being 20 kw-hr. for billing purposes. Then we have to draw a vertical plane $PQQ'20$ normal to the axis of energy and at a distance from the origin O which is equal to the stipulated minimum number of kilowatt-hours (20 kw-hr. in our example). From every point of the line of intersection between this vertical plane and the rate surface we draw a horizontal line parallel to the energy axis. All these horizontal lines together then form a set of planes $pp'q'q$, $qq'r'r$ and $rr's's$ (Fig. 20) which determine the minimum charge. Fig. 20a shows the range of the minimum charge in this case.

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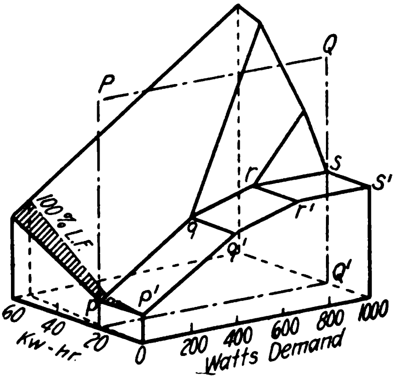
[9] Assumed charges are 12 cents per kw-hr. for the first 30 hours' use, 8 cents per kw-hr. for the next 20 hours' use and 2 cents per kw-hr. for the excess.

[10] This range in which the minimum bill applies can be found either graphically from the tracing of the model itself or, if greater accuracy or a check would be required, it can also be computed, in the following way:

In the primary range where $a = 12e$ we get by setting $a = 100$ cents, $12e = 100$ and $e = 100/12 = 8\ 1/3$ kw-hr.; this means a straight line AA (Fig. 19a), normal to the axis of energy consumption (kw-hr.) and at the distance 8 1/3 kw-hr. from the origin. In the secondary range we have the equation of the rate surface $a = (12 \times 30d) + 8(e - 30d) = 120d + 8e$ and setting this = 100 cents we get $120d + 8e = 100$. This is the equation of the straight line BB in Fig. 19a. The portion bb between the 30-hour and the 50-hour load-factor lines determines the range of the minimum charge in the secondary range of the rate. To construct this line $120d + 8e = 100$ we find its intersection points with the axes of abscissae (watts) and of ordinates (kw-hr.) by setting first $e = 0$ and then $d = 0$. $e = 0$ gives $120d_o = 100$ or $d_o = 100/120 = 0.8333$ kw. = 833 1/3 watts on the axis of demand. $d = 0$ results in $e_o = 100/8 = 12\ 1/2$ kw-hr. on the axis of energy consumption. In analogous manner we can find the range of the minimum charges in the tertiary range of the rate.

We see from this at once the difference between a minimum bill and a minimum number of kilowatt-hours for billing purposes (see end of Section 108).

The stipulation of a minimum demand which is also sometimes found in practice results in an analo-



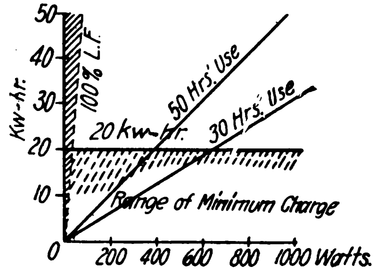
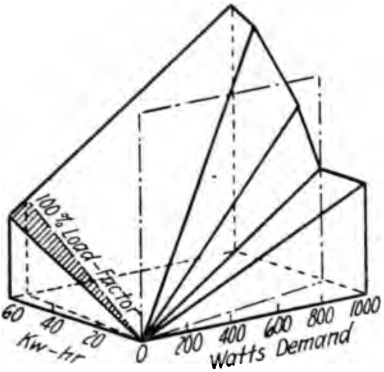


Fig. 20.—Minimum Energy Consumption.
Fig. 20a.—Plan of Fig. 20.

gous set of planes to those shown in Fig. 20, but on the kilowatt-hour side of the model instead of on the kilowatt (or watt) side.

The stipulation of a minimum load-factor to be taken into account for billing purposes results (with the same simple Wright demand schedule) in a model like Fig. 21 (no load-factor considered for billing purposes smaller than 20 hours' use per month).

It requires no further detailed explanation to show



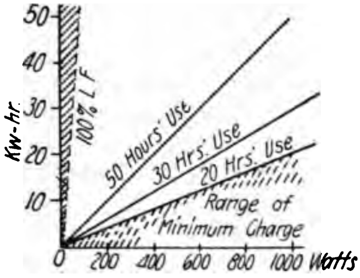


Fig. 21.—Minimum Load-Factor.
Fig. 21a.—Plan of Fig. 21.

how a combination of minimum charges expresses itself in space representation. For instance, it may be stipulated that under the Wright schedule just discussed (Figs. 19 to 21) the minimum number of kilowatt-hours to be paid must correspond to a monthly use of the maximum demand of at least 20 hours, and that moreover no bill shall be made out at less than \$1. The two minimum-charge steps of Fig. 19 and Fig. 21 will then combine in such a manner that that step applies which is the higher one of the two at the respective point of the bottom plane, in other words on the left portion of the model (small demands) the \$1 minimum step (Fig. 19) will apply and for larger demands the load-factor-minimum applies. The demand at which the two "minimum" steps intersect one another can be found either graphically or by computation as

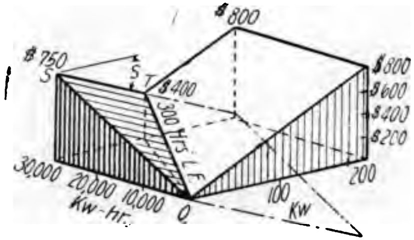


Fig. 22.—Negative Demand Charge.

$$d_1 = \frac{\text{100 cents per month}}{\text{12-c/kw-hr.} \times \text{20 hours per month}}$$

$$= 0.4166 \text{ kw.} = 416\ 2/3 \text{ watts.}$$

169. Fig. 22 shows a model of the rate referred to and analyzed in Section 163. It demonstrates clearly how the plane for load-factors above 300 hours (plane *OST*) slopes in the wrong direction so that a negative angle *s* (that is, a negative demand charge) results.

170. Only simple rates have been dealt with here as far as their space representation is concerned. The *more complex* a rate system is the greater is the

advantage of a graphic representation for a clear insight into the meaning and the character of the rate.

More about this system of rate representation is contained in the author's article on "Space Representation of Central-States Rates" in the *Electrical World* of Nov. 4, 1911, which also contains many examples of various kinds of rates and numerous photographs of a large number of actual models of rates as had been made for the purpose of investigating these rates. It is, of course, not strictly necessary to have the models actually made in three dimensions. An axonometric drawing, of the type of Figs. 17 to 22, or even pure imagination how the model looks, will also prove very helpful.

PART V

ACCURACY OF RATES

171. We have seen that, regardless of whether we make the prices on the basis of the cost-of-service principle or the maximum-earnings principle or the value-of-service principle, there are always certain prices for every particle or for every class of service. These prices vary with the class of service according to the variation of the cost and, except in case of the cost-of-service principle, according to the variation of the profit.

Referring to the system of graphical rate representation explained in the preceding Sections (164-170) we can say that the *cost* of the various consumers (at least within one certain class of service) is roughly a Doherty plane. If the percentage of profit were to be constant for all sizes of customers this would result in another Doherty plane for the *prices* (rates) which is everywhere higher than the cost plane by just that percentage.

According to Sections 97-99 it will be of advantage for the producers as well as for the consuming public if the small consumers and the large ones get prices which are based on smaller profits than those charged to the consumers of medium size. Then the price (rate) surface will become curved, with the convex side pointing upwards.

We cannot follow this ideal surface exactly by the rate schedule, in the first place because we do not know the surface exactly and secondly because we do not want the rate system to become too complicated. The surfaces of most rate systems show a more or less close approximation to this vault-like curvature of the ideal surface by a substitution of a combination of planes in lieu of the curved surface. We see this approximation to the curvature for instance in the Wright demand rates, especially in those with a greater number of blocks, and still better in the double-block Hopkinson rate. (For photographs and drawings of models of this system, see H. E. Eisenmenger, "Space Representation of Central-Station Rates," *Electrical World,* Nov. 4, 1911, Figs. 13 and 14.)

The very fact that we have only an approximation to the ideal rate surface implies that we have certain inaccuracies of the rate. The rate will be just at its ideal or theoretical value at certain points of the bottom plane, that is for certain combinations of demand and energy consumption, but it will be higher than it ought to be at other points, and again lower at others. The smaller these inaccuracies are (with reasonable simplicity of the rate schedule) the better for the consumer and the producer.

172. The reason why every inaccuracy of rates is of disadvantage to both the consumer and the producer is this: Suppose certain customers (to be called Class A) are charged more than what is intended and would be theoretically correct, that is, more than what is to the best interest of all parties concerned (see Part II, Section 78 *et seq.*, "The Value-of-Service Principle"); other customers (Class B) are charged less than their proper amounts. The result will be that some of the customers of Class A will drop out or change the character of the service they require, in such a manner that they become Class B customers or approach to Class B customers. An inaccuracy of rates has therefore the following effect: Whatever class (or classes) of customers or service is affected in such a way by the inaccuracy that its rates and therefore its profitableness is reduced below the theoretical and intended amount, it will always be just that class which is increased in number and volume, and conversely just that class whose profitableness would be increased by the inaccuracy will—at least partly—disappear. Therefore every inaccuracy of the rate will of necessity reduce the profits and thus create a tendency towards raising the prices.

173. A separate and frequent case of rate inaccuracy is the case where one or two of the three charges (customer, demand, energy) are left out. In fact, as has been shown (Section 10) even rates which embody all three charges are necessarily inaccurate because an accurate rate would require an indefinite number of charges.

If, for instance, we suppress the demand charge (as in pure meter rates) the customers will get the use of the demand for nothing and consequently they will not care how large their demand is; they will be wasteful with their demand, so to speak. A customer using, for instance, 50 kw-hr. per month under a pure meter rate will have a larger maximum demand than a customer using 50 kw-hr. under a rate system where a charge is made for the demand. The consequence will be a smaller load-factor. For a given kilowatt-hour consumption the central station has to keep in readiness a large generating capacity, etc., which reacts unfavorably on the cost and therefore on the price per kilowatt-hour.

On the other hand, if no energy charge is made (flat demand rate) the customers will be careful to keep down their maximum demand, or its rating-equivalent, respectively (capacity connected or connected load). In any case they will be far less careful about their number of kilowatt-hours (burning hours) than under a system of charging which makes a charge *for every* additional kilowatt-hour actually consumed.

'v *check* which remains against absolute extravagance (outside of a feeling of fairness towards the central station on the part of a portion of the customers) is the cost of lamp renewals where these must be borne by the customer. The result will be that the load-factor of customers with a flat demand rate will be large.[1] A superficial observer who has accustomed himself to think in terms of average cost per kilowatt-hour may consider this an advantage because he knows that a large load-factor results in a low cost per kilowatt-hour. It must not be forgotten, however, that just in this case (flat demand rate) the central station is not paid for the additional kilowatt-hours furnished. The element that builds up the revenue is in this case exclusively the maximum demand and the larger the load-factor is for a given maximum demand the larger is the energy consumption for which the central station has to provide without getting any return.

Again, taking the case where the customer charge (and its substitute, the minimum charge) is left out of the rate schedule, this means that the very small customers will pay less than it costs to serve them; they can be carried only at a loss and, on the other hand, we have to give them service if they demand it. A customer charge or a minimum charge will either keep these unprofitable customers away or else raise their bills high enough so that they are turned into profitable customers. We can, in case of a suppression of the customer charge, not say that the *individual* customer is wasting anything, as in the case of the pure meter rate and of the flat demand

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[1] According to the information obtained by S. E. Doane and the author in Milan, Italy, it has been found in that city by using check meters that the average load-factor for small flat-rate customers was 2270 hours per year. The demand charge must therefore be large enough to cover the cost of this waste of kilowatt-hours, but as the increment energy cost per kilowatt-hour is very small the large reduction in the customer cost (see Section 128) and the other advantages make this system economical in spite of the waste of kilowatt-hours.

rate, but the general average of the customers is wasteful, so to speak, without the customer cost, inasmuch as the small customers (whose customer cost would be a large percentage of their total cost) are unduly attracted.

It is always the element which is offered gratuitously or excessively cheaply with which the customer will be extravagant and in this manner the public service company will suffer damage from an inaccuracy of the rates and as the losses must, at least to a certain degree, ultimately be paid out of the customers' pockets the customers will suffer also.

174. It must not be overlooked, however, that the inaccuracies of the rate weigh differently in different places of the bottom plane. For illustration, if we have a residence rate it is necessary that it be accurate for small consumers but it does not matter if there is even a large inaccuracy for a residential customer of, say, 10 kw. demand and of some 2000 kw-hr. energy consumption per month because there will be very few, if any, residential customers who have a demand of that size and certainly none with a load-factor of that magnitude. A retail power rate, on the other hand, must be accurate for just that type of customers and its accuracy for the very small customers is not of such importance.

Fig. 23 shows a model constructed by the author several years ago in the course of an investigation of the rates of a company operating in a large city on the Pacific Coast. This model refers to the distribution of the residential customers of that company with reference to the size of the customers. The two horizontal axes of this model are kilowatts and kilowatt-hours, the same as in the rate models, but the vertical dimension is used for stepping off the number of residential customers. The ground plane (bottom plane) has been divided into squares (or rectangles) by using *certain steps* for the kilowatts as well as for the kilowatt-hours and then the number of residential customers to be found in each square in the territory of the company has been stepped off as, vertical ordinate.

Fig. 23.—Distribution of Consumers of Different Sizes.

The model shows that in one of the "squares" the number of customers in existence is very much larger than in any other square and it further shows that practically all residential customers are crowded together—so to speak—in a very few, perhaps six, of the squares so that anything outside of these six squares is of little or no importance and the accuracy of the rate outside of these six squares is of no great consequence. A similar model has also been constructed, showing in the same manner the income from the various sizes of customers instead of their numbers in each group. The shape of this model which has a still greater bearing on the accuracy of the rates is, of course, in general similar to the one shown in Fig. 18 except that the maximum is moved a little further away from the origin.

PART V

PUBLIC SERVICE RATE REGULATION

CONTRIBUTED BY S. F. WALKER,
Formerly Associate Editor of Rate Research.

I. PUBLIC UTILITIES AND THE PUBLIC INTEREST.

WHAT ARE PUBLIC UTILITIES?

There is no generally accepted or legal definition of a public utility, or, looking at it another way, there is no generally accepted authority showing just what business enterprises may be brought into the program of public utility regulation and what industries may not be subjected to public service regulation as it has been set up in the various states.

In practically all of the state laws, public utilities have been defined by enumeration, the law stating, for example, that railways, electric, gas, telephone and water companies are to be subject to the regulatory powers vested in the commission. The policy of state regulation of public utilities has had a rapid extension. It has gone from state to state until every state, except Delaware, has some form of a railroad or public utility commission, and in various states the law has been amended year after year to include additional business enterprises. The question is, just where must this extension stop because the remaining unregulated businesses are not public utilities or are not subject to public regulation.

Franchise Companies.—Some authorities have held that public utilities are those companies which hold franchises to use the public streets. But regulation has gone beyond that limit.

Noncompetitive Enterprises.—Franchise companies were protected from competition, more or less. At least the field was not easily accessible to every one willing to come in and compete for business. On this basis some authorities have said that public utilities were subject to regulation because competition did not regulate their rates. But there are many monopolistic enterprises able to nullify the effect of competition in naming their prices that are not thought of in the same breath with public utilities. And, on the other hand, competition is a factor, often a ruling factor, in the rates of public utilities. There have been rate wars between public utilities that have driven rates lower than any commission would feel justified in imposing upon the companies, and it has often been one of the first tasks of a commission to bring such rate wars to a close and raise the rates to enable the companies to operate properly. The public utility rates are also affected by competition with other products. For example, electric companies are in competition with gas companies and with private plants for certain classes of service.

Commissions that have adhered strictly to the cost-of-service theory and have recorded their disapproval of "value of service" and "what the traffic will bear" methods, have often encountered situations where the theory cannot be followed, but where the rates for certain classes of the service are determined by the competition of other sources of fuel, light, power or whatever it may be.

Public Interest.—Taking refuge in a broader definition, public utilities are those companies whose business is clothed with a public interest.

The principle of public regulation was applied in early times to innkeepers, ferries, turnpikes, blacksmiths, and surgeons, because public welfare at that *time was dependent* upon obtaining these services at *sonable rates.* This power to regulate in the

interest of the public is held to be one of the inherent powers of the state, so much so in fact that the states [redacted]n held
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interest of the public is held to be one of the inherent powers of the state, so much so in fact that the states cannot bargain this power away. It has been held that contracts, special charters and franchises granted a company or institution cannot debar the state from later imposing regulations contrary to the terms of such special grant, if it is in the interest of the public so to do.

EXTENSION OF PUBLIC REGULATION.

The state of Massachusetts established a Railroad Commission as early as 1869, and before 1890 there were railroad commissions in California, New York, and Iowa. The Interstate Commerce Commission was established in 1887.

Commission control of public utilities as it is known today may be said to have started with the Wisconsin and New York laws passed in 1907. Wisconsin established its Railroad Commission in 1905, and extended its power by the public utilities act of 1907. The New York law passed in the same year provided similar regulations of public utilities and other states have followed rapidly, keeping in general to the regulation of electric, gas, water, telephone, street railway, railroad, telegraph, and express companies, until the idea of public utility has been associated with these industries in a peculiar degree.

These industries might well ask, "Why is it so particular with us?" The business of furnishing food and clothing is certainly "clothed with public interest," and in fact, during the war period especially, has been subjected to regulation by administrative commissions in the interest of the public.

Commission regulation is not limited to natural monopolies and companies holding franchises, but may be extended at any time to such industries as public interest may dictate. There is no legal holding to the *contrary*.

In Oklahoma the Corporation Commission, provided for in the constitution of that state, was given general powers of regulation over all industries clothed with a public interest.

The provision of the Oklahoma constitution and the statute laws of that state relating to the establishment of the Corporation Commission and defining the powers of that body, while mentioning particularly railroad and transmission companies, include the following section going beyond any attempt at enumeration of public utilities subject to regulation.

Chapter 38, Article I, Section 13, Oklahoma Statute Law (1908): "Inevitable monopolies declared subject to price regulation by the state. Whenever any business, by reason of its nature, extent, or the existence of a virtual monopoly therein, is such that the public must use the same, or its services, or the consideration by it given or taken or offered or the commodities bought or sold therein are offered or taken by purchase in such manner as to make it of public consequence or to affect the community at large as to supply, demand or price or rate thereof, or said business is conducted in violation of the first section of this act (restricting combination in form of trust or conspiracy in restraint of trade or commerce within the state), said business is a public utility and subject to be controlled by the State, by the Corporation Commission or by an action in any district court of the State, as to all of its practices, prices, rates and charges. And it is hereby declared to be the duty of any person, firm or corporation engaged in any public business to render its services and offer its commodities, or either, upon reasonable terms without discrimination and adequately to the needs of the public considering the facilities of said business."

An enumeration of industries in the different state laws has led to a more artificial idea of public utility regulation. While in one city or town the people are in need of having their electric company or street-railway company regulated, perhaps in other towns the people are more in need of having a meat market, a grocery, a warehouse, stockyard or perhaps even the dentist, the doctor or the lawyer regulated than the so-called public utilities.

Sometimes the commission regulation planned for companies known as natural monopolies has been extended to other classes of business under the same law—warehouses and cotton gins, for example, while other methods of regulation have appeared as anti-trust laws, food and drug acts, and special laws regulating the rates of insurance companies.

Just what the future limits of public utility regulation by commission will be cannot be foretold, but the tendency appears to be toward extension of the regulatory policy rather than toward restriction.
to serve the best interests of the public and is given

PURPOSE OF REGULATION.

The main purpose of regulation may be briefly stated as the securing of adequate service at reasonable rates, without unjust discrimination in either the furnishing of service or in the rates charged.

Proper regulation is concerned with the protection of the company, on the one hand, as well as the protection of the public, on the other.

Service.—The commission may determine what extensions and improvements in service are required the power to order such extensions and improvements where the company has failed to make them of its own initiative.

However, the company is protected from unreasonable demands which would increase the investment in its property out of proportion to the public benefit secured.

The protection of the company against unreasonable service demands to the benefit of the general public has been most marked during the period of the war. Service improvements have been postponed, further extensions discontinued, requirements lightened, all at a time when the high cost of materials and labor would have imposed a high investment as a permanent burden upon the utility service.

Rates.—The public is entitled to service at reasonable rates, and the utility is entitled to a fair return on its investment and to compensation for good management. Rate regulation is only one of the phases of regulation, although it has been given the first place in public attention. Poor service at low rates may not be as good a bargain for the public as good service at high rates. The rates must be such as to afford proper inducement to capital and managing ability and to secure the benefits of new inventions and improvements in the industries concerned.

Rate Reductions.—The introduction of regulation was followed by a period of rate reductions. The public expected this of the commissions established. There were many factors making it possible, perhaps, for the various utilities regulated to withstand these rate reductions without harm. Under regulation the rights and duties of these companies were more clearly defined, thus giving them a more stable position. The stocks and bonds of the companies issued under commission approval had a ready market. The experimental period of development was over and the companies were entering upon a period of regular development. The companies were in many instances freed from losses from free service and discriminatory rates held over from the days of bargaining. And finally, a most important factor, rate reductions brought increases in business.

Increasing Rates.—During the period of rising prices, under war conditions, the commissions have found it necessary to grant increases in public utility rates. While there have been disappointments and delays in granting what companies have considered justifiable increases in some cases, it is undoubtedly true that increases in the emergency were secured more generally under commission regulation than could have been secured by bargaining with local authorities.

Reasonable Rates.—It is the duty of a commission to determine reasonable rates—not just nonconfiscatory rates. The courts had the power to prevent the exaction of a confiscatory rate, but the commission must go further and prescribe reasonable rates affording a fair profit to the company above all legitimate expenses of operation.

II. Regulation by Commission.

UTILITY COMMISSIONS MUST BE DISINTERESTED AND UNBIASED ADMINISTRATIVE BODIES.

Commissions are not courts of punishment, but are administrative bodies intimately related to many phases of the utility business. A commission should not usurp the position of management in determining the development and operation of a utility, but as an administrative body, act in a corrective and advisory capacity. The underlying authority for the regulatory powers vested in commissions is the common-law principle that all business affected with a public interest is subject to public regulation.

Unbiased Judgment versus Home Rule.—The difficulty in regulation of public utilities by local franchises was that the municipality was a party to the contract and at the same time acted as the regulatory body. Therefore, free service to the municipality was exacted of the companies, heavy paving requirements, special taxes, unprofitable extensions and other unjustifiable burdens were placed upon the companies to their detriment and also to the detriment of the general public served by the utilities.

A statewide interest as against the companies operated in several states, such as railroad companies, natural-gas companies, or interstate power companies, may affect the judgment of the state commission to shade the findings in favor of a low rate for service within the state.

In order to secure an unbiased point of view the regulating body must be the agent of a larger unit than that affected by the regulation. State commissions pass upon the rights and privileges of the city and the local companies; the Interstate Commerce Commission deals with matters affecting the often conflicting interests of different sections of the United States.

In the future certain public utilities may outgrow the limits of state commissions and provision may be made for the delegation of matters affecting interstate operation of power-transmission companies, natural-gas companies and similar industries to an interstate or national commission.

Another consideration which favors regulation by state commission rather than by the municipality is that men with the desired education and training, capable of exercising the broad administrative powers affecting various industries, cannot readily be secured for local commissions. Furthermore, extensive investigations are often called for in determining proper standards of service, in adopting a policy of rate making or in deciding the advisability of adopting new inventions and improvements in operating conditions, which investigations can be conducted by the state body for the benefit of the industry in the entire state. The necessary equipment, such as meter-testing equipment is not often available to the local body, but may be acquired by the state commission for use throughout the state, or the state commission may use certain equipment jointly with other state departments or the *state university.*

Special Laws.—Prior to the establishment of commissions, special laws had been passed in some states, fixing a rate or standard of service for individual companies or for the railroads, gas, electric or other public utilities throughout the state.

This sort of regulation is too haphazard and limited in scope. Furthermore, it has the objection obtaining in the case of regulation by local franchise, that of inflexibility. The rate or service requirement, if reasonable when the special law or franchise became effective, may not continue reasonable and suitable to developments throughout the term of the franchise or the period during which the law remains in effect.

After the establishment of commission regulation there should be no occasion justifying the passage of a special law of this kind; and the special laws already passed should be nullified to give the commission entire freedom to make changes when they are found to be justified, the special rates and regulations to remain in effect only until reasonable rates and regulations can be determined by the commission.

COMMISSION ORGANIZATION.

The majority of the state laws provide for three members on a commission, but there are commissions of four, five and seven members. The majority of the laws also provide for appointment of the commission by the governor, by and with the consent of the senate. In some of the laws, provision is made for the election of the commissioners by the people, but the appointive commission has grown in favor, and this plan is adopted in the laws most recently enacted.

Term of Office.—The different state laws provide for a term of office for the commissioners varying from two to ten years, but the term most common is that of six years.

The body is given a greater degree of stability by providing for the expiration of the term of but one commissioner at a time, and there are many instances where men, who have proved their special adaptability for the work, have been returned for successive terms to the best interest of the regulatory system. There is an unwritten obligation that the governor maintain a commission divided as to party affiliations so that it may act entirely in a nonpartisan manner.

Qualifications.—In practically all of the laws, men appointed or elected to commissions are not to have any direct or indirect pecuniary interest in the industries to be regulated, and the requirement is often exacted that the members and employes of the commission are not to engage in any other business or hold any other political office.

The training of the men eligible to the office of commissioner is specified in some of the laws, while in others certain standards have been established by precedent. The men making up a commission should have different qualifications and training. For example one member may be a lawyer, and associated with him may be a business man familiar with railroad and public utility problems, an engineer of broad training and a man familiar with the financial and economic aspect of the regulated industries.

Provision is often made in the law for a secretary or clerk of the commission and also for a special attorney.

The Staff of the Commission.—There is a commission staff, the members of which are appointed by the commission, to supplement the work of the commissioners, make investigations of operating conditions, make appraisals, compile data for submission to the commission, and conduct investigations in the field. The staff usually is organized by departments; *the engineering* department, accounting and statistical *departments, and* the tariff or rate departments.

These are in turn subdivided. The engineering department, for example, is made up of subdivisions presided over by a railroad engineer, electrical engineer, gas engineer and experts in whatever line is included in the commission's jurisdiction.

COMMISSION PROCEDURE.

The commissions are in practically every case free to adopt their own rules and practice of procedure in making investigations and in conducting formal hearings.

The commission need not wait for the presenting of a formal case. Some of the most valuable work of commissions has not been recorded in formal opinions and cases. Especially is this true in the case of small companies who have been required to keep proper accounts, directed to make improvements in operating machinery, and advised in the development of their business under a favorable rate system.

Informal Cases.—A great part of the complaints made to the state commission are handled informally by correspondence and by visits of some member of the commission's staff.

Formal Cases.—Generally speaking, the less like a court the sittings of the commission in formal cases are the better the results. Matters may be brought up which are outside of the original case presented on complaint, petition or application. The scope of the case may be broadened to include parties other than those bringing the case to the commission and the final order of the commission is not necessarily limited to the initial proceedings but may present a full and complete disposition of all matters brought up during the progress of the case. For example, complaint against a rate charged one individual for electric power service may result in a change in schedule for all classes of service by the company involved.

The commission is given power to require the furnishing of records and data considered necessary to the case and has power to summon witnesses similar to the powers of a court in that respect.

Review by Court.—The judgment of the commission as to the proper findings in a case is practically paramount. The commission's decision may be taken to court for review, but the judgment of the court may not be substituted for that of the commission. If the court finds that the commission has exceeded its powers or made a decision contrary to law and fact, the case must be remanded to the commission for modification in accordance with the court's finding. If new and additional evidence is introduced when the case is presented to the court, the case is returned to the commission for consideration of the new evidence.

The finding of the commission is *prima facie* reasonable, and the burden is upon those carrying the case to the court to prove that the commission's finding is otherwise.

SCOPE OF REGULATION.

Without taking up separately the regulation provided for in the different states, it may be said that commission regulation of public utilities includes regulation of rates, service, accounting practice, the issue of securities, and the valuation of public utility properties.

Rates.—In a few of the state laws the commission is given the power to fix maximum rates only, and in Ohio the rates are in the first instance fixed by municipal ordinance subject in all cases to appeal to the state commission. In the large majority of states, however, the commission is charged with the duty of determining reasonable rates, and may order changes in individual rates or may prescribe entire new schedules. In about half of the states where the commission is given the more complete jurisdiction over rates, the rates of municipal utilities are exempt from regulation.

Provision is made for the commission to make valuations and obtain whatever data it may consider necessary to the determination of proper rates.

Rates Fixed by Contract.—When the utility services were first established, the utility companies were left free to make what agreement they could to gain permission to operate and to induce the customers to take the service. Many promises were made to individuals and to communities naming rates for the service, and these promises were often embodied in special contracts between the company and the customers. With the establishment of regulation, the interesting question arose as to whether or not the company should be permitted or required to carry out these original agreements. It has been generally established that these special contracts must be abolished as constituting unjust discrimination, and the reasonable rates adopted by the commission should be generally applied to all customers regardless of whether these special agreements called for a higher rate or a more advantageous rate to the customer than those found by the commission to be reasonable.

Franchise Rates.—Likewise rates fixed by franchise have been changed after investigation by commission. The franchise rates are binding upon the company only until modified by the commission.

Such orders of the commission have been upheld in cases of review by the courts upon the general ground that the state's power to regulate in cases affected with public interest cannot be bargained away by such special agreements.

Increases above rates fixed by franchise are allowed as well as decreases in such rates, since this is only just to the company, and rate increases may be as much in public interest as rate decreases.

Publicity in Rates.—All rate schedules are to be filed with the commission and all changes in such schedules are to be filed prior to their going into effect. Duplicate schedules and rate changes are to be filed in a place and manner readily accessible to the public. Such provisions securing publicity of rates are made in the interest of elimination of discrimination.

Different rates may be prescribed for different classes of service, but all customers whose service comes under a certain class are entitled to the same rates, rules and regulations.

Modification in rates.—Rates once determined by the commission may be reviewed at any time, either on the commission's own motion or upon complaint of either the company or the customer.

Emergency Rates.—Flexibility of rate regulation was secured by special provision in some of the laws permitting the commission to fix rates in an emergency without recourse to the extended investigation made in usual cases and the advance notice to the public of the proposed increase. Emergency increases have been resorted to in many cases under such provisions to meet the unprecedented increase in operating costs attendant upon war conditions.

Service.—The regulation of the service of public utilities is one of the most important branches of regulation.

Standards of Service.—In the case of gas and electric services, commissions have quite generally worked out statewide standards of service, covering, in the case of electric companies, such matters as accuracy of meters, meter tests, voltage regulation, variations in frequency; and in the case of gas companies, meter accuracy, heating value and illuminating value of gas, tests for purity of product and maintenance of uniform pressure. In some states following the establishment of such rules, regular inspections are made by the engineers in the gas and electric departments of the commission and the companies are required to keep operating records affording a check upon their operation under the standards set by the commission.

Other rules have been adopted covering the method and manner of construction of electric transmission lines, crossing of power lines, street car and railroad crossings, prescribing the intervals of service on car lines and frequency of car stops, regulating telephone and water service and many other matters affecting the safety and adequacy of service in these various utilities.

Extension of Service.—The commission has power to determine what extensions of service are necessary, and when the cost of such extensions should be borne by the company. In some cases the commission has required extensions to be made only upon condition that part or all of the cost be borne by the customers to be served from the extension or only upon such customers guaranteeing a specified return upon the company's investment in the extension.

Joint Service.—The supervision of service covers the rendering of joint service by utility companies. The companies may be required to co-operate in rendering joint service to the public and the compensation for such service may be divided between the companies on a basis determined by the commission.

Refusal of Service.—Public utilities cannot arbitrarily refuse to serve certain customers, but must render service without discrimination. If an application for service is refused by a company it must be in accordance with rules on file with the commission and meeting with its approval. Upon complaint of the applicant to the commission, a determination will be made as to whether or not service should be given in that particular case.

Abandonment of Service.—A utility once having undertaken a service to the public, cannot abandon that service without approval by the commission after proper cause has been shown.

Neither can public utilities change hands by transfer or lease without commission approval; and reorganization and mergers are also made under commission supervision, the theory being that the public is an interested party to all such proceedings to the extent of determining that no unnecessary financial burdens are placed on the public service and that those acquiring control of the property are competent to conduct it in such manner as to afford adequate service to the public.

Accounting.—Commissions have been given the power to establish uniform systems of accounts for all utilities subject to regulation and to require regular company reports.

The information thus on file in the commission's office is of use in capitalization and rate cases and, being uniform, these accounts afford valuable sources of comparisons between operating companies. Complete accounting records for a number of years may afford a better basis for rate making than a valuation prepared for the immediate purpose at great expenditure of time and labor. Such data have been used in regular rate cases as well as in many of the recent emergency increase cases due to the great increase in operating and maintenance costs.

Issue of Securities.—The public utility laws vest in the commission power to regulate capitalization, varying from a general provision stating that no public utility should issue securities without the commission's approval, to an elaboration of the commission's power until regulation affords a practical guarantee that there is a property value upon which the company is entitled to a return for all securities issued.

The continued regulation of capital expenditures by the same body regulating rates makes it possible *to use the* capitalization as a rate base. The company is assured before making an expenditure that it will meet the commission's approval and that it is such that it should be allowed to earn a return upon the amount expended. In the past, companies have made expenditures which have not been justified by future developments. Extensions have been made which never have been profitable and perhaps never will be, and plants have been enlarged to an extent not justified by the later growth of the town. But if these expenditures had been approved by a commission, the judgment of the company at the time having appeared reasonable to the commission, there would be less opportunity to penalize the company in a future rate case on the ground that poor judgment had been shown in the expenditures which proved to be unnecessarily high.

Valuations.—Appraisals of public utility property may be made by the commission in capitalization and rate cases. In most laws where valuations are mentioned it is left optional with the commission to determine what and when valuations will be made. In one or two laws where it was made mandatory upon the commission to value all public utility property it was found to be practically impossible to determine a basis for such general valuations and it proved to be an unjustifiable burden and expense upon the commission and the utilities.

In some of the states, the staff of engineers organized by the commission has made property valuations for purposes of taxation.

Valuation, especially as concerned in rate cases, is later treated at length in the last two articles of this series as the subject most closely related to the discussion of rates.

Protection From Competition.—The companies most generally subjected to regulation are those in which competition increases the cost of service and is otherwise undesirable. Electric, gas, telephone, water and railway companies are called natural monopolies, because it is possible to have competition only at the expense of running duplicate systems of wires, mains and tracks through the city's streets, as well as maintaining greater plant capacities than otherwise necessary. Regulation by competition is secured by duplication of expenses in other industries, but the objections to duplication in the case of natural monopolies are more obvious. There is absolutely no excuse for duplication of expense under commission regulation.

Before the establishment of commission regulation, municipalities often followed the policy of granting franchises to as many companies as applied, thinking to secure the benefit of low prices during the rate war which invariably followed the establishment of duplicate properties, but the resulting waste, unwise expenditure, and failure to earn an adequate return resulted in the elimination of the weaker competitors or led to a general price agreement, and the price of the service had to be high enough to retrieve the losses during the rate war. Measures to prevent this state of affairs under regulation have been incorporated in many of the state laws. Companies desiring to establish new plants or established companies desiring to extend their service into new fields are required to first obtain a certificate of convenience and necessity from the regulating commission; and practically without exception the disadvantages of duplication have been recognized by the commission and the certificate of convenience and necessity has been withheld when the territory in question is already served by another company.

With one company in the field, if the service is inadequate the commission may order it improved, and if the rates are unreasonable new rates may be prescribed by the commission, leaving no excuse for resorting to the old unsatisfactory method of regulation by competition.

The greater stability given investment in a public utility under protection from competition makes it just that much easier to secure necessary capital for the development of these utilities and is reflected in a lower cost of capital. This protection should be against the establishment of municipal utilities as well as private utilities.

Municipal Utilities.—In about half of the states having regulation of utilities, municipal utilities are subject to the same regulation as private companies under the law. Regulation of the municipal utilities in these states has shown that they are greatly in need of revision of their accounting practice (in many cases the utility accounts are confused with the other municipal accounts until it is impossible to tell what the costs of the utility service are), revision of rates to eliminate discriminations and to afford a proper system for developing the business, and supervision of operation requiring improvements in equipment and operating conditions.

Only in those states placing municipal and private utilities upon the same basis can fair comparisons be made between municipal and private ownership and operation of utilities.

Indeterminate Permit.—In the case of the short-term franchise, capital is secured under the uncertainty as to what will happen when the franchise expires. Capital is under the practical necessity of getting what compensation it can within the term designated in the franchise, often at the expense of quality of the service rendered and upkeep of its property.

Bargaining for renewal of the franchise is often drawn out over a period of years and is a disturbing factor in local politics and company enterprise.

Under proper state regulation there appears to be no reason for continuing the term franchise. Grants to operate indefinitely do not prejudice public welfare as long as regulation requires adequate service at reasonable rates with power to enforce all reasonable requirements in public interest.

In Massachusetts the term franchise was abandoned for what were termed "grants of location," which were in fact franchises granted for an indefinite period during good behavior. Indeterminate-franchise forms have been adopted by the Federal Government: for example, franchises granted by Congress to public service companies operating in the District of Columbia, Porto Rico and the Philippine Islands. The indeterminate-permit provision was written into the Wisconsin law and has proved satisfactory to the companies and the public. In that state all franchises were arbitrarily made indeterminate by law in 1911. Indiana adopted the indeterminate-permit provision, leaving the acceptance of the indeterminate permit optional with the utilities. Commissioners in both states have publicly pointed out the advantages of the indeterminate permit.

Holding Companies.—Under proper regulation, all possible benefits to be derived from the organization of public utility holding companies may be secured without fear of the abuse of their power as against public interest.

Among the advantages to be gained from the holding company is the aid in securing capital for the further development of the subsidiary properties, in securing managing ability to advise in the development of the local enterprises, in building up better accounting practice and in the furnishing of legal and engineering advice from a central office. The expense of furnishing such services is shared by all the subsidiaries, and the pooling of the various risks and hazards makes it easier to secure necessary capital through the central holding company than by the local companies at a similar expenditure of time and money.

With the financial transactions subject to commission regulation, the accounts of the local companies open to public inspection, the service and rates regulated by the commission, the public interest is protected in the cases where holding companies are involved the same as in any other instances.

Consolidation.—Similarly, combinations of public utilities under regulation are not against public welfare. In fact, combination of business under these conditions is attended with the benefits of economic operation without fear of monopolistic power to be used against the public. The consolidations are under commission supervision as to capitalization, reliability of parties and similar matters of public interest; and after the consolidation has been effected regulation is still just as efficient, not being dependent upon the maintenance of competition. The consolidation of electric utilities, for example, results in more economical operation because the load is more uniform and the reserve maintained at one consolidated plant is not as great as the sum of the reserves necessary in separate plants. The desirability of effecting this saving in electric utility operation has been most marked under the shortage of coal and other war conditions.

The Massachusetts Gas and Electric Commissioners in their thirty-third annual report included in their recommendations for legislation the following request for an extension of their powers tc make it possible to order consolidations that would be in public interest:

"Considerable progress has been made by the companies in tying together the electric generating stations, but in a great national emergency a matter so vital to the public interest should not rest entirely on the initiative of the companies. What has been said of power plants applies with equal force to gas works. With a view to prevent unnecessary duplication of investment and to conserve natural fuel resources, the

Board recommends that it be given authority, after proper proceedings, to require the physical connection of power stations and gas works, the supply of gas or electric energy to other companies and municipal lighting plants, and the shutting down of such power plants and gas works as may be unnecessary, upon such terms and for such periods as may be just and reasonable, and to exercise the right of eminent domain wherever essential to this end."

Such a law was passed by the Legislature in accordance with the Board's suggestion as Chapter 152 of the General Acts of the year 1918.

Interconnection of electric systems, the pooling of electric power, and its distribution under the direction of a power administrator were secured in other states by the co-operation of the companies and under the direction of the state commissions and the Federal authorities, the aim being to overcome the shortage of power, eliminate wherever possible its uneconomical generation and assure the furnishing of adequate power for the essential industries.

III. Procedure in Rate Cases.

The commissions on first taking up their responsibilities, without precedent to guide them, with little chance of pleasing both parties to a controversy and the probability of pleasing neither, sought to base their decisions upon carefully prepared data so that the justice of their orders might be proved in mathematical terms.

In rate cases the responsibility of the commission lay in securing for the public rates as low as were consistent with good service and at the same time in securing to the company a reasonable income covering all operating expenses, including allowance for the inevitable depreciation of the property and, over and above this a reasonable rate of return.

VALUATION OF UTILITY PROPERTIES.

To determine upon just what amount the company should be permitted to base the return, the commission made extensive and detailed valuations of the properties. These were often drawn out over many months. In some of the earlier rate cases it would be two or three years after the filing of the complaint before the final rate order was entered.

When a rate case opened up, the commission's valuation engineers went out to make a valuation of the company's property, and, because a rate investigation was a new and untried experience, the company's engineers were also set to work making a separate and complete valuation and, in most cases in a city of any size, a third valuation was prepared by engineers for the city. The different interests of these three sets of engineers led to widely different results and at the hearing before the commission the different arguments were presented, substantiating the findings of the different appraisers. When it came to the writing of the commission's opinion this extensive debate was reflected in a lengthy opinion passing upon the merits and demerits of the different valuations and the arguments presented on the different values making up the total. As a matter of fact, a careful reading of the opinion would not always disclose just what had been used as the final rate base. But the practice of making these extensive valuations was continued, showing the desire of the commissions to determine mathematically the justice of the decision on rates. Arguments and counter arguments were heard, and different bases of valuations were proposed. It would appear that a valuation worth making at all would be worth making well, but the difficulty is that the whole matter is fluid—the property values do not stand still, unit prices fluctuate, depreciation goes on, improvements and extensions are necessary, new developments cause property to become obsolete.

Excessive Accuracy in Valuation Vitiates Its Timely Value.—If the rates under investigation are unnecessarily high or unreasonably low, the injustice to the public or the company is continued during the period of investigation. It is practically impossible to make refunds or adjustments over past charges, as the company's patrons are constantly changing and the transactions are numerous and involve small amounts. For this reason again the rate determinations should not be extended by needless argument over the theoretical differences of opinion. A rate investigation is more or less a disturbing factor in the company's business. The sooner settled the better, as a rule, so that the correction in rates, if a change is found to be justified, can be made promptly.

After expending time and effort to arrive at an exact valuation, the lapse of time makes it necessary to make allowances all along the line to adjust the findings to the present. After all, the new rates are not expected to be reasonable as to past conditions, but they must be reasonable for future operation.

Assuming that an absolutely perfect determination of value for rate-making purposes has been made, the commission is then met with practical difficulties in the way of fixing the rate on a strict cost-of-service basis. Perhaps the utility is still in the developmental stage and cannot be made to bring in a reasonable return immediately or some other circumstance calls for a modification of the rate finding. Competition is a factor, value of the service must be considered, and judgment must be exercised at every step of the investigation, regardless of all attempts to place rate making on an exact mathematical basis.

Changing Attitude as to Valuation.—It would seem that now after some ten years of regulation of public utilities a theory and method of procedure would be clearly mapped out, if such is possible of determination, but the subject is still in a confused state. The holdings of courts on various elements of value in taxation cases or in cases where valuations have been made to determine whether or not there has been confiscation of property, and the holdings of commissions in capitalization and purchase cases have been cited in rate proceedings without proper distinction being made. After much discussion on a certain point commissions have failed to state the final conclusion and value used. So in following precedents one comes to blind alleys, many branching off places and puzzling crossroads.

Valuation has been overemphasized, has been made an end in itself. It should be recognized that the commission is not seeking a value, but is seeking a reasonable-rate base. With a value right at hand, a commission will go out around it, add on and subtract from, in order to arrive at a base which in its judgment is proper in that case. Even if the records of a company were absolutely complete, showing the cost of the property just as it was acquired, the cost, or the first cost less depreciation, would not necessarily be taken as the base for rate making.

In the above discussion it is not intended to present an argument for the entire disregard of valuation in rate cases, but for a practical treatment of the work of making appraisals. The commission must make an investigation and get together facts upon which to base an opinion, but it would seem that, with more complete operating records before the commission and greater familiarity of the regulating commissions with the work given them to do, a more practical handling of valuation may be expected than that found in the earlier decisions.

A very marked change has in fact taken place. One advance was made by one commission which secured more co-operation between the commission's engineers and the engineers of the company and the city. The position was taken that the items of property either were or were not there and an inventory of the property was made up which was acceptable to all the parties making the appraisal. Then the proposition was asserted that the property either had a value in a rate case or did not have a value, and they sought to secure a practical co-operation of the various engineers in determining common-sense values which would appear reasonable to all parties for all practical purposes. In this way many of the differences in engineering practice were worked out before the data were presented to the commission.

In other cases, to avoid the costs of making complete duplicate appraisals, the commission's engineers, representing the neutral body, made the only complete appraisal in the case, their findings being subject to check by the city and the company. Or in still other cases the burden was on the company of showing the reasonableness of the rates complained of and a complete inventory and appraisal was required of the company, subject to check by the commission's engineers.

In the case of an application for an increase when the record showed a deficit from operation under the rates in force, the New Jersey Board prescribed new rates and ordered the company to file an appraisal of its property with the Board at a subsequent date fixed in the order, failure to file the inventory cancelling the permission to increase the rates.

More recently, during the war period, much less importance has been given to the determination of value in rate cases. This is true especially of the emergency rate increases, but it is very doubtful indeed if, even after a return to more normal times, valuation will ever be taken as seriously in a rate case as it has in the past. A number of causes have contributed to recent omission of extensive appraisals.

The rapid increase in prices has made it difficult to determine what should be considered reasonable unit prices for such valuations. There has been a marked withdrawal of engineers from valuation work to construction work or other work in connection with the demands of war. The emergency rate increases needed to meet the sudden increase in costs of operation could not be postponed for complete investigation. And still another reason lies in the fact that in many cases the commission is familiar with the company's financial standing through previous rate investigations, accounting supervision, or supervision of security issues and cases of purchase and sale. Or if not that particular company, the commission has a practical knowledge of other similar companies to serve as a guide.

Recent Opinions as to Importance of Valuation.—The attitude of the various commissions may be shown by the following excerpts from recent opinions.

In the Queens Borough Gas & Electric Co. case, in which the company applied for increases in gas and electric rates, the New York First District Commission in an opinion rendered June 3, 1918, discussed the question of whether or not a valuation of the properties should be required. Referring to the making of an inventory, the ascertainment of proper unit prices, and examination to determine the present condition of the property and the extent of depreciation, the Commission said:

"All of these matters involve controversial elements which mean delay and mean expense; and after all, proof of present reproduction cost, with or without deduction of depreciation, is not an indispensable element in ascertaining whether the quantum of net operating revenue yielded by present rates and expenses affords to the company's investors an inadequate return upon that which the law and the constitution says an adequate return must be afforded.* * *

In a time of war, there should be no avoidable expenditure of money, expert skill or materials, which does not serve the purpose of integrating these local agencies in the successful conduct of the war. To require the present depleted valuation staffs of the Commission or company engineers to bend their energies for several months to a reinventory and appraisal of this company's property, at a time when unit prices for construction work and materials are so abnormally high as to be misleading and of little aid in ascertaining 'value for rate purposes,' would seem to involve unnecessary delay, unnecessary expense, and undesirable diversion of labor and skill."

The New York Second District Commission said, in the Empire Gas & Electric Co. case (June 11, 1918):

"All parties to the controversy are to be congratulated and commended for taking such action as will enable this Commission to fix emergency rates without a prolonged investigation involving perhaps a valuation of the company's properties in each community and which would be conducted at a time when costs are shifting so rapidly that, whatever time might be taken as the basis of the calculation, the situation would probably be materially changed while the investigation was in progress."

The Illinois Public Utilities Commission determined rates in the Galva Electric Light Co. case (decided June 4, 1918) without a detailed and exact appraisal of the property being made. The Commission said:

"The valuation of the engineering staff is based upon the original cost of the property. This valuation does not purport to be an accurate valuation of the property and was made by the engineering staff without giving detailed consideration to the costs of each individual item of property. The reason for this method of valuation appears to be the emergency nature of the case and the evident desire to render an opinion as promptly as possible and still present data which represents with reasonable accuracy the property values. The engineering staff testified to its belief that a more careful valuation would not alter the results by more than 5%."

When a large number of applications for emergency increases had to be handled promptly this Commission prepared a questionnaire form which the applicant might fill out, submitting to the Commission the facts in outline regarding the operating revenue and expenses and general condition of the business as compared with the conditions prior to the marked increase in operating costs which were the direct cause of the application for rate increase.

There were many indications of a coming change in opinion, however, before these strictly emergency cases arose.

That the changes in conditions during the time which it would take to make a complete valuation might seriously affect the accuracy of the final figures, is taken into account by the New Jersey Board of Public Utility Commissioners in the New York Telephone rate case. The Board in a decision in that case rendered Nov. 20, 1917, said:

"Since the rates to be prescribed by order are to be operative in the future, it follows that, in the exercise of the power of regulation, past conditions and experience thereunder alone cannot be taken into account. The future and the conditions under which the rates prescribed are in fact to be applied must be considered.

"As was said by Justice Harlan in Smyth vs. Ames (169, U. S. 466), 'the probable earning capacity under particular rates prescribed * * * and the sum required to meet operating expenses are * * * matter for consideration.'

"Especially is this so, where, as here, a long period of time necessarily elapsed in making the inventory and appraisement of the property and in hearings, and consequently the data which form the basis of the exercise of the power relate to a date already some time in the past. In the meantime conditions have materially changed; annual taxes have increased; special war taxes have been and will be imposed; the trend of the cost of labor and materials has been substantially upward; and the proof, in fact, shows that operating expenses have been and are on a steadily ascending scale."

The Supreme Court of Pennsylvania said:

"The ascertainment of the fair value of the property, for rate-making purposes, is not a matter of formulas, but it is a matter which calls for the exercise of a sound and reasonable judgment upon a proper consideration of all relevant facts." (Borough of Ben Avon *et al. vs.* Ohio Valley Water Co.)

After discussing the valuation data submitted in the fare case of the Chicago, North Shore & Milwaukee Railroad Co., the Illinois Commission in an opinion under date of Sept. 5, 1917, said:

"Finally, it must depend upon the good judgment and impartiality of the regulating body which has before it all facts bearing upon the matter. In all proceedings of this nature there will always be found outstanding certain points which will serve as guideposts by which the amount shall be determined that is fair both to the corporation and its patrons. More than this cannot be expected, and justice is satisfied if this be done."

The simplifying of valuation problems through continued regulation of accounts and capitalization is illustrated by the following New York and New Jersey cases.

In the New York & North Shore Traction Co. fare case, before the New York First District Commission, the records as to investment and return were unusually free from dispute for the reason that the company had been continuously subject to commission supervision. The Commission in that case (decided Jan. 7, 1918) said:

"The compactness and convenience with which it was possible to present the petitioner's case in this proceeding, and the absence of confusion or controversy as to the accuracy or significance of the items appearing in the various exhibits submitted to the Commission, may be ascribed almost wholly to the fact that the petitioner's financing, accountancy and operations, except in respect to the sufficiency of its depreciation reserve, have been conducted in conformity to the Commission's rulings and directions. In consequence, it was possible, with little difficulty, to present the petitioner's property costs, operating statistics, revenue needs, and the like, with complete fairness and clarity."

The New Jersey case referred to above is the New Jersey Northern Gas Co. rate case (decided Dec. 19, 1917) in which the Board said:

"In lieu of an inventory and appraisal, the company submitted a table showing its issue of bonds, the net proceeds therefrom and the amount of stock issued at par, which securities were issued under the authority of this Board. It does not necessarily follow, however, that a valuation made *ex parte* for the purpose of approving an issue of securities would be conclusive for purposes of developing a schedule of rates. For the purposes of this report, however, the difference will not be such as to affect the validity of the conclusions which may be arrived at."

Valuations in Capitalization and Sale Cases.—Valuations have been made by commissions in capitalization cases and in cases involving purchase and sale of utility properties. The commissions have held in such cases that the process of valuation is not the same as in a rate case. The purpose for which the valuation is made is controlling. But it would appear that under regulation there should be little difference between the value for a rate case and value as a basis for capitalization or purchase. The earning power of a utility would ordinarily have weight in fixing a purchase price, but the earnings are subject to modification by a commission's order. If the commission approves the purchase of a property at a higher value than the value upon which the company would be allowed a return in a subsequent rate case an injustice will be done. A similar circumstance would arise if securities were issued under commission approval on a larger basis than would be used in a rate case. Provision might be made for amortization of an excess value capitalized or allowed in a purchase case, or the buyer might be willing to pay a higher price for the property, expecting to improve the property by more efficient management and the working out of economies in operation, trusting that the commission would permit him to share in the resulting benefits. But, as a rule, the approval of a different and higher value in capitalization or purchase cases than in a rate case subjects the industry to an element of instability which is unnecessary under regulation.

There may continue to be cause for uncertainty in the determination of a proper rate base in cases where the company has been established long before commission regulation was instituted, but it is of greater significance to determine the attitude of commissions toward new capital which has been invested in these properties with their approval and where the company's accounts have been open to inspection and regular reports made to the commission. Any uncertainty as to the treatment of this new capital by commissions in rate cases will be reflected in the attitude of capital toward investment in the public utility field and will have much to do with fixing the interest rate which it is necessary to offer such capital for development and extension of these industries.

BASES OF VALUATION IN RATE CASES.

Earning Value.—Commissions have pointed out that earning value cannot be considered in a rate case, for the proceeding is for the purpose of determining what earnings are reasonable.

Market Value.—Market value is not available as a base in public utility cases, as properties do not change hands very frequently as a rule, and under regulation the usual considerations affecting a purchase price are modified by the very fact that the property is under regulation.

Original Cost.—The original cost of a property determined from actual company records or an estimate of original cost built up by the use of unit prices which were probably paid at the time the property was constructed may be used as the measure of value or one of the bases of value in a rate case. When regulation was first established it was seldom possible to find actual cost records. Properties had changed hands, and perhaps the present property was the result of consolidations and original records were not available, but with regulation continued over a considerable period, in the case at least of properties more recently established, complete cost data will be on record.

Present Value.—The present value of the property may be taken as a basis, judged by present-day prices of labor and material necessary in building up the property, the possibilities of development of the business in the field, the adaptability of the property as erected to serve the demands of the community, and the depreciated condition of the property. This value should reflect appreciation as well as depreciation.

Reproduction Cost New.—Lacking exact records, the commissions have built up estimates of the cost to reproduce the property. In some cases the aim has been to determine the cost of an identical property.

in other cases the cost of a property planned to meet the present needs of the community. In some cases the estimate has been made using unit prices such as might have gone into the property at the time it was constructed, in others present-day prices have been used, while in others the prices used do not belong to any definite date but are averages covering a number of years.

Actual-Performance Method.—A basis called the actual-performance method was used in a recent case purporting to bring a new angle to bear upon the question of value. The method was used by the Pacific Telephone & Telegraph Co. in presenting cases before the Washington and Oregon Commissions. The Oregon Commission discusses the method as follows:

"The utility has strongly urged the acceptance of this theory, contending it is superior to the 'reproduction-cost method,' which, we believe, all will admit has not proven entirely satisfactory in its application when considered as a determining factor of value.

"The company maintains an elaborate system of cost accounting which is of particular value in investigations of this nature, and by reason of which it is enabled to present this 'Actual-Performance' estimate. Briefly stated, this estimate was made up by an analysis of the entire cost of doing work, including supervision, general and other so-called overhead expenses, as shown by the company's cost records over a period of years, from which analysis unit costs were derived. These unit costs were then applied to the various items of property as shown by an inventory of the system. In the case of land, studies were made to determine the cost of acquisition over and above the price paid the seller, and this cost was added to the value of each parcel as determined by appraisals made by real estate dealers and land appraisers.

"The result, the company contends, is not 'Original *Cost,*' *neither* is it 'Book Value,' nor 'Reproduction

Cost,' but is rather an 'Appraisement on the Basis of Actual Performance' and presents 'an array of facts as distinguished from an array of opinion, expert or otherwise, that ought to be the recourse for constructive and efficient regulation.'

"The theory commends itself to us very strongly. It embodies many features which are lacking in the reproduction theory and, in a measure, meets many of the meritorious objections to original-cost figures. Taken in connection with original-cost statements and reproduction estimates, it supplies information which is of inestimable value in arriving at a correct solution of a problem which, at best, is surrounded with uncertainties and frought with technical considerations. We do not wish to be understood, however, as accepting this theory as a substitute for the reproduction method. Neither do we think it should supplant original-cost figures. It conflicts with neither, and we are inclined to view this new presentation rather in the light of a valuable addition to those theories which have been accepted as bases for the determination of values."

INVENTORY OF THE PHYSICAL PROPERTY OF THE UTILITY.

The first step in making an appraisal is to make a complete inventory of the physical property of the utility. In this connection a number of matters may come up for decision in determining what property to include.

Donated Property.—Often a certain part of the properties used and useful to the furnishing of the service has been donated. For example, land has been donated to induce street railways and railroads to furnish service over a certain route. The question of whether or not donated property should be included in an appraisal for rate making has been variously decided, but the rule has been, unless there are certain modifying circumstances, to include all properties however acquired that are now used and useful to the furnishing of utility service.

Service Paid For by Consumers.—When public utility companies were first established it was customary to require consumers to furnish their own service connections. Where this has been done, commissions in making valuations have sometimes held that the value of such services should not be included, that the customer having paid for the property should not be made to pay a return to the company on the value of that property. But usually the customers who furnished the property will not be the ones affected by the rates to be established in the case. The records do not always show just what services were furnished by the customer and what by the company. The commissions have usually included this property, holding that the ownership is in the company the same as in the case of gifts and the manner of its acquisition, having belonged to a period prior to regulation, is one with which the commissions are not concerned.

Discarded Property.—Discarded property, such as may be found in any utility system due to construction of a larger property than later developments in the community justified, or more often due to the consolidation of competing properties, is another matter for consideration. It has generally been conceded that such discarded property must be included until provision can be made for its gradual amortization in cases where the investment in such property has been made in good faith and with a reasonable exercise of judgment to best serve the interests of the community.

In one case where the property under investigation was the result of consolidation of a number of competing companies the Pennsylvania Public Service Commission refused to allow the value of parallel lines and duplicate equipment in the rate base, but in *reviewing* the Commission's decision the Supreme

Court pointed out that such holding was not in public interest. The court said:

"If the utility companies in organizing into one operating plant a number of smaller ones must be deprived of a fair return on the value of the properties purchased and are also limited to the reduced cost of operation by reason of such consolidation, it is evident that the more practical way should be to permit the companies to remain separate operating concerns.

"The public is entitled to a fair benefit from every such move made by a utility concern and where they assemble a number of plants with separate overhead and operating charges into one plant with one overhead charge and one or more operating plants, while the operating plant of some of the separate units may be rendered useless, still the new concern has paid for it, and the capital was originally and is now invested on its account. Investors could not be inducd to traffic in such an unsatisfactory and unstable security, where the risk is not failure by reason of operation, but failure through confiscation. The benefit that should come to the public is through the reduced cost of operation; but the value of the dismantled plant should not be taken from the books as a capital charge until such time through sale, or other equitable arrangement, such value has been reamortized. So, too, with respect to the parallel lines in question." (Ben Avon Borough *et al.* *vs.* Ohio Valley Water Co. Decision of Supreme Court of Pennsylvania, Oct. 8, 1917.)

Paving Over Mains.—In making an estimate of the cost to reproduce a property, allowance has not been made for the cost of paving over mains where the history of the plant shows that the mains were laid before the streets were paved. Such costs would, however, have to be reckoned with in a true estimate of reproduction of the property as of the present time.

Unit Prices.—To the list of properties included in the complete inventory unit prices are applied. The determination of unit prices depends upon whether the basis of the valuation is original cost, present value or some reproduction value. Usually to determine a fair value for rate-making cases, the unit prices used represent an average of prices over a period of years (in most cases five years is taken). The sudden advent of present high prices of machinery and equipment of all kinds has led commissions to postpone the making of valuations "until prices and times are normal," and in fact this marked fluctuation in unit prices has been a factor in reaching the conclusion that complete valuations are not essential in a rate case.

Overhead Charges.—To the property cost obtained by the application of prices to the item of property in the inventory is added, usually as a percentage allowance, an amount to cover overhead charges—that is, costs which are attached to the construction of the property as a whole and not included in unit prices—such costs as promotion, legal expense, engineering, incorporation and organization, interest during construction, early experimental work, contractor's profit, costs of piecemeal construction, contingencies, and allowances for possible omissions in inventory.

INTANGIBLE ELEMENTS OF VALUE.

In addition to the determination of value of tangible property, consideration must be given to intangible values, principal of which is going value.

Going Value is the value which is added to a property by the fact that it is a going concern. In some cases the commission has not made a separate determination of going value, holding that a property with an attached business is of more value than the property without such business, and stating merely that the property has been valued as a going concern.

Other commissions have made a definite addition as a percentage of the physical value or as a separate sum for this element. The estimate has been based upon the actual or estimated cost of developing the business, including costs of soliciting the business and deficits in earnings during development of the business. The objection to this method is that an unprogressive company slow in attaching business would have a greater going value than a more progressive company. Adjustments can be made to a reasonable amount, however, or an estimate may be made of the cost to reproduce the business just as an estimate is made of the cost to reproduce the physical property.

Both courts and commissions have varied widely in the treatment of this intangible element, considering it as a value in one case and as a cost in another.

Considered as a value, the authority may hold that a plant which has an established business is clearly of greater value than an identical property without an established business and where it has not been shown that it is possible to secure sufficient patronage to make a success of the undertaking. It is often added as a conclusive argument on this point that any buyer would pay more for the plant and going business than for the plant without such business.

Considered as a cost, the authorities have pointed out that a business can not be secured without expenditure of time and money. On this basis, as stated above, an allowance may be based on deficits in earnings during the period of business development, or the company may show what it costs to connect a new customer, including such costs as advertising and soliciting new business and the attention necessarily given to the prospective customer in acquainting him with the adaptability of the service to his needs, proper use of the equipment installed on the customer's premises, and similar considerations. This cost per customer is then multiplied by the number of customers attached to the going concern at the time of the valuation.

Good Will.—Going value is not the same as good will, which is recognized as an element of value in competitive undertakings where the patronage can be built up and a business given value by securing the good will of the customers. Good will has never been included in a valuation of public utility property, although the element is a factor in such undertakings. A public utility which has the good will of a community will develop faster, get more business, have less controversial expense, and to that extent be more valuable.

Franchise Value.—While the cost of securing a franchise which the company has actually borne in the form of a special tax or free service to the municipality is allowed, no intangible value attaching to the right to operate on the public streets is included in the rate base on the ground that such right is a grant of the public and should not be capitalized against a service to the public.

Working Capital.—An allowance should be made in the valuation for working capital sufficient to maintain the company's credit, permit purchases to be made at advantageous times and maintain company supplies sufficient for efficient and uninterrupted conduct of the business. The amount of working capital necessary may be determined in each case by an examination of past expenditures, consideration of the value of supplies on hand, the tendency toward increase or decrease in the cost of such supplies, and the time and frequency of the company's collection of bills for service rendered. The cash reserve and value of supplies is as much a part of the valuation as the investment in the property installed, as it represents capital used in the interest of the business of the public utility and withheld from other investment.

ELEMENTS OF VALUE GIVEN SPECIAL CONSIDERATION.

Land.—The same theory or basis of valuation followed in valuing physical property other than land is seldom applied to the value of the land. Land has perhaps most often been valued at its present value estimated by those familiar with real estate transactions in the community or judged by the value placed upon adjacent land.

The present value of the land is in many cases higher than the price paid by the company in acquiring the land. The amount of the appreciation has been considered in some instances as sufficient to cover certain overhead charges incurred in connection with the purchase of land and as reward for the exercise of good judgment in selecting a suitable location and securing property adapted to the proper development of the undertaking.

If the property is particularly well adapted to the public use to which it has been put a special value may be allowed as representing that added value to the public service, and as a reward for the exercise of the business sagacity that saw the possibilities in the unimproved land and put it to public service.

The actual purchase price of the land has been considered instead of present value in some cases where the land has depreciated in value. If the purchase price of the land is known and the property values in the community have gone down, the company has not been penalized where it appears that the investment was made in good faith and with reasonably good judgment.

Again in cases where the purchase price of land is used, the company may be denied the return on the appreciated value on the theory that the property is held in public trust and that so long as the land is held for that use the purchaser is entitled only to a return on the money actually invested. But this practice clearly prejudices property in public use as compared with property held where the owner is free to sell at an advantageous price or do what he wishes with the property to bring a return on increased real estate values.

Water-Power Rights.—Another matter which has received considerable discussion is that of value of water-power rights. Some authorities have held that such rights should be entered at the original cost of acquisition. In electric rate cases a comparison has often been made between the cost of operation of the hydroelectric property as compared with steam operation and a value found by capitalization of the saving in operation made possible by the company's water-power rights.

OPERATING EXPENSES.

Investigation to determine reasonable operating expenses, provision for depreciation and the fixing of a proper rate of return are important in any rate case and this work has often been passed over lightly where the importance of valuation has been overemphasized.

The Colorado Public Utilities Commission said: "In the opinion of the Commission the operating expense of a public utility should be as carefully considered as the amount allowed for fair value of the property of the public utility." (Suburban Light & Power Co. rate case, decided Nov. 16, 1917.)

Future operating expense, which must be provided for in the rate ordered, cannot always be judged alone by past records. Allowances must be made for the probable trend of material and labor costs.

An interesting example of rate regulation in which operating expense was made an important factor was that of the decision of the Oregon Commission in the

Portland Railway, Light & Power Co. case (decided Jan. 5, 1918). The company applied for an increase in fares; the employes were asking for an increase in wages; and the city was opposed to any increase in fares.

The Commission found that the company was in need of relief and it was suggested that the city grant a modification of certain paving requirements and service exactions which were considered by the Commission to be unjustly burdensome upon the utility service. The decision of the city with respect to lightening operating expenses was obtained, and the wage disputes were submitted to arbitration before the Commission took up the matter of rate adjustments. The Commission said:

"States should so regulate that neither an increase in profits to the utility nor lower rates to the consumers will be given without considering the wages of the employes."

The reasonableness of salaries paid, the reasonableness of payments of subsidiary companies to a holding company for certain service rendered, the possibilities for instituting economies in operation, the proper maintenance and upkeep of the property, the necessity of provision for unusual expenses caused by sleet storms, ice jams, or whatever the hazard may be to which the particular utility under consideration is subject, may enter into the discussion of what are reasonable operating expenses in any rate case.

Increase in Operating Expenses.—The sudden increases in operating expenses of public utilities, due to war conditions, have emphasized the need for greater flexibility in rate systems to maintain a proper relationship between expenses of operation and the rate schedules.

Provision for the automatic variation of the rate with fluctuations in the cost of fuel became quite common during recent times. Coal clauses have been adopted by electric and gas companies and have been generally approved by commissions, which provide that the rates shall be increased or decreased each month to correspond to an increase or decrease from the price as of a certain date for coal or fuel oil as the case may be.

In at least one case, a company applied to a commission to extend this automatic adjustment of rates to provide for fluctuations in total operating expenses. The commission had made a complete investigation of the value of the company's property and operating accounts and had prescribed rates for the company's business in 1912. The company recently asked that the commission take the rates thus established as a base and provide for automatic increase or decrease in those rates corresponding to the increase or decrease in operating expenses from those of that date. The commission refused to adopt such a radical departure from former practice under the exigencies of the war period.

However, there has been at least one new adaptation of this principle which has come to the writer's notice in the form of a wage clause. Starting with the rates as of a certain date and the scale of wages as of that date, the rate supplement provides for increase or decrease in the rate for electric service corresponding to the increase or decrease in the scale of wages paid to the company's employes.

DEPRECIATION.

As soon as property is constructed depreciation must be considered. Ordinary maintenance and upkeep costs do not cover the depreciation of the property. In addition to the regular wear and tear from operation there may be special agents to be reckoned with, such as electrolysis, corrosion, incrustation or decay, which are peculiar to the equipment of the diffe s.

Obsolescence.—Aside from the wearing out of the property there is the more uncertain element of obsolescence. Machinery new or in good operating condition may be hastened on its way to the discard by the appearance on the market of a greatly superior type of machinery. The plant and system may also become inadequate to meet the demands of the community for service which may require the building of a new plant and the abandonment of much of the old.

Accrued Depreciation.—The question of whether or not the estimated first cost or reproduction cost of the property should be depreciated to correspond to the present depreciated condition of the property has received much discussion in rate cases. Where the deduction for accrued depreciation has been made it has been generally decided that the amount of the depreciation should be determined by actual inspection of the property and not on a theoretical basis considering merely the number of years the property has been in use.

It has been contended by the company in many cases, that if the property has been efficiently maintained so as to furnish 100% service no deduction should be made for depreciation. In support of this view it has been pointed out that a property which has been adjusted and in working order is of more value than an absolutely new plant from the standpoint of quality of service rendered. This claim for undepreciated value has been sustained by the regulatory commission in some cases.

It would seem that the treatment of accrued depreciation is necessarily bound up with the question of whether or not the company has, in the past, earned sufficient to provide for depreciation, and what sort of appropriation the commission will make in the rate case for future accruals to a depreciation fund.

Depreciation Funds.—Considering the inevitableness of depreciation of property, good business practice requires the building up of an adequate reserve against the day when the different portions of the plant and system must be scrapped and new equipment purchased.

Instances have arisen where large varied properties have found that replacements occur with regularity, the property having struck its gait as is said, and that no reserve fund need be carried, the depreciation allowance simply appearing as an enlarged upkeep or maintenance cost, but in most properties a depreciation reserve must be provided.

There are two methods used by commissions in creating a depreciation fund, called the straight-line method and the sinking fund method. The straight-line method calls for the setting aside annually of an amount which, at the time of the expiration of the life of the property, will be sufficient, without interest accruals, to replace the property.

Applying the other method, the sinking-fund method, the amount set aside annually is such that the amount together with interest will replace the property. Commissions have discussed the merits of these two methods and their adaptation to short-life property, a telephone property, for example, or a comparatively long-life property, such as a water utility.

No broad general rule can be deduced from the holdings of the commissions in this respect, it being a matter for determination in each case depending upon the life and present condition of the property, the treatment of depreciation by the company in the past, the maintenance and upkeep of the property taken care of in operating expenses, and the holding of the commission as to whether or not a deduction should be made for accrued depreciation in determining the amount upon which the company will be allowed to earn a return.

In determining an adequate depreciation reserve, account must be taken of the fact that the trend of prices has been upward for a number of years. Where replacements have been necessary during the war period an additional burden has been placed upon the replacement fund because of the high prices of machinery, materials, and labor prevailing. It is not always possible to delay such replacements until prices are favorable as public interest demands safety of service and continuous operation. Allowance for depreciation should be liberal so that the company may be required at all time to maintain its property in good operating condition.

RATE OF RETURN.

The rate of return which the company is to be allowed to earn in fixing rates is more than interest on invested capital. There should be a margin of profit as an inducement to capital to continue to flow into the public utility industries, and a compensation or reward for efficient management and skillful economic operation.

Other factors which have been given consideration by commissions in fixing rate of return are hazards and risks in the business, bond discounts, effect of political interference and popular whim, and the necessity for allowance to cover possible undervaluation of the property.

Attracting Capital.—Capital cannot be forced to invest in public .utilities, but must be attracted in the open market by the offer of security of investment and an adequate rate of interest. During the war period, the utilities have had to compete in a restricted market with other industries offering high rewards.

The California Commission recognized the advantages accruing to the public from the adoption of a liberal policy in fixing the rate of return. The following excerpt from the Palo Alto Gas Co. case (Vol. 2, Opinions and Orders of the Railroad Commission of California, pp. 300, 317) has been cited by the

Commission in subsequent decisions and may be quoted as an expression of its policy.

"The Commission in fixing a rate of return must be liberal, lest too strict a policy result in turning capital to other fields of enterprise. California needs development of public utilities, and this Commission's policy should be a broad and liberal one, so as to encourage capital to develop the state by legitimate public utility enterprises where needed. The Commission should be careful not to permit an inflation of prices in ascertaining the value of the property of a public utility used and useful for the public purposes; but should be liberal in establishing the rate of return on that value."

The Michigan Railroad Commission in the Michigan State Telephone Co. rate case, discussing the need of capital in the public utility field, said:

"It is important, therefore, to the development and extension of public service, that the exercise of the power of public regulation be such as will not discourage the investment of new capital in such enterprises. The destruction or serious impairment of the value of securities issued by such companies, by undue limitation of rates, would strongly tend to discourage such investments, and this would result in inevitable and serious public injury through the impairment of the service adequate to the constantly expanding needs of the public. The exercise of the power of public regulation should promote the confidence of the public in public service securities, rather than constitute a menace to such securities. The State does not guarantee public utilities against losses nor insure them any specific rate of return upon their investment. While costs of operation and maintenance—costs of new material and labor—fluctuate with changing conditions of production and use, there should be some margin allowed to public utilities, over *bare present* costs, and those happenings which can be foreseen and estimated with reasonable certainty." (Decision rendered Jan. 30, 1918.)

The Oregon Public Service Commission, in the Portland Railway, Light & Power Co. case, decided Jan. 5, 1918, said:

"If the rates fixed by the Commission, while sufficiently high to escape condemnation by the courts as confiscatory, will yield only a return insufficient to attract capital into needed public service, it is the public and not the investor who will suffer."

The Illinois Commission in the Chicago, North Shore & Milwaukee Railroad Co. fare case, decided Sept. 5, 1917, said:

"Legislation cannot compel the investment of funds in a concern, and only the opportunity to earn sufficient to attract investors will avail in obtaining support for any enterprise. Without its support the enterprise will perish and a community may thereby be deprived of a necessary service."

Business Risks and Reward to Management.—The Illinois Public Utilities Commission in the rate case of the Central Illinois Public Service Co., fixed the rate of return, "after giving due consideration to the circumstances governing this case, including the character of the service, the hazards connected with the business, and the cost of securing capital; and further taking into consideration the testimony regarding the degree of ability displayed by the management which was found to compare favorably with that shown by other small gas utilities in the state of Illinois."

The Maine Public Utilities Commission said:

"The rate of return on the value of the property in a rate case must be sufficient to compensate and attract money to such undertakings under the conditions as to business risk, etc., which existed in the beginning. If the rate of return is reduced from time to time as it appears that the risk in that particular undertaking in the light of developments was less than might reasonably have been expected, or has been eliminated by successful management, it would practically amount to a penalty for skillful administration." (Maine and New Brunswick Electrical Power Co., *et al.*, purchase case, decided Dec. 4, 1917.)

The Illinois Public Utilities Commission in the Union Gas & Electric Co. rate case, decided Jan. 16, 1918, found proof that the utility had been ably directed in its policies and efficiently managed, and the Commission said:

"Where causes of an opposite character have produced poor results the Commission has not hesitated to conclude it had cause to penalize a utility by reason of such poor management. A rule of this kind should operate both ways, and it is considered just in this case to recognize, in a substantial way, that good management should not be penalized but ought to be further encouraged. It is realized that a utility can very materially control the cost of production in more ways than one, and if the expense per unit of service is reduced by reason of an increase above the normal in the sales per customer and per capita, the company should be given some incentive to maintain its good record. Any proof that is made of excellence in the operation of a utility ought to entitle the operators, as well as the holders of the securities, to a reward for their meritorious work."

Undervaluation of the Property.—In the Addison Gas & Power Co. case, decided Feb. 28, 1918, the New York Second District Commission estimated that the rates prescribed would yield a return equal to 8.6% of the value of the property, and the Commission said: "This is not inordinate in itself, and permits a small margin to cover any possible undervaluation of the property."

Bond Discount.—In some cases commissions have allowed for bond discounts in fixing the value of the property by considering it as a necessary cost of building up the property, while in other cases bond discount has been considered as an interest charge to be taken care of in the rate of return.

The Supreme Court of Pennsylvania, in reviewing the decision of the Pennsylvania Commission in the case, Ben Avon Borough *et al. vs.* Ohio Valley Water Co., said:

"Concerning the item of brokerage, the courts and commissions of other states have held that discounts on securities should be allowed, as utilities, like other companies, are not able to make their financial arrangements without allowing such discount. The difference between the amounts derived from the sale of its bonds and the amount which the company must eventually pay on the bonds has been regarded as a part of capital charges for construction. While corporations should not be permitted to capitalize their lack of credit, still, where bonds are sold at a reasonable discount and bear a fair rate of interest, such discount should be allowed."

In the Chicago, North Shore & Milwaukee Railroad Co. fare case, decided by the Illinois Public Utilities Commission, Sept. 5, 1917, the company submitted a valuation of its property to the Commission in which an amount was included for bond discount during the construction period. The Commission said:

"It is believed, however, that a more equitable disposal of the matter of bond discount may be had by considering it as a form of interest. Doubtless, a utility, by making its interest rate sufficiently attractive could dispose of its bond securities at or above par, but this basic principle would not be vitiated should it choose to fix a lower rate of interest and thereby be compelled to dispose of its bonds at less than par. The effect has been merely to change the rate of return which it provides upon such securities.

This idea is well expressed by the District of Columbia Public Utilities Commission, *in re* Potomac Electric Power Co., wherein it is said 'Bond discount, constituting a payment for the use of money, is in the nature of an interest payment; that is, it is not a proper capital charge but rather an adjustment of the interest rate to the existing market conditions and chargeable to interest account, and not capital.' The Commission is of the opinion that brokerage and bond discount are matters to be considered in connection with, and to be reflected in, the rate of return allowed; that such costs should be made up out of income by the creation of a sinking fund or reserve sufficient to cover the cost during the life of the bonds; and that, therefore, such costs should not in equity be considered a part of the cost of reproduction or of the 'fair value' to be taken as a base for the fixing of rates.

"This Commission is of the opinion that the doctrine thus enumerated is sound and affords a proper disposal of the matter of bond discount."

Surplus.—The wisdom of following a liberal policy in the fixing of a rate of return has found ample illustration under war conditions. Where the companies have been held down to a close margin hardship has been experienced under the increase in operating costs which could not be prevented entirely by emergency increases. While in the case of other properties it has been possible to tide over the period of high costs to some extent by the surplus built up under more normal conditions, commissions in granting emergency increases have held that it is impossible to maintain the rate of return earned previous to the war period and have held that the hardships must be borne jointly by the public and the utility. They have also pointed out that utility rates should not fluctuate with every rise and fall in costs. The steady rate level cannot be maintained by the company in a period of high costs unless a surplus has been built up in a preceding period, and if such surplus is not sufficient a portion of the loss in net earnings during the period of high costs may be spread over the years following a reduction of operating costs to normal.

The Oregon Commission in the Portland Railway, Light & Power Co. case, decided Jan. 5, 1918, said:

“Under state regulation of rates no utility is permitted to earn a surplus during good times by which to carry itself over the lean years which may lie ahead of it. Rates must at all times be kept down in conformity with the value and the cost of service rendered. Justice, therefore, requires that when costs go up, rates should do likewise. * * * No starved horse ever pulled a heavy load. The utilities have been deprived of the power to make unjust profits. They must also be protected against unjust losses. If a utility is driven into a position where its credit is impaired and it can obtain money for operations and extensions only at unreasonable cost, the public must share the loss.”

However, some commissions have adopted a policy of providing for the building up of a surplus in the rate of return, to provide for unforseen contingencies and to guard against the necessity of adjusting the rate to correspond with every fluctuation in price. For example, in the case, City and County of San Francisco *vs.* Pacific Gas & Electric Co., decided by the California Railroad Commission, Oct. 8, 1917, the Commission said:

“In permitting this return, we do so with a frank realization that it allows a liberal margin over the cost of money. We are animated in doing so, not merely by a desire to be fair to the company, but also in part by the uncertainty as to whether the price of fuel oil will not further advance and by the desire to create a margin of profit which will take care at least for a time, of such further advance, if it occurs.”

APPENDIX I

(To Section 10)

(For the Non-Technical Reader.)

Explanation of the Terms "Horsepower," "Kilowatt," "Kilowatt-hour," Etc.[1]

If a weight is lifted up by some force it requires "mechanical work" to do this, and if it is lowered mechanical work is set free. Work is measured in foot-pounds, which means: the unit of work is the foot-pound. A foot-pound is the work necessary to lift one pound one foot high. To lift 1 lb. 2 ft. high 2 ft.-lb. are necessary, and to lift ½ lb. 2 ft. high, evidently one-half of that amount is necessary, namely 1 ft.-lb. In the same way we can say, one foot-pound is the work necessary to lift 10 lb. 1/10 ft. high, or 1/100 lb. 100 ft. high, etc. It is seen from these examples that the work is given by the product, force × distance.

The rate at which work is being done is called *power*. Power is the work done *per second*. It could be measured in foot-pounds per second, but this unit is inconveniently small for the purposes of the engineer and therefore a unit has been chosen which is 550 times greater, or which, in other words, amounts to 550 ft.-lb. *per second*. It is called the *horsepower* (hp.). A horsepower is, therefore, the power of one pound falling 550 ft. *every second*, or, for instance, 100 lb. falling 5.5 ft. every second. Conversely, 1 hp. would be required to lift 100 lb. 5.5 ft. every second. A water fall, for instance, which pours 880 cu. ft. (55,000 lb.) of water *every second* over a head of 100 ft. would, if harnessed, be able to produce 10,000 hp. (provided that no power would be lost in the turbines).

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[1] The object of this appendix is not to give a scientific explanation of the terms "kilowatt" and "kilowatt-hour," which are so frequently encountered in central-station and rate practice and which are not always fully understood, but to give the commercial man a clear working insight of what is meant by these terms. It deals, therefore, rather with examples than with definitions, and scientific exactness of expression is sacrificed for brevity and clearness.

The electrical engineer uses other units besides the horsepower: the *watt,* which is about 0.738 ft.-lb., and the *kilowatt,* which is 1000 times as great as the watt. A kilowatt is, therefore, the work done by 1 lb. falling 738 ft. *every second.* One kilowatt = 738/550 = 1.342 hp. and one horsepower = 550/738 = 0.746 kw. The kilowatt, the watt and the horsepower are, therefore, different units for the same thing, just as the mile, the inch and the kilometer, or the dollar, the cent, and the shilling are different units for the same thing.

One kilowatt produces 738 ft.-lb. every second, or 2×738 =1476 ft.-lb. in two seconds, 2214 ft.-lb. in three seconds, etc., and 7380 ft.-lb. in 10 seconds. We can also obtain the same work of 7380 ft.-lb. from 10 kw. and then it will take us only one second, instead of ten. We see that the work is given by the product of power × the time during which the power is being applied.

We have thus been moving in a circle. We have seen that if a certain work has to be done within a given time—for instance, a second—this means a certain "power." And now we see that if a certain power is applied for a certain length of time the result must be again mechanical work.

The work done by one horsepower every second is called a horsepower-second and that done by a kilowatt every second is called a kilowatt-second. According to the preceding paragraph a horsepower-second and a kilowatt-second are units of mechanical work just as the foot-pound. Indeed the horsepower-second equals 550 ft.-lb., as is easy to understand, and the kilowatt-second equals 738 ft.-lb.

Similarly, the work done by one kilowatt during one whole hour is called a *kilowatt-hour.* A kilowatt-hour is evidently 3600 times as big as a kilowatt-second, because an hour contains 3600 seconds. A kilowatt-hour has, therefore, 3600 × 738 = 2,656,800 ft.-lb. In exactly the same manner we can talk about a horsepower-hour, which is 3600 × 550 = 1,980,000 ft.-lb.

It is also clear that we can obtain one kilowatt-hour either by using one kilowatt for one hour, or two kilowatts for one-half hour, or 60 kw. for one minute, or 1/10 kw. for 10 hours, etc.

Electrical work is generally not called "work" but "energy."

It is important for the student of electric rates to get the difference between kilowatt and kilowatt-hour quite clearly fixed in his mind. The following may prove helpful. A salary of $100 per month is not the same as a capital of $100. The relation between the two is the same as between the kilowatt and the kilowatt-hour. A salary is the rate at which a man gets his money. It is money per month, mea ured in dollars *per month,* just as the kilowatt is *the rate* at which work is being done (energy being supplied), measured in foot-pounds *per second.* If we multiply salary by months we get a certain amount of money, which may be capital, and if we multiply kilowatts by hours we get work or energy, kilowatt-hours. Thus $500 per month during two months results in the same amount of money as $100 per month during ten months, namely $1000. Just so 5 kw. for two hours result in the same work as 1 kw. applied during 10 hours, namely 10 kw-hr. or 26,568,000 ft.-lb.

Power (kw.) corresponds to .... Salary (earning power), $ per month.

Work or energy (kw-hr.) corresponds to.. Capital (accumulated salary), $.

Just as the power is the rate at which work is being done or consumed, so velocity or speed might be defined as the rate at which a distance is increased or decreased. The speed of a ship, for instance, is the rate at which the ship is moving away from a certain point (or towards it) and it is measured in (nautical) miles per hour, just as the power is measured in foot-pounds per second. The velocity of one mile per hour is called "one knot." Now the term "one knot" does *not* mean "one mile." The expression sometimes heard, "a speed of twenty knots per hour," is nonsensical. The knot is a certain speed, a rate of progress in miles per hour and a mile is a certain length, a certain total progress, so to speak, irrespectively of time. Instead of "one mile" we could also say "one knot-hour," although the term is not customary. Just so the kilowatt or watt or horsepower is a certain power, or rate at which work is being done or consumed, corresponding to the knot, and the kilowatt-hour or watt-hour or horsepower-hour, corresponding to the "knot-hour," is a certain work *irrespective of time*[2], and can be measured, for instance, in foot-pounds.

Kilowatt, watt or horsepower means the rate at which an engine or some other device is generating or consuming work. An engine is running at 1000 hp. at a certain moment, a ship is using 5000 hp. for its propulsion at a certain speed, an electric generator is putting out 2000 kw., an electric furnace is consuming 5000 kw. at a given moment. Frequently, however, the terms kw., etc., are used to denote the maximum or the normal output which a generating device (engine) is capable of supplying or a consuming device (lamp,

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[2] Although the term "hour" occurs in the denomination.

motor) is capable of consuming, just as the term "speed of a steamship" may mean either the speed at which the ship happens to be running at the moment to which one is referring, or the term may mean the maximum or the normal speed of the ship. So the terms kilowatt, horsepower, watt, etc., may also be used to denote the size of an engine or other device. We talk of a 10,000-kw. generator and of a 250-watt incandescent lamp. We talk of a 200-hp. boiler, meaning a boiler which, if normally operated, gives just enough steam to produce 200 hp. in the engine which is run from the boiler.

To summarize: Kilowatt-hour means *work* or *energy;* kilowatt means the *rate* at which work is being done and in a secondary meaning the term kilowatt denotes the capacity of machinery, etc. We can get the same number of kilowatt-hours with a *large* number of kilowatts operating for a *short* time or with a *small* number of kilowatts operating for a *long* time.

APPENDIX II

(To Section 15)

(For the Beginner.)

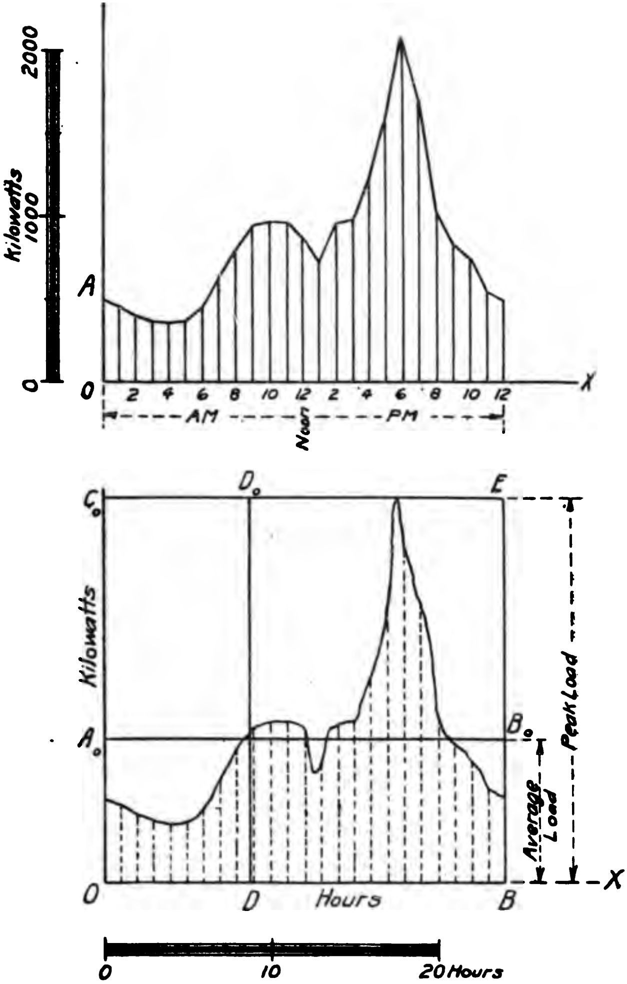
Explanation of the Terms "Load Curve" and "Load Factor."

A.—LOAD CURVE.

Step off on a horizontal line *OX* in Fig. A, 24 equal sections, each one representing one hour of the day. From the end of every one of these sections draw a vertical line upwards and step off on these, beginning at their respective intersection with *OX*, the load in kilowatts of a certain central station at that particular hour, using some arbitrary scale, for instance 1 in. = 1000 kw. If, for example, at 12 o'clock midnight the load is 500 kw., step off ½ in. vertically from the point *O*, thus reaching point *A*. Do the same thing for every full hour of the day (1 o'clock, 2 o'clock, etc.). Joining the point at the upper end of each vertical with both its neighbors to the right and left by a straight line we get a continuous series of straight lines. We are not restricted to entering the loads at the full hours only. We can enter them in the same way for any intermediate time; as quarter hours, minutes, etc., and the series of lines will then more and more approach in any desired degree a steady curved line (Fig. B).

This curve is called the *load curve* of the central station for the respective day. The load curve can, of course, also be extended over any other interval of time, but usually a 24-hour interval is chosen for reasons of convenient representation. This curve shows clearly at a glance how the load varies within the day. Where it is high above the horizontal axis *OX* the load is large and vice versa. The high parts of the curve are called the peaks and the low parts the valleys. The highest peak represents the peak load, hence the name of the latter.

In exactly the same manner we can draw a load curve for any individual customer or any group of customers.


Kilowatts
2000
1000
0
A
0
2
4
6
8
10
12
2
4
6
8
10
12
X
AM
Noon
PM
D_o
E
C_o
Kilowatts
A_o
B_o
Peak Load
Average Load
O
D
Hours
B
X
0
10
20 Hours

Figs. A and B.

The general shape of the diagram, Fig. B, is typical for what we can expect in an average central station on an ordinary winter day. Beginning at midnight we see the load is comparatively low, naturally so, because most of the customers have shut off their current entirely. The load is mostly street lighting and street-car service, provided that the latter is not furnished from the street-railway company's separate power house. A few late birds who are still up at midnight go to bed by and by and the load reaches a minimum until the first early risers turn on their lights. The demand for this kind of lighting rises until daylight comes and then it may decrease. At that time, however, the factories begin to work, drawing a rather heavy power demand. Such stores and offices as need artificial illumination in daytime turn on their lights and thus we get an increase in the load during the earlier part of the forenoon. The motors in the factories which consume a considerable portion of the power generated are then shut off temporarily during the noon hour and accordingly we see a valley in the load curve at noon. At about 4 or 5 oclock in the winter time the electric lights are turned on everywhere and we get the highest peak of the day. Soon after that time stores and offices close, causing a rapid falling off of the curve.

Of course the daily load curves of central stations differ from each other quite considerably according to the season of the year and to the local conditions.[1]

The load curve gives us more information than simply whether the load at a particular time is high or low. The area under the load curve is a direct measure of the number of kilowatt-hours delivered, in other words, it is proportional to the energy delivered (or consumed, respectively). To understand this let us begin with a simple case. A central station is assumed to be delivering 1000 kw. uniformly during 24 hours of the day. Then the number of kilowatt-hours delivered during the day will be 24,000. The load curve will be a rectangle, the height of which is 1 in. (if measured in the scale selected above for the purpose), that is, 1000 kw. The base of the rectangle is, of course, a length representing 24 hours. Since the area of the rectangle is given by the product of base and height, or in our case by 24 × 1000 = 24,000, we see that in this simple case the area under the load curve is equal to the number of kilowatt-hours. The same will be true not only for a 24-

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[1] Fig. L (Appendix VII) shows the load curve of a December day and of a June day of the same central station. It shows how the evening lighting peak in June not only is smaller (due to less commercial lighting) but also comes much later than in December.

hour period, but also for any other period of time. For instance, if we have 100 kw. during $\frac{1}{2}$ hour, the resulting energy is $100 \times \frac{1}{2} = 50$ kw-hr. and at the same time the area under the rectangle, which is the load curve in this case, is also $100 \times \frac{1}{2} = 50$ units.

If we have different loads in successive periods of time of equal length, for instance, the load changing every 15 minutes. Fig. C, the load curve, will be a number of rectangles arranged side by side and the total number of kilowatt-hours will be the sum of the kilowatt-hours delivered (or consumed) in the various periods. The area under the load curve will be the sum of the areas of the individual rectangles. Therefore, in this case also the number of kilowatt-hours is given by the area under the load curve. Now, if in a load curve of that kind the rectangles become narrower and narrower, at the same time increasing in number so that the total horizontal length of the diagram remains constant, the same law will always hold good, no matter how short the periods of time are.

Now we can disintegrate the area under *any* load curve into a number of narrow high rectangles and their aggregate area will equal the area under the original curve the more exactly the larger the number of the rectangles and the smaller consequently their width (Fig. D). We can make the error smaller than any desired limit (however small that may be) by making the rectangles numerous enough. Therefore, we can also say quite generally that the area under any shape of load curve is proportional to the number of kilowatt-hours, that is, to the energy. The unit area is a rectangle with the base, 1 hour, and the height, 1 kw.; it corresponds to 1 kw-hr.

B.—LOAD-FACTOR.

The easiest way to arrive at a conception what the term "*load-factor*" means seems to be to start from the "average load" (or, as it also may be called, the "average power") of a certain period. That is a load of such size that, if uniformly and without change supplied by the central station (or drawn by the consumer, as the case may be) over a certain period, it results in the same number of kilowatt-hours as are supplied (or consumed) under actual conditions during that period.

If we transform the area under a given load curve into a rectangle OA_0B_0B (Fig. B), having the same base, OB, as the load curve, the height OA_0 of that rectangle is the average load during the period over which the load curve extends. The reason why this is so is easy to see. This load, OA_0, if supplied during the period OB, results in the number of kilowatt-hours given by the area of the rectangle OA_0B_0B, which means it results in the same number of kilowatt-hours as have been actually supplied or consumed; in other words, it is the average load.

a.—First Definition of Load-Factor.

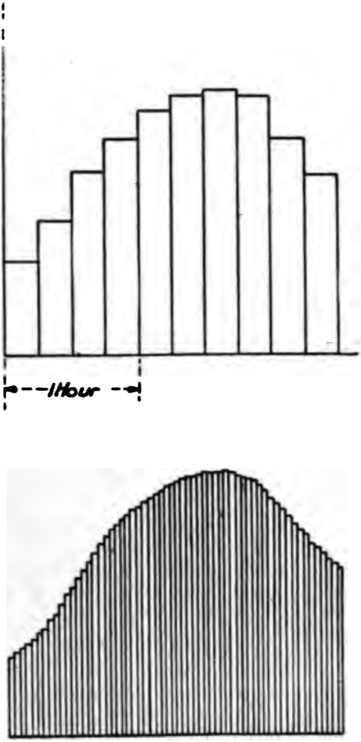
The ratio $\frac{\text{average load}}{\text{peak load}}$, or, in other words, $\frac{\text{average power}}{\text{maximum power}}$, is called the *load-factor* of the central station (or consumer, or group of consumers) for the given period of time. It is usually given in per cent. The load-factor belonging to the load curve Fig. B would, therefore, be the ratio, $\frac{BB_0}{BE}$ or 37%. This would be the daily load-factor of a certain day, because the load curve extends over one day. In practice, however, the load-factor is much more frequently referred to either the year or month, so that we have a yearly or a monthly load-factor.

Since the load curve extending over a month, not to say a year, with its numerous large fluctuations, is a very complicated curve and it is impossible to find a convenient scale for its practical use, the following definition—which in effect is identical with the above—is preferable.

b.—Second Definition.

If we multiply the numerator and the denominator of the above ratio by the number of hours of the whole period (month or year) the value of the ratio is not changed but the ratio takes another form. The average load multiplied by the period over which it extends is, according to the definition of the average load, equal to the number of kilowatt-hours actually supplied (or consumed). The peak load multiplied by the same period is the maximum number of kilowatt-hours which could possibly be supplied (consumed) with the given maximum power in that period, that is, the number of kilowatt-hours which would be the result of the maximum amount of power applying during the whole period. Therefore, the load-factor can be also defined as a percentage expressing the ratio of the energy actually supplied (consumed) to the energy which could be supplied (consumed) in the same time if the maximum load was utilized during the whole time.

The load-factor according to this definition can be easily determined. The actual number of kw-hr. is determined by an ordinary integrating watt-hour meter, and the energy in kw-hr. which could be supplied (consumed), is the maximum load was utilized during the whole time, is



Figs. C and D.

simply found as the product of the maximum load in kw. by the length of the time over which the load-factor is to be taken, in hours. The maximum load can be found by an instrument known as a demand meter (described in Section 140 and in Appendix XVI).

c.—Third Definition.

Finally, we can also assume that the actually supplied (or consumed) energy has been produced by the maximum load uniformly and steadily applied. It will require a certain period of time until the energy is thus supplied and the ratio of this time to the total time over which the load-factor is taken (one month or one year) is also equal to the load-factor, as can be easily shown graphically (see below). The load-factor is then expressed in terms of "hours per year" or "hours per month," respectively. A load-factor of 10% can therefore be expressed as a load-factor of 876 hours per year (since the year has 8760 hours) or as a load-factor of 73 hours per month (since the average month has 730 hours), depending on whether the load-factor in question is a yearly or a monthly one.

The graphic representation of and proof for this third form of defintion of the load-factor is given by transforming the area under the actual load curve into a rectangle of the same area with the maximum load as height. This rectangle OC_0D_0D (Fig. B), therefore, is equal to the energy actually supplied (or consumed); the large rectangle OC_0EB represents the maximum energy which could possibly be supplied (consumed) in the given period with the given maximum power OC_0. The ratio of the rectangles is the load-factor according to the second definition given above of the load-factor. The ratio of these two rectangles is also given by the ratio $\frac{OD}{OB}$, therefore $\frac{OD}{OB}$ is the load-factor.

Where the maximum load is replaced by one of its substitutes (as is shown in Section 141 *et seq.* of the main text—"Substitutes to Approximate the Measured Maximum Demand") the load-factor can also be based on one of these substitutes, notably the connected load instead of on the actual maximum load.

APPENDIX III

(To Sections 18 and 20)

Interest and Depreciation.

A. INTEREST ON THE CAPITAL INVESTED.

(To Section 18.)

The capital invested in fairly sized electric central stations is practically never the property of an individual, but belongs almost always to a corporation and we have to distinguish various ways in which this capital or its parts have been raised, as this influences the distribution of the interest.

A (physical or juristic) person desiring to invest money in an enterprise of that kind may do so either by becoming part owner of the company's property or by lending money to the owners at a certain prearranged rate of interest without acquiring any rights or duties of ownership in the latter case. The company's property serves as security for the loan. In the first case the person investing money receives *shares of stock,* which are certificates that he shares in the ownership, and in the second case he receives *bonds* (or notes), which are receipts for money and a promise to pay together with interest at a rate agreed upon. When a person buys stock he becomes in a sense a partner of the enterprise and if he buys bonds or notes he becomes a creditor and as a creditor has no proprietary interest.

The capital may be raised entirely by bonds (municipal enterprises), or entirely by stocks, or (usually) partly by stock and partly by bonds. The revenue of the enterprise is used to defray the running expenses (wages, salaries, raw material, etc.), after this the fixed amount of interest on bonds and the other capital charges, like depreciation funds, sinking funds, etc., (see Sections 19-22) are taken care of. The stockholders are entitled to the entire balance, which may be large or small or zero, according to the prosperity of the enterprise.

The stock may be all *common stock* or a part of the capital may have been raised by the issuance of *preferred stock*. The difference between common stock and preferred stock is the same as that between stocks and bonds, as far as the payment of interest is concerned. The holders of the preferred stock receive a certain prearranged rate of interest from the balance of the earnings after the bond interest has been deducted and, unless the preferred stockholders can receive their full rate of interest, the holders of the common stock will not be paid any return on their investment. Sometimes the preferred stock is subdivided into first and second preferred, so that the interest on the second preferred is paid out of the balance remaining after the deduction of the interest on the first preferred.

The preferred stock may be "participating" or "non-participating." In the first case, after the preferred stock has received the dividend provided for, and after the common stock has received a certain prescribed dividend, the balance of the profit is divided between the preferred and the common stock. In the second case, which is the usual one, the whole balance goes to the common stock.

The dividend on the preferred may be "cumulative" or "non-cumulative." This means if the enterprise has not had earnings enough to pay the full percentage on the preferred stock in one year the holders of cumulative preferred stock will be indemnified at the expense of the common-stock dividend as soon as the earnings become better in subsequent years, whereas with non-cumulative preferred the holders have no claim for indemnification out of the earnings of later years.

B. A FEW GENERAL REMARKS ABOUT DEPRECIATION FUNDS.

(To Section 20.)

The reasons why the depreciation fund does not, as may seem from the simple example of Sections 19 and 20 of the main text, fluctuate between 0% and 100% of the net replacement cost but within much narrower limits are briefly the following:

In the first place, the plant contains different parts with different lengths of life so that the fluctuations of the respective parts of the depreciation fund overlap each other.

In the second place, the growth of the plant in the course of the years must be considered; in other words, the fact must not be lost sight of that the entire existing plant has not been built at the same time, but at different periods. This causes another overlapping and consequent smoothing out of the fluctuations of the various parts of the depreciation fund[1].

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[1] *Very interesting* investigations on the subject of deprecia-

In the third place, where we have a greater number of parts of the plant, even if all have the same average life and all have been installed at the same time, they will not all have to be replaced in the same year, because the actual life of the individual part is generally not equal to the average life. If, for instance, we have installed in a certain year a new transmission line with wooden poles having an average life of 10 years, though we will have to replace a large number at the end of the 10th year and in the years near it (for instance, between the ends of the 6th and 14th years), yet when the transmission line is old enough so that every pole has been exchanged several times, the varying periods of life of the individual poles evidently will overlap finally to such a degree that we have to exchange the same number of poles every year, namely, 10% of the total number.

That the depreciation fund is far from reaching 100%,

TABLE 1.

I. End of Year.	II. Percentage Poles Requiring First Replacement in Respective Year.	III. Aggregate Percentage of Poles Requiring First Replacement up to Respective Year.	IV. Percentage Payment Into Fund in Respective Year.	V. Aggregate Percentage Paid Into Fund.	VI. Balance Remaining in Fund. (Column V — Column III.)
1	..	..	10	10	10
2	..	..	10	20	20
3	..	..	10	30	30
4	..	..	10	40	40
5	..	..	10	50	50
6	3	3	10	60	57
7	5	8	10	70	62
8	12	20	10	80	60
9	15	35	10	90	55
10	30	65	10	100	35
11	15	80	10	110	30
12	12	92	10	120	28
13	5	97	10	130	33
14	3	100	10	140	40
15	..	100	10	150	50

Note.—Commencing at the end of the 12th year the second replacements begin for which corresponding amounts are taken out of the fund. As the second replacements are not considered in this table, the values of Column VI beginning with the 12th year are slightly too high.

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tion in connection with the growth of the plant are published under the title "Growth and Depreciation," by Julian Loebenstein in the 1916 Transactions of the A. I. E. E., pages 1389 to 1407.

even at the time when the first replacement is due, is shown by Table 1, which is based on the arbitrary but verisimilar assumption that the number of poles which require their first replacement in each year is as given by Column II of the table. (The interest accruements to the fund and the consequent reduction of the annual payments are disregarded in this table as non-essential for the present purpose.)

The maximum amount ever contained in the replacement fund is then 62% of the net replacement value and this maximum occurs in the 7th year. As shown above, the replacement fund is generally reduced in the course of the following renewal periods to the limit of 10%, so that 52% are set free.

It is very interesting to observe that the transition from the 62% to the 10% is not a steady reduction but is made up of oscillating increases and decreases. Column VI of the above table shows this tendency very clearly. The oscillating character of the decrease is still better brought out by the following considerations: Assuming, for instance, the first renewal ("renewal of the first order") to be spread out over four years from the ends of the 8th to the 12th years, inclusive, the first poles will have to be replaced at the end of the 8th year while others will last as long as 12 years. Now, some of the most short-lived poles (8-year poles) will be replaced by equally short-lived ones. This may be an accidental coincidence or the local conditions may be so unfavorable that in that location the renewed poles will generally require replacement after every 8 years; at any rate, the period of the second replacement of poles will begin at the end of the 16th year. On the other hand, some of the most long-lived poles (12-year poles) will last another 12 years after replacement before a third pole has to be installed in that locality. Thus the second replacement period is spread out between the ends of the 16th and the 24th years—that is, over 8 years as against 4 years of the first replacement period. Following the same line of thought further, we get the data of Table 2.

TABLE 2.

Replacement.	After	Years Duration.
1st	8th to 12th year	4
2nd	16th to 24th year	8
3rd	24th to 36th year	12
4th	32nd to 48th year	16
5th	40th to 60th year	20

Since in every renewal period the same total number of poles has to be replaced, namely 100%, and as the renewal period spreads out over greater and greater lengths of time, it is obvious that, as the renewals progress, less and less poles e to be renewed in every year as far as one certain order of renewals (first, second, third, etc.) is concerned. On the other hand, the renewals of different orders overlap more and more. For instance, in the 34th year we have a number of poles to be renewed for the third time and others for the fourth time (see third and fourth lines of Table 2). These conditions are shown in Fig. A. The lower part shows how around each renewal period (10th, 20th, 30th, etc., year) renewals have to be made and how the lengths of the renewal periods spread out from one period to the next, whereas the number of renewals of the same order to be made during one certain year decreases as time goes on. The upper part of Fig. A then shows the sum of the renewals of all orders in every year and demonstrates clearly the fluctuations with decreasing amplitude (damped oscillations), converging to the 10% limit[2].

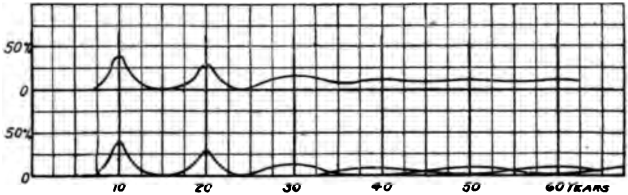

Fig. A.—Renewals of Wooden Poles Having an Average Life of 10 years, Showing the Overlapping of the Successive Renewals (Lower Curve) and Their Sum (Upper Curve).

The interesting subject of depreciation is sufficient for a treatise in itself, but the above remarks are all that is considered necessary for the scope of this book.[3]

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[2] M. D. Cooper, of the National Lamp Works, has made interesting investigations on a closely allied field, the replacement of burnt-out incandescent lamps (Electrical World, Jan. 12, 1918, pages 93 to 95) in which he shows that the number of renewals of lamps installed at the same time and always burning simultaneously is subject to periodic fluctuations until, after a number of lamps have burned out in each socket, the stable condition is reached.

[3] Besides pages 246-249.

APPENDIX IV

(To Footnote of Section 24)

The Influence of the Central-Station's Capacity (Peak Load) on the Operating Expenses.

The example of Section 24 deals with a four-hour load of 5000 kw. and a 20-hour load of 200 kw. every day, and it is assumed that to carry this load a 5000-kw. generator would be installed.

We will, of course, try to improve on this and choose the size and the number of the generators in such a manner that a better 24-hour efficiency results. Other factors beside the efficiency curves also take a part in this problem; for instance, the reserve capacity, changes of load to be expected in future, the prices of generators of different sizes, the value attached to efficiency of generation as compared to savings in the capital invested, etc. The price of the fuel, oil, etc., will in its turn influence the relative value attached to the efficiency of generation and we might continue this analysis of influencing factors until we arrive at the mood in which the designer of the plant found himself when he made this decision about the size of the sets and whether at that time he happened to take a more or less optimistic view of the future growth of the plant, etc.

Assuming that we could be fairly positive about the constancy of the load curve in the future, special investigations might show that it is advisable to install two units of 5000 kw. (one of them as reserve) and one of 200 kw., or possibly three 2500-kw. sets, or some such combination. Now comparing, for instance, the first one of these two combinations (2 × 5000 + 200 kw.) with the 1000-kw. set for uniform 24-hour service (see Section 24) it is evident that the efficiency will probably be greater than at uniform 24-hour service, because the bulk of the energy (20,000 kw-hr. or more than 83%) is generated by an economical 5000-kw. generator at full load—that is, by a generator five times greater than in case of the uniform 24-hour service, whereas less than 17% is generated by a (fully loaded) generator only one-fifth as large (200 kw.) as in case of the uniform 24-hour service. This means that the demand will have a *negative* influence on the fuel and oil consumption; in other words, the larger demand will result in a *reduction* of the fuel and oil cost. (The same will probably be true for the 3 × 2500 kw. combination.)

This seems peculiar. In the first instance [using 5000-kw. units throughout (Section 24)] we saw that an increase of the peak load from 1000 to 5000 kw. resulted in an *increase* of the fuel, etc., cost and now the same change of the peak load results in a *decrease* of these costs. The explanation is that a certain portion of the fuel, etc., cost is neither proportional to the demand nor to the energy consumption, but depends on some other elements which are not among the three fundamental items considered in our system of cost computation (compare Section 10). A portion of the fuel consumption depends, for instance, on the shape of the load curve, another portion on the size of the machines, etc. We try to apportion this part of the fuel cost somehow into the three items, just as we squeeze in the salaries of the executive officers, the advertising expenses, etc., somewhat sacrificing accuracy for simplicity and practicability, as explained in the general discussion of costs. The result of these little inaccuracies, which are entirely permissible, are little inconsistencies as the one just quoted above—the relation of the demand and the cost of fuel. To be exact we would have to say: Under the given conditions with regard to changes of the load curve, size of machinery, etc., a certain given portion of the fuel, etc., costs is proportional to the maximum demand and the factor of proportionality has a certain given—positive or negative—value. More particulars about this question are given in Appendix VIII about the determination of the numerical values of the three items of cost.

APPENDIX V

(To Section 25)

Approximations Made to Compute the Demand Cost of the Plant.

The approximating assumptions which permit us to set the demand cost of an electric plant as proportional to the peak load of the plant (see Section 25 and footnote 4 of Section 17) are shown in comprehensive form in the diagram of Fig. A. These approximations bring about much less inaccuracy than would seem at first sight, because we have to deal always with the changes produced in one *individual* central station only, by the changes of its peak load and these

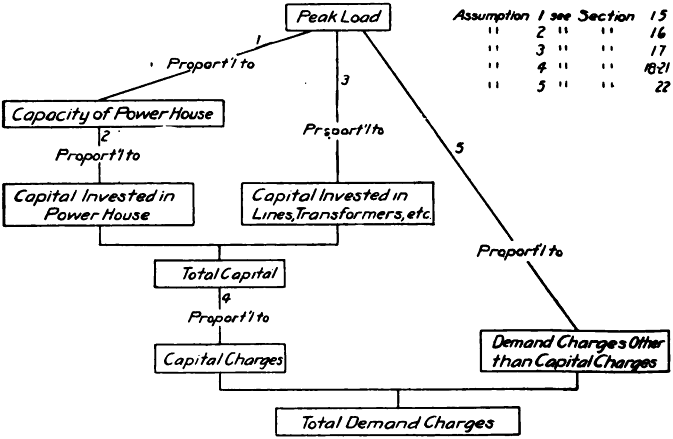

Fig. A.—Diagram Showing Assumptions as to Proportionality of Demand Charges to Peak Load.

latter changes remain within certain limits during a given period.

Thus, for instance, if the peak load of the central station increases from 5000 to 6000 kw. in one year, the other conditions, such as the geographic distribution of the load over the territory of supply (approximation 3), or the percentage of reserve capacity, etc. (approximation 1) will not have to be changed very materially. If, however, these conditions do vary to a large degree in the course of years, this will simply mean that the factor of proportionality—that is, the annual capital charges per kilowatt of peak load—will have to be changed.

Supposing, for instance, that in a certain territory the use of electric service is not very widely spread; then the copper section of the lines is not fully loaded (because the lines must have a certain minimum of copper section for mechanical reasons). If now the use of electric service becomes more general in that town the wires will first become fully loaded and then we will have to string additional wires, but the existing poles can be used for the new wires, so that the cost of the lines does not increase proportionally with the peak load. Larger generator sets will be installed, which

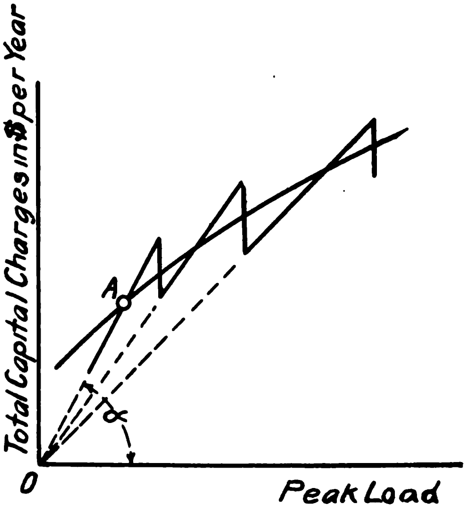

Fig. B.—Relation of Capital Charges to Peak Load.

cost less per kilowatt of capacity than those originally installed. For these and many other reasons the capital, and consequently the total annual capital charges, will under these conditions grow more slowly than the peak load does. Yet we assume that they grow at the same rate, for a number of years at least—that is, that they remain proportional to the capital. If the discrepancy in the course of the years becomes too great we simply change the factor of proportionality—that is, we change the number of dollars per kilowatt per year to another one for the following period.

Readers who are familiar with the elements of graphic representation may get further illumination on this point from Fig. B. (An example of the principles of graphic representation employed therein will be found in Appendix XII on Step Meter Rates.) The central station's peak load is stepped off on the axis of abscissae (horizontal) and the ordinates give the corresponding total yearly capital charges per year, in dollars per year. If we connect a certain point A of that curve with the origin O of co-ordinates, the angle α between this connecting line and the axis of abscissae will be the greater the greater the capital charges per kilowatt of central-station peak load ("unit capital charges") are. (Readers who are familiar with elementary mathematics will understand that the unit capital charges are given by tan α.) Assuming the unit capital charges as being constant means that we replace the curve of the actual unit capital charges by a straight line starting from the origin with an inclination depending on the size of the assumed constant unit capital charge. If this angle of inclination is chosen properly—that is, if the value for the constant unit capital charge is chosen properly—the straight line will more or less closely follow the curve, at least for a certain distance, because the curve must necessarily rise from left to right. Where the deviation of the straight line from the curve becomes too great we substitute another straight line. This means that when the peak load in the course of the growth of the central station becomes so great that the constant unit capital charge used so far becomes too inaccurate, we revise the calculation of the unit capital charge and substitute a corrected value.

In normal times this revision will always be a revision downwards, owing to the smaller unit capital which is necessary for a large plant. But in times of war, or when money is very scarce, etc., it is conceivable that in spite of the growth of the plant, the unit capital charges may rise.

APPENDIX VI

(To Section 28)

Apportionment of the Demand Cost to the Individual Consumers, Considering the Shapes of the Load Curves (Diversity-Factor).

(Containing Elementary Mathematics.)

It is easily shown that the "Peak Responsibility[1]" is not the theoretically correct measure for the demand charge of a certain consumer.

Starting with the assumption of the simple case that the central station has only two consumers *A* and *B* with a plain rectangular load curve each, as per Fig. A, it might be believed at first sight that consumer *A* has to carry the demand charges alone, whereas *B* is using only a part of the central-station capacity which has to exist anyway in order to serve *A*. The possibility of serving *B* will therefore be only a by-product.[2]

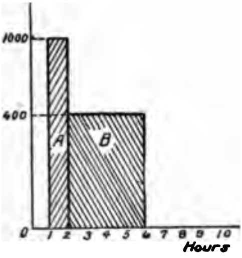

Fig. A.

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[1] For the definition of this term see Section 26 of the main text.

[2] This would be as logical as if we would charge the first-coming passenger on a car with all the expenses of running that

That this way of reasoning is wrong can be easily seen if we assume that *B* is increasing his demand gradually up to 800, 900, 990 and 999 kw. As soon as he has gotten so far, the slightest increase of his maximum demand would throw the whole demand charges[8] onto his shoulders and *A* would just as suddenly be entirely relieved of demand charges, if the above way of reckoning were correct. If *B*, then, should slightly reduce his demand by one or two kw. the whole conditions would be reversed again.

Evidently these unstable conditions are not only theoretically incorrect but also commercially unsound. The correct point of view can be obtained from the following reasoning:

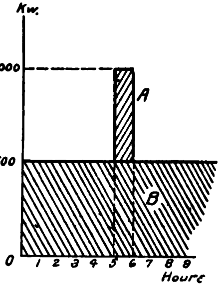

Fig. B.

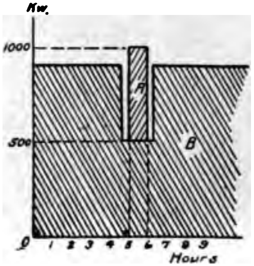

Fig. C.

Let us assume again that the central station has only two consumers *A* and *B*, each of which is using uniformly 500 kw., but, whereas *A* is using his demand only during one hour of the day, from 5 to 6 o'clock, *B* is using his load continually 24 hours a day (Fig. B). Evidently *A* should then be charged less for demand than *B* because such new customers as can be taken on without increasing

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car and let the others ride free, arguing that the car has to run anyway in order to serve the first-coming passenger and the resulting possibility of carrying the later-coming passengers is only a by-product.

[8] Where the term "Charge" occurs in the text of this investigation it is always to be understood in the accounting sense of the word, that a consumer is considered to be the cause of a certain amount of demand cost but not in the sense that he ought to be made to pay that or any other amount. The cost, moreover, is the average cost and not the increment cost. Price-making is distinctly different from cost apportionment on account of the entering of the element of "Value-of-Service." (This will be taken up in Part II of this book—Price of Electric Service.")

the capacity of the central station will draw their demand from that part of the central-station capacity which is being used by *A*, whereas *B* uses his part of the central station all for himself. If both the consumers, *A* and *B*, had the same load curve as *B*, the central station would not have the possibility of taking on any new consumers at any time without increasing its capacity. This principle becomes still more clear if we assume the load curve of the two consumers to be as represented in Fig. C. The "Peak Responsibilities" of *A* and *B* in this case still are 500 kw. each and yet it is evident that *B* should pay higher demand charges than *A* since by the shape of his load curve he is crowding out practically all possibility of taking on any new consumers with the given capacity of the central station. This possibility has been left open to a large extent by consumer *A*, but is utilized by *B* for his own purposes[4].

Finding the Proper Demand Charge Based on the Consumer's Load Curve.

The value of the proper demand charge which ought to be assessed to a customer as cost, considering the shape of his load curve, will be found by the following line of reasoning.

Let us again assume two consumers only being on the lines, with a maximum demand of, say, 1000 kw. each, and the shape of the load curves of these two consumers be as indicated in Fig. D (a) or D (b). The total capacity of the central station is then also 1000 kw., and the "Peak Responsibility" of each one of the consumers amounts to the same number of kilowatts. It does not need any elaborate proof that in this case 1/5 of the total charge will have to be charged to *A* and 4/5 to *B*[5].

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[4] Readers who have not given much attention to this problem may find it paradoxical that a consumer with a large load-factor (referring the load-factor to the peak responsibility instead of to the maximum demand, as usual) shall be charged with a higher demand cost per kilowatt than another consumer with a smaller load-factor. It has been our mental habit to consider the consumer with a high load-factor a "favorable" one who causes smaller cost than the low-load-factor consumer. We must, however, not confuse the "demand cost per kilowatt" with the "total cost per kilowatt-hour." Even under the theoretically correct principle, explained above, the total cost per kilowatt-hour will be considerably lower for the consumer with large load-factor than for the consumer who with a comparatively large peak responsibility consumes only a small number of kilowatt-hours.

[5] The reader should keep in mind that this appendix is dealing with the distribution of the cost and not with the apportionment of prices.

As regards price-making, the possibilities of using the same -

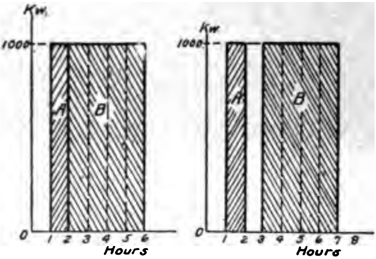
The matter becomes a little more complicated if we assume two customers with the load curves as indicated above in Fig. A. There will be no doubt as to how the demand charges are to be distributed in this case, if we consider the central-station capacity as consisting of two parts, one of 600 kw. which is being used by both *A* and *B*, and another one of 400 kw. which is for the use of *A* only. *A* will then be charged for

$$(1/5 \times 600) + 400 = 520 \text{ kw.}$$

and *B* for $4/5 \times 600 = 480$ kw.

Together 1000 kw.

We see that the demand cost which is to be charged to a certain consumer may always be expressed by a certain number of kilowatts. This number of kilowatts will be called the *"equivalent demand."* If, for instance, we had



Figs. D(a) and D(b).

two 24-hour consumers, one with 520 kw. continuous demand and the other with 480 kw. continuous demand, the demand cost would be distributed between the two consumers in the same way as in Fig. A and for that reason the term "equivalent demand" has been chosen. Obviously, the sum of all equivalent demands must be equal to the sum of the peak responsibilities or, in other words, to the central-station peak (disregarding in that latter case the transmission and distribution losses).

Some doubt might arise, however, as to whether the given load curve of *B* should be really drawn for that pur-

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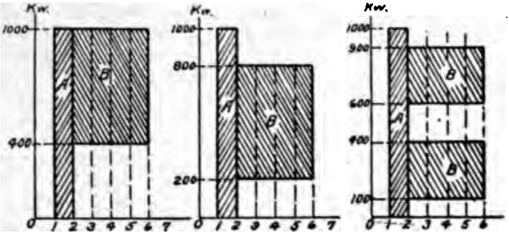
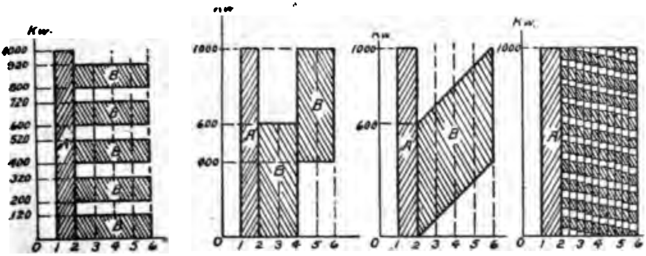
central-station capacity at different hours for different customers should be considered as co-ordinated by-products (see Section 4) and the price should be made according to the Value-of-Service principle (see Part II of this book.)



Figs. A (a) to A (c).

pose as indicated in Fig. A, since there is an infinity of other ways in which a load curve might be drawn for the same load. Some typical examples of the different ways of drawing the load curve of customer *B* of Fig. A are shown in Figs. A (a) to A (g). In the "Note" at the end of this Appendix it will be fully explained what effect these different methods of drawing the load curves would have on the distribution of the demand charges, and the proof is given there that the method as used Fig. A is the correct one. In other words, we arrive at the principle that all curves should be drawn in the customary way, that is starting all ordinates from the axis of the abscissae.

Let us assume as the next step that we have four consumers on the lines, and not more, all of which again shall have a plain rectangular load curve. Three of these consumers A_1, A_2 and A_3 have this in common that they are drawing their current all at the same hour, as shown in Fig. E. These consumers will, for convenience, be called



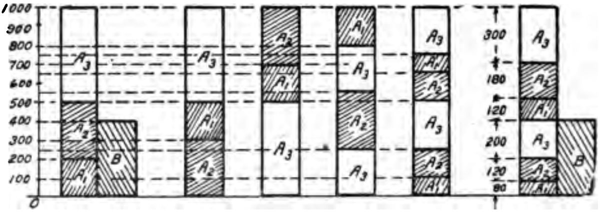
Figs. A (d) to A (g).

"*A*-consumers" hereafter. The fourth consumer *B* is using his demand at some other hour for the same length of time. The amounts of the demand of each consumer in kilowatts can be seen from Fig. E. We can again subdivide the total capacity of the central station of 1000 kw. into two parts as has been done in Fig. A, one part-station of 400 kw. (lower part of Fig. E) being used by the *A*-consumers first and later by *B*, whereas the other part-station, of 600 kw. (top part of Fig. E), is used only by the *A*-consumers and then stands idle during the rest of the day.

The demand cost for the first part of the central station will have to be divided into equal parts, between *B* on the one hand and the *A*-consumers on the other hand. It is unknown so far in what proportion the three customers *A* have to divide their half of the charges for 400 kw. between them.

The second part of the central station has a capacity of 600 kw. and the demand charges for this have to be paid by the *A*-consumers to the exclusion of B. Here again it is unknown as yet in what relation A_1, A_2 and A_3 ought to divide up these expenses between themselves.

We can no longer, of course, resort to the expedient of just choosing the lower part of the load curve of the *A*-consumers as the part which is to be charged to the latter and to *B* in common (as has been done in Fig. A), since we can pile the load curves of the individual *A*-consumers upon each other in an infinite variety of ways, some of which are shown as examples in Figs. E (a) to E (d). Obviously, each of these methods would then have a different effect upon the distribution of the demand charges.



Figs. E, E(a), E(b), E(c), E(d), E(e).

Evidently those *A*-consumers who are using that part of the central station which is also used at another hour by consumer *B* will find themselves at an advantage, so to speak, since they are charged only one-half of the amount per kilowatt as if they had to use that part of the central station which is used only by consumer A[6].

It appears logical, therefore, to allow each one of the A-consumers to partake of that favored 400-kw. portion in proportion to his own maximum demand. In other words, we divide the total capacity of 1000 kw. into two part-stations, one of 400 kw. which is used by the A-consumers and by B, and the other one of 600 kw. which is used by the A-consumers alone. We then allot a proportional share of each one of these two part-stations to each one of the A-consumers. Thus, for instance, consumer A_1 who with his 200 kw. is using 20% of the total capacity of 1000 kw. shall get 20% (=80 kw.) of the 400-kw. station, and 20% (= 120 kw.) of the 600-kw. station. A_2 shall get 30% of each one of the two stations (120 and 180 kw., respectively) and A_3 50% (200 and 300 kw., respectively). This distribution is indicated in Fig. E (e). It is easy now to find out what the charges for each one of these four consumers ought to be, as follows:

A_1............ 80/2 + 120 = 160 kw.
A_2............120/2 + 180 = 240 kw.
A_3............200/2 + 300 = 400 kw.
B ....400/2 = 200 kw.
1000 kw.

Let it now be assumed as the next step that we have a larger number of consumers on the central station, as per Fig. F. During the first hour we have any number of consumers A_1, B_1, ... E_1, but all with rectangular load curves, using current during the whole hour; these load curves are piled on each other. During the second hour there are also a number of consumers, let us say three: A_2, B_2, C_2, with the same character of load curve as just explained. In the same manner we have a certain number of consumers in every following hour up to and including the n^{th} hour (A_n, B_n). Let the small letters a_1, b_1, a_2, b_2 .... indicate the number of kw-hr. (or maximum demand, or average demand, which in this case is the same) of every respective customer. Let it be further assumed that h_1, h_2, h_3

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[6] It should be distinctly understood that this investigation deals with the computation of the demand cost which the consumer causes to the central station and not with the demand charges which the central station should make to the customer as a price. The cost, however, is an important, though not the only factor for determining the price (see Part II of this book: "The Price of Electric Service") and the consumer who causes a smaller cost is liable to be charged a smaller price on the average.

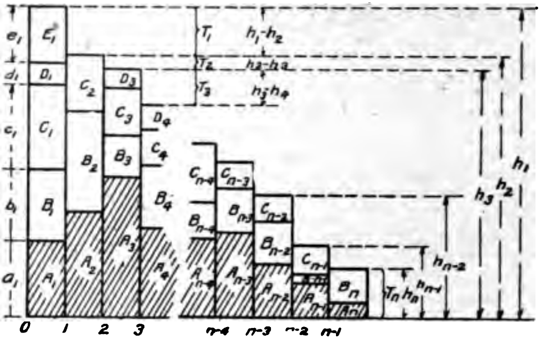

Fig. F.

be the aggregate number of kilowatts used during each one of these periods as indicated in Fig. F. Let it be further assumed that the groups of consumers as indicated by corresponding indices are arranged in the order of their aggregate size so that $h_1 > h_2 > h_3 > h_4 > \dots\dots$

What will then be the equivalent number of kilowatts, K_{a1}, for which customer A_1 will have to be charged?

The total capacity of the central station necessary for all customers mentioned will be h_1. Let the central station of this size be subdivided again into n small part-stations T_1, T_2 ......T_n of the respective sizes $(h_1—h_2)$, $(h_2—h_3)$, ... $(h_{n-1}—h_n)$, h_n (see Fig. F).

The first-named central station T_1, which has the capacity of $h_1—h_2$ kw. and is represented as the uppermost part in Fig. F, is being used not longer than during one hour, as can be seen from Fig. F. The second station T_2, of the size $h_2—h_3$ is being used during two hours. The nth central station or part central station of the capacity of h_n kw., which occupies the lowest part of the diagram Fig. F, is being used during all the n hours. According to what has been said above, we will have to apportion a certain part of the demand of A_1 to the first-named central station T_1, another part of A_1 to the second T_2 and so on, a certain part to each one of those past central stations. The size of the part of A_1 apportioned to the top central station T_1 will be $a_1 \times \frac{h_1 - h_2}{h_1}$. The part allotted to the second central station T_2 will be $a_1 \times \frac{h_2 - h_3}{h_1}$, etc., and the part belonging to the last central station T_n will be $a_1 \times \frac{h_n}{h_1}$. The demand cost for the part apportioned to the first central station will have to be charged to A_1 alone. Of the demand cost of the central station T_2, only one-half will have to be charged to customer A_1, whereas the other half is charged to such other customers as are using central station T_2 during the second hour. In the same manner we will continue and finally find that the charges for that part of a_1 which is allotted to the nth or bottom central station are divided into n parts of which only one is to be paid by customer A_1.

The total charges of A_1 expressed in equivalent kilowatts (that is, the "equivalent demand" in kilowatts) are therefore

$$K_{a1} = \frac{a_1}{h_1}\left[\frac{h_1-h_2}{1}+\frac{h_2-h_3}{2}+\ldots\ldots+\frac{h_m-h_{m+1}}{m}+\ldots\frac{h_{n-1}-h_n}{n-1}+\frac{h_n}{n}\right]$$

In the same manner the equivalent maximum demand for A_2 will be found as

$$K_{a2} = \frac{a_2}{h_2}\left[\frac{h_2-h_3}{2}+\frac{h_3-h_4}{3}+\ldots\frac{h_m-h_{m+1}}{m}+\ldots\frac{h_{n-1}-h_n}{n-1}+\frac{h_n}{n}\right]$$

or, in general, of the customer A_m

$$K_{am} = \frac{a_m}{h_m}\left[\frac{h_m-h_{m+1}}{m}+\ldots\frac{h_{n-1}-h_n}{n-1}+\frac{h_n}{n}\right]$$

Let now all the consumers characterized by the letter A (that is A_1, A_2, ... A_n) be combined into one new consumer A, whose load curve is represented by the shaded part of Fig. F. Then evidently the demand charge of this consumer A is the sum of the demand charges of A_1, A_2, ... A_n, and the equivalent maximum demand of A is the sum of $K_{a1}+K_{a2}+\ldots+K$[illegible]

If we call the quotients $\frac{a_1}{h_1}$, $\frac{a_2}{h_2}$, ... $\frac{a_n}{h_n}$ for the sake of brevity $q_1, q_2, \ldots q_n$, then we get by carrying out the addition of $K_{a1}+K_{a2}+\ldots+K_{an}$:

$$K_a = (h_1-h_2)q_1 + (h_2-h_3)\frac{q_1+q_2}{2}+\ldots+(h_m-h_{m+1})\frac{q_1+q_2+\ldots q_m}{m}+$$

$$\ldots+(h_{n\ 1}-h_u)\frac{q_1+q_2+\ldots+q_{n-1}}{n-1}+h_n\frac{q_1+q_2+\ldots q_n}{n}\ldots(1)$$

In order to get a method of geometrical construction for this formula we proceed as follows: q is an abstract number and in order to represent it geometrically we have to select a certain length as unity. For reasons which will become obvious later it is most convenient to select the capacity of

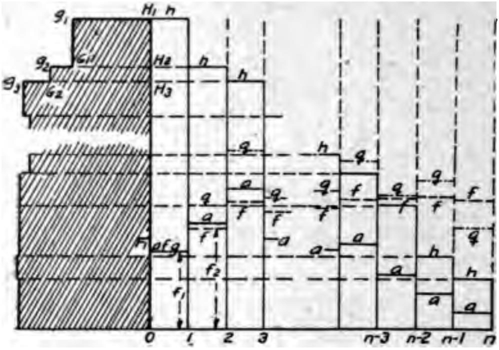

Fig. G.

the central station h_1 as unity. The construction of the value $q_m = a_m/h_m$ is then a simple elementary procedure.

Those parts of Fig. F which are essential for the following have been repeated in Fig. G, that is, the total load curve (staggered curve *h*) and the load curve of customer *A* (staggered curve *a*). The amounts for *q* are entered into Fig. G as dash and dot lines. Evidently for the first interval of time *O1*, *q* and *a* are identical on account of the choice of the unity.

The term $(q_1 + q_2 + \ldots\ldots + q_m) / m$ obviously represents the arithmetical mean of the ordinates *q* between the abscissa *O* and the abscissa of the end of the m^th^ interval of time. This term will, for the sake of brevity, be called f_m hereafter. Thus, for instance, f_3 in the third interval of time is found as the arithmetical mean of q_1, q_2 and q_3 or $(q_1 + q_2 + q_3) / 3$. The curve of *f* is also entered in Fig. G.

Introducing the term $f_m = (q_1 + q_2 + \ldots\ldots q_m) / m$ into equation (1) we have

$$K_a = (h_1 - h_2)\ f_1 + (h_2 - h_3)\ f_2 + \ldots\ldots\ldots\ (h_m - h_{m+1})\ f_m + \ldots\ldots (h_{n-1} - h_n)\ f_{n-1} + h_n\ f_n \qquad (1^*)$$

If we step off now (Fig. G) the length $f_1 = OF_1$ horizontally to the left from the point H_1 (situated on the axis of ordinates) as $H_1\ g_1$ the rectangle $H_1\ g_1\ G_1\ H_2$ will by its area represent the first term $(h_1 - h_2)\ f_1$ of equation (1*).

In the same manner, if $H_2\ g_2$ is made equal to f_2 of the second interval 1-2 of time, the rectangle $H_2\ g_2\ G_2\ H_3$ represents by this area the second term $(h_2 - h_3)\ f_2$ of equa- (1*).

By stepping off the lengths $f_1, f_2, f_3 \ldots\ldots f_n$ horizontally in the manner indicated, we get the shaded surface in Fig. G, the area of which is a measure of K_a according to equation (1*). The length OH_1 has been assumed as unity. If, therefore, the shaded area in Fig. G is transformed into a rectangle over the base OH_1, the height of this rectangle will be equal to the equivalent demand K_a of customer A. The kilowatt scale will be the same as that of the load curves.

The intervals of time 0-1, 1-2, 2-3,......(n—1)-n, in Figs. F and G have been assumed so far for the sake of convenience to be of one hour's duration. Evidently, however, the absolute lengths of these intervals have nothing to do with the above reasoning as long as the intervals are equal among each other. We can, therefore, without influencing the result, reduce the lengths of these intervals more and more, at the same time increasing their number, until we get down to differentials. This will change the staggered curves of Fig. F and G into smooth continuous curves (Fig. H).

Nor has the order in which the intervals of time follow upon each other any influence on the amount of the equivalent demand. So far it has been assumed that the load curve of the central station is of such nature that it starts with the peak load and that the load curve of the central station is steadily falling from the left to the right. This, however, is an assumption which in practice will hardly ever be fulfilled. The load curves go up and down in an irregular manner and have one or more peaks and valleys (maxima and minima).

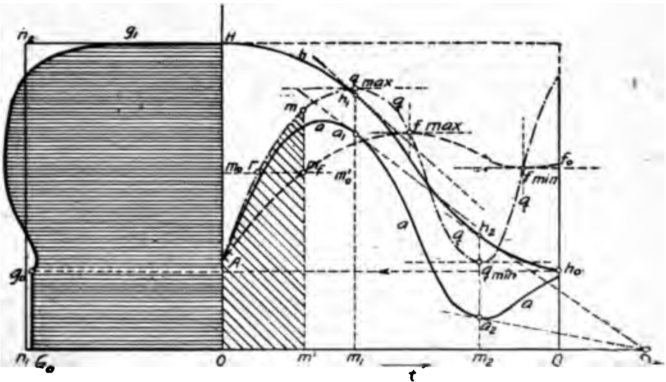

Fig. H.

We might now just as well have had the time intervals of Fig. F arranged in any other order than what Fig. F shows without causing any change thereby in the amount of the equivalent demand, as long as the demands themselves are not changed. For instance, the time intervals might be arranged so that the staggered duct of lines first rises and then, after reaching a maximum h_1, falls again or they might be arranged to form several peaks and valleys in the central-station duct of lines. The resulting equivalent demand will not be changed by this, because it is not essential that, for instance, the interval with the central-station load h_2 is to the right of the interval with load h_1 (Fig. F) nor that it immediately adjoins it. We can follow exactly the same line of reasoning and come to the same conclusions if this is not so. But it is more convenient to have the intervals arranged in the order of the magnitude of their respective central-station load, especially where the load curve is no longer a duct of horizontal straight lines but a steadily curved line, in other words, where we have an infinitely large number of infinitesimally small time intervals.

The first problem to be solved in a practical case is, therefore, the transformation of the load curve of the central

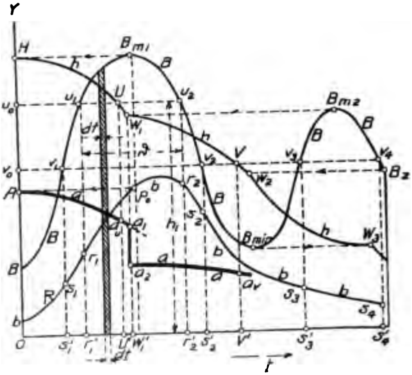

Fig. 1.

station into another curve by a transposition of the time elements so that the latter are arranged in the order of the corresponding central-station loads, beginning with the *highest load* (peak load) at the extreme left and ending with *lowest.* Designating the original central-station load curve as curve *B* (Fig. 1), we will by this rearrangement transform the *B* curve into a curve corresponding to the duct of lines with the ordinates h_1, h_2, ... (Fig. F). The resulting curve will for that reason be designated as the *h*-curve. It will steadily fall from the left to the right, or at least rise nowhere to the right.

This transformation of curve *B* will also result in a transformation of curve *b*, that is, of the load curve of the individual consumer whose equivalent demand we wish to determine. Every moment of time—to whichever position it has been shifted—must always remain connected, not only with the same central-station load which it had originally as per curve *B*, but also with the same load of the individual customer as given by curve *b*.

The customer's load curve thus changes into another curve to be called curve *"a"* hereafter because it corresponds to the shaded part of Fig. F, that is, to the aggregate of the load curves of all consumers *A*.

From *"h"* and *"a"* the curves *"q," "f,"* and *"g"* will be found successively, the meanings of which have been explained before. Finally, by transforming the area under curve *"g"* into a rectangle the equivalent demand of the customer with the load curve *"b"* is obtained as the height (or rather width) of that rectangle in the horizontal direction.

These operations, although not very simple, still are not as intricate as it may appear at first sight and they are made in less time than it takes to describe them.

1. Curve "h."

Given curve *"B,"* that is, the load curve of the central station, it must be transformed in such a way that the ordinates are arranged from left to right in the order of their lengths, to get the curve *"h."*

The construction of this curve is as follows (see Fig. I).[1] We have to cut up, as it were, the whole area under the curve *B* into vertical strips of very small (infinitesimal) width *dt* and then rearrange these strips in the order of their magnitude from left to right. One of these strips is shown shaded in Fig. I.

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[1] To avoid misunderstandings it should be kept in mind that Fig. I deals with the construction of the *h*-curve and of the *a*-curve from a given *B*-curve and *b*-curve, whereas Fig. H deals with the construction of the *q*, *f* and *g*-curves, beginning with an assumed *h*-curve which is not identical with the *h*-curve arrived at in Fig. I. Fig. L (see later), being an example from practice, will then show how the different operations tie together if carried out consistently on the same set of curves.

From the highest maximum of B, that is from the point B_{m1}, a horizontal line is drawn towards the axis of ordinates; it intersects the latter at the point H. This point evidently is the starting point of the h-curve. Considering now some other ordinate h_1, there will generally exist at least two points u_1 and u_2 on curve B which correspond to this ordinate. If now we select all those of the above-mentioned vertical strips which are higher than h_1, in order words, which correspond to ordinates larger than h_1, and arrange them side by side beginning from the axis of ordinates in order of their magnitude, these strips will take up a space of the width $= u_1u_2$. The curve h will therefore contain a point with the ordinate h_1 and the abscissa $u_0U = u_1u_2$. In the same manner any number of points of the curve h can be found. If the respective parallel to the axis of the abscissae intersects the load curve of the central station in more than two points—for instance, in v_1, v_2, v_3, v_4, then v_0V must of course be made equal to the sum of $v_1v_2 + v_3v_4$. It will be noticed that the h-curve has several breaks, at W_1, W_2 and W_3. Every maximum (other than the peak load B_{m1}) and every minimum of the B-curve causes such a break, as is easily understood from the construction of the h-curve. Usually the B-curve extends over a complete cycle, which means the end ordinate (point B_2) has the same height as the initial ordinate (point B). If this is not the case, as has been assumed for the sake of getting a general case in Fig. I, the end ordinate at B_2 or the initial ordinate at B, both, will also cause such a break.

In Fig. H, which starts from an assumed h-curve, this curve is so chosen that it does not contain any such breaks, but the method of the subsequent construction is exactly the same where such breaks occur. (Also see Fig. L.)

2. *Curve "a."*

According to the definition (see Fig. F) a is the instantaneous demand of the respective customer at the moment when the total demand on the central station equals h. Fig. I shows, however, that usually there is more than one value of instantaneous demand of the customer corresponding to a certain given amount of load of the central station, since that given amount of central-station load will generally occur more often than once. Thus, for instance, in Fig I when constructing the point U of the h-curve which has the ordinate h_1 we find that there are two points in the central station's load curve B which have that same ordinate h_1; con-
ently, there will be also two ordinates (generally of dif-
it lengths) of the customer's load curve b corresponding at ordinate h_1 of the h-curve. They are r'_1r_1 and r'_2r_2.

The question arises which one of these two ordinates should be chosen. Resuming the above-mentioned method of cutting up the area into vertical differential strips it is clear that the strip attributed to point U actually is composed of two parallel strips side by side, each one having a width $dt/2$, one of which owes its existence to $r'_1 u_1$, the other one to $r'_2 u_2$. It is evident from this that the ordinates of the a-curve must be equal to the arithmetical mean of the different ordinates of b belonging to the respective point of the curve h.

Thus $U'a_u = (r'_1 r_1 + r'_2 r_2)/2$.

Every one of the breaks of the h-curve mentioned above (W_1, W_2, and W_3 in Fig. I) is connected with a sudden drop (or rise) in the ordinate of the a-curve. Thus, for instance, for the abscissa OW'_1 the a-curve takes a sudden jump downwards from a_1 to a_2. The reason for this is that at this abscissa the ordinates of the second peak (B_{m_2}) of the B-curve enter into the h-curve and consequently the ordinate of the a-curve is no longer the arithmetical mean of *two* ordinates of the b-curve, situated on either side of the peak B_{m_1}, but it suddenly becomes the arithmetical mean of *four* ordinates of the b-curve, one on either side of B_{m_1} and one on either side of B_{m_2}.

3. *Curve "q."*

Whereas the ordinates of the two curves "h" and "a" dealt with heretofore are given in units of kilowatts, the term of $q = a/h$ is an abstract number and a certain length must be chosen as unity. It has been mentioned before that the maximum demand of the central station (OH in Figs. H and I) should be chosen as unity.

The construction of curve $q = a/h$ is obvious and hardly needs any explanation. It is shown in Fig. J for a certain abscissa OP_1. H_1 and A_1 are the points on the curves h and a, respectively, belonging to the abscissa OP_1. O' is chosen somewhere on the axis of abscissae in a convenient position. H'_1 is found as the intersection of the extension of line $O'H_1$ with a horizontal line drawn from the starting point H of curve h (compare Fig. I). Q'_1 is found as the intersection of $P'_1H'_1$ and $O'A_1$; a horizontal line from Q'_1 furnishes in its intersection with P_1H_1 the required point Q_1 of the curve of q.

$$\frac{P'_1Q'_1}{P'_1H'_1} = \frac{P_1A_1}{P_1H_1}$$

and, since $P'_1H'_1 = OH$ has been assumed as unity,

$$P'_1Q'_1 = \frac{P_1A_1}{P_1H_1} = \frac{a}{h} \text{ or } P_1Q_1 = \frac{a}{h} = q.^*$$

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*An additional feature of the construction of the curv

Where possible, the location of O^1 should be chosen so that $P'_1H'_1$, coincides with the axis of ordinates, which saves a little labor.

Every break in the *h*-curve mentioned above with consequent sudden drop or rise of the *a*-curve corresponds to a sudden drop or rise in the *q*-curve (see Fig. L).

4. *Curve "f".*

According to the definition, the ordinates of the *f*-curve for a given abscissa *Om'* (see Fig. H) are represented by the height of a rectangle erected over the base *Om'* and enclosing

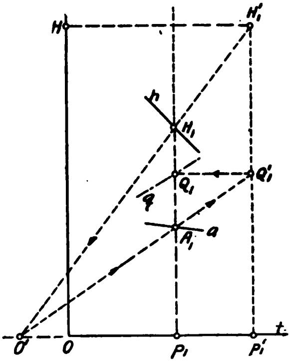

Fig. J.

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the following: $q = a/h$, differentiate with respect to the time t (the abscissa):

$$\frac{dq}{dt} = \frac{h\frac{da}{dt} - a\frac{dh}{dt}}{h^2}$$

From this we see that q reaches a maximum (or minimum) if

$$\frac{\frac{dh}{dt}}{\frac{da}{dt}} = \frac{h}{a},$$

in other words, if the tangents of such points of the two curves *h* and *a* as have the same abscissa intersect each other on the axis of abscissa (for instance, abscissae Om_1 or Om_2 in Fig. H). By means of this principle it is easy to predict with a fair amount of precision the position of the maxima and minima of the

the same area as the surface enclosed by the curve q, the ordinate over the point m' and the two axes of co-ordinates. This last named area is shown shaded diagonally. If we desire therefore to find that point m_1 of curve f which belongs to a certain abscissa Om' we have to select a horizontal line $m_0m'_0$ at such a height that the two three-cornered figures Am_0r and mm_1r have the same area. The use of a planimeter will generally not be necessary and a person with fairly good eye-measure will be able to locate these horizontal lines accurately enough by the eye.

A few additional characteristics as explained hereafter of the curve of f will facilitate an exact and rapid construction of that curve.

Maxima and Minima.—Since the ordinate of every point of f is the arithmetical mean of all preceding ordinates of the curve q, it is evident that f will have either a maximum or a minimum every time it intersects curve q. Thus, for instance, in the left part of the f-curve in Fig. H the ordinates of q are larger throughout than those of f, therefore with increasing abscissae the arithmetical mean of the ordinates of q will grow, that is, curve f will rise. As soon as q falls below f, that is, to the right of the intersection point f_{max} of the two curves, every addition of a new ordinate of q will decrease the average and f will assume a falling tendency until the next intersection point f_{min} is reached, etc.

Direction of tangents of curves.—Expressing the definition of f mathematically we get

$$f = \frac{\int_0^t q\,dt}{t}$$

where f is the ordinate of the curve of the same name and t is the abscissa. From this we get $ft = \int_0^t q\,dt$ and differentiating

$$f\,dt + t\,df = q\,dt$$

$$\text{or } \frac{df}{dt} = \frac{q-f}{t} \quad \dots\dots\dots\dots\dots\dots\dots (2)$$

this means with reference to Fig. K that $t_1m_1t_1$ (the tangent to the curve of f) is parallel to m_0mt_0 where m_0 is again (as in Fig. H) the horizontal projection on the axis of ordinates of the selected point m_1 and m is the point on curve q with the same abscissa as m_1; we thus get easily and quickly the direction of the tangent of every point of curve f. From the definition of f it follows that the curve of f will start from the same point A of the axis of ordinates as the curve of q.

(Figs. G, H and K.) For this point *A* ($t=0$, $q=f$) equation (2) changes into

$$\frac{df}{dt} = \frac{0}{0} = \frac{dq}{dt} - \frac{df}{dt} \dots\dots\dots\dots\dots\dots (2^*)$$

Now $\left(\frac{dq}{dt}\right)_{t=0} = \tan\phi_0$ (see Fig. K) and $\left(\frac{df}{dt}\right)_{t=0} = \tan\psi_0$.

Substituting this in equation (2*)

$$\tan\psi_0 = \tan\phi_0 - \tan\psi_0$$
$$\tan\psi_0 = 1/2 \tan\phi_0.$$

This equation allows us to rapidly and easily find the angle at which *f* rises from its starting point *A* on the axis of ordinates.

Every sudden drop or rise of the *q*-curve as mentioned before causes a sudden change in the direction of the *f*-curve,

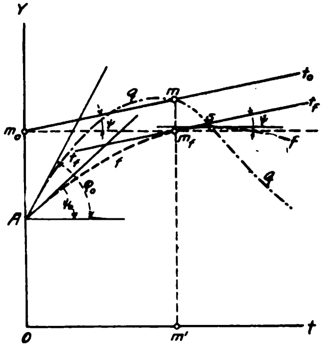

Fig. K.

but in practice these changes in direction, especially for large abscissae, will be imperceptible (Fig. L).

5. *Curve "g."*

The construction of the curve of *g* is so simple that it hardly needs any explanation. Select a point on curve *h* (Fig. H), draw a horizontal line from the same to the left beyond the axis of ordinates and on that line from its intersection with the axis of ordinates step off that ordinate of the *f*-curve which belongs to the same abscissa as the originally selected point of *h*. Repeating this for a number of points we arrive at the curve of *"g."*

After we have thus reached, from left to right, the point ich corresponds to the lowest value O_0h_0 of the *h*-curve (at the extreme right of the latter) the ordinates of the g-curve are constant and equal to the terminating value O_0f_0 of the f-curve. The g-curve, beginning from the point T_0, will therefore change into a straight line parallel to the axis of abscissae OH of the g-curve, that is, to the axis of ordinates of the rest of the curves. (That this must be so is easily understod if we go back to Fig. G.)

Summary.

This whole procedure of establishing the equivalent demand is not a very simple one, but it is more simple than it appears at first sight. It must also be considered that the problem itself, as has been shown above, is an inherently intricate one and involves a great many factors.

We see that the consumer's demand cost is an intricate function of the ***entire*** load curve of the central station ***and*** of the ***entire*** load curve of the respective consumer, not only of certain parts of those curves. The equivalent demand and consequently the demand cost, is represented by the shaded area of curve "*g*," Fig. H, and instead of being simply proportional to the peak responsibility it is a true integral extending over all elements of time over which the load curves extend[3]. As will be demonstrated in the following, however, at those times when the central station's load is low the influence of the consumer's load is small and ultimately even becomes practically negligible compared with the influence of the consumer's load at or near the central station's peak load time. Yet all these infinitesimal influences of the various differentials of time are of the same *order* of magnitude.

It is obvious that the f-curve (which represents by its ordinates the respective areas under the q-curve) has the tendency of approaching more and more a horizontal straight line as the abscissa increases, that is, as the instantaneous load h of the central station decreases. (Supposing, for instance, that the f-curve stands at $0.1 \times OH = 0.1$[†] at the end of the first hour and that $q = 0.3$ throughout the following hour; then at the end of that hour, f will have risen to $(0.1 + 0.3)/2 = 0.2$, or by 0.1, or by 100%. If, however, the f-curve stands at 0.1 as above, not at the end of the first hour but at the end of the 23rd hour, and q is again $= 0.3$ throughout the following hour, then at the end of that hour f will

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[3] This integral is, of course, not identical with the integral which gives the energy consumed $\int_0^t b\,dt$ (Fig. I) and it is of a much more complex nature. Further investigations (which are omitted here) show that it is a double integral.

[†] It should be remembered that OH has been chosen as unity.

be only $(23 \times 0.1 + 0.3)/24 = 0.1125$, or it will have risen only by 0.0125, or by 12.5%.)

This means that the values of the consumer's load for small central-station loads (large abscissae of *h*-curve), where the *f*-curve is already steadied, will not be able to affect the equivalent demand and the demand charges as much as the consumer's load at those particles of time, when the central station's load is near its peak. The consumer's loads near the central station's peak load have a greater influence than where the central station's load is small, so much so that in the generality of cases at all moments when the central station's load is not near peak load the amount of the consumer's load need not be considered. Nevertheless, as will be shown by an example from practice a little later (Fig. L), *the equivalent demand may be very far off from the peak responsibility.*

Theoretically the load curve would have to be taken over

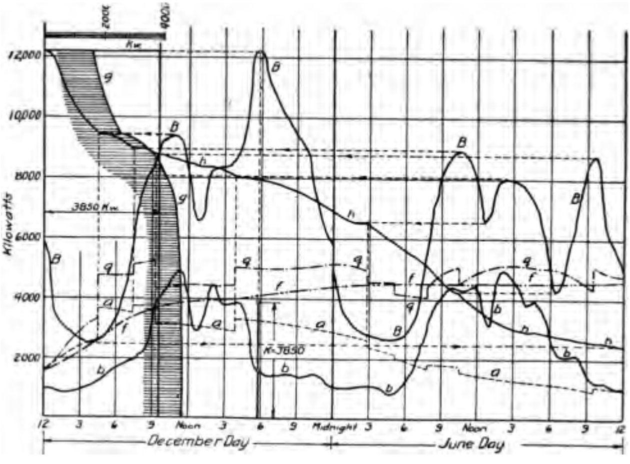

Fig. L.

the whole year. But in practice this is neither possible nor necessary. In ordinary cases it will be sufficient to take one typical day around the winter and one around the summer solstice.

Fig. L is an example from practice showing the curves of a large central station which was the first one to employ this method devised by the author. The diagram has the purpose of determining how the aggregate peak load for light and power should be apportioned between the power consumers and the lighting consumers, in other words, how large the aggregate equivalent demand of each one of these two groups is. Fig. L shows the total lighting and power curve *B* and the power load curve alone *b*. A December day and a June day were selected as representatives of the extreme seasons of the year. We see plainly how every peak and every valley of the *B*-curve causes a break in the direction of the *h*-curve, as explained before, and a sudden jump up or down in the *q*-curve. The resulting breaks in the direction of the *f*-curve are so small that they are hardly discernible in the left-hand part of the curve and disappear entirely to the eye after the first quarter of the total abscissa. We see from this practical example also how the *f*-curve as well as the *g*-curve[3] become practically level, beginning with about the second half of the respective total abscissa.

The peak load of the central station occurs shortly before 6 p. m. in December and amounts to about 12,200 kw.; the peak responsibility of the power load is only about 1600 kw. The equivalent demand, however, in consequence of the great power demand at other hours is not less than 3850 kw. or nearly 2½ times as high as the peak responsibility. The balance of 12,200 — 3850 = 8350 kw. is charged to lighting.

Note.[4]

A glance at Figs. A(a) to A(g) shows that in each case customer *B* has used 600 kw. continuously during the hours between 2 o'clock and 6 o'clock. If the equivalent demand for *B* is figured by the same method as given above (see page 279) but on the basis of Figs. A(a) to A(d), the results will be the same as before in Fig. A. On Fig. A(e), however, we get the following result from the equivalent demands:

Customer A.....$\frac{1}{3} \times 400 + \frac{1}{5} \times 200 + \frac{1}{3} \times 400 = 306\frac{2}{3}$ kw.
Customer B.....$\frac{2}{3} \times 400 + \frac{4}{5} \times 200 + \frac{2}{3} \times 400 = 693\frac{1}{3}$ kw.

1000 kw.

This is very different from the result given on the basis of Fig. A or Figs. A(a) to A(d) on page 279. If *A* and *B*

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[3] In order to save space in the drawing, the ordinates of the *g*-curve have been stepped off to the right of the main axis of ordinates instead of to the left as on the other illustrations. The *g*-curve is emphasized by shading.

[4] Referring to the paragraph on page 280/281.

were to deal directly and independently with each other about the use of the central station (for instance, in case they are joint owners of the central station) *A* might contend that the distribution of the demand charges should be made according to Fig. A(e), whereas *B* would probably claim that the load curves should be drawn as in Fig. A and not as in Fig. A(e). Who is right, *A* or *B*?

The 1000-kw. central station might be imagined in this case to consist of five units of 200 kw. each (it does not matter for the purposes of this deduction whether these five units are actually in physical existence as such or not). According to Fig. A, customer *B* would use none but the first three of these units (counting from the bottom up in the drawing). If Fig. A(a) is made the basis of the computation, he would be using only the three uppermost units, etc. In Fig. A(e) customer *B* would use the three lowest units for the first two hours and the three uppermost units for the last two hours.

Let it now be supposed that this latter assumption [Fig. A(e)] be permissible; then evidently there is no reason why the total time between 2 and 6 o'clock should be divided into only two periods of that kind. With the same right, four subdivisions of time might be assumed, of one hour's length each, or eight, etc.; in fact, there is no upper limit to the number of subdivisions of that period. We can, therefore, assume an infinite number of very small periods and suppose that during each one of these periods *B* is drawing his 600 kw. from another part of the central station. This may result for instance in Fig. A(f). If Fig. A(e) is permissible as a basis of computation, moreover, we are certainly at liberty to assume the subdivision of the demand of *B* also in the manner indicated in Figs. A(c) and A(d). This subdivision in vertical direction can also be carried out to finer and finer degrees, and can be combined with the manner of subdivisions shown in Figs. A(e) and A(f); this will result in a distribution of the demand about as shown in Fig. A(g). If these combined subdivisions are carried out to sizes of infinitesimal order this obviously means that the demand charge be distributed proportionately to the number of kw.-hr. used, independently of the shape of the load curve. This evidently is absurd; therefore, the basic assumption of the whole conclusion must be wrong. This initial assumption has been that the distribution according to Fig. A(e) is permissible and we arrive, therefore, at the conclusion that the distribution according to Fig. A(e) is wrong. With two customers only the distributions of the load curve of the type of Figs. A, A(a) to A(d) are the only ones which are correct, A(e) and A(f), and it is easy to see that they all have the same result as to the equivalent demand charge. An analogous deduction shows that with three or more customers the load curve of every single customer must be drawn in the customary way in order to obtain correct results; that is, with the axis of abscissae as a basis from which to step off all ordinates of every single load curve, as in Fig. A.

APPENDIX VII

(To Footnote of Section 45)

Mean and Weighted Average.

Where we have to deal with averages of fractions or ratios, we have to distinguish between the "mean" average and the "weighted" average. The *mean* average is formed in the same way as the average of all other values, that is, by adding the values and dividing the sum by the number of values which are to be averaged. For instance, the mean average of $\frac{1}{2}$ and $\frac{1}{4}$ is $\frac{1/2 + 1/4}{2} = \frac{3}{8} = 0.375$. The *weighted* average is the sum of the numerators divided by the sum of the denominators. The weighted average of $\frac{1}{2}$ and $\frac{1}{4}$ is therefore $\frac{2}{6} = 0.333..$, but the weighted average of $\frac{50}{100}$ and $\frac{1}{4}$ is $\frac{51}{104} = 0.4904$; of $\frac{1}{2}$ and $\frac{25}{100}$ the weighted average would be $= 0.2549$.

The weighted average applies to such cases where the fractions are ratios and the numerators as well as the denominators are the results of physical measurements or countings or other observations. It is clear that before taking the weighted average, the fractions must *not* be reduced by the elimination of common factors. The larger are the values of both numerator and denominator of a certain individual ratio resulting from a certain observation the greater is the relative importance or weight of the respective ratio if we are concerned with the "weighted average," hence that name. With the mean average every fraction or ratio has the same "weight" as every other one.

An example will make this clear. Supposing, for instance, we would be interested in the average density of population in two neighboring towns—a city and its suburb—the city *A* having a million inhabitants on an area of 50 square miles and the suburb *B* having 10,000 inhabitants on 2 square miles. *A* has therefore a density of 20,000 inhabitants per square mile and *B* 5000. It would be obviously incorrect to say that the average density of population in the two cities is the average of these latter two values, *i. e.*, 12,500 inhabitants per square mile (mean average). The lower density of *B* will have much less weight in influencing the average density because *B* is so small in comparison with *A*. The correct value for the average density is the aggregate population of the two cities divided by their aggregate area, that is the weighted average, $\frac{\text{sum of numerators}}{\text{sum of denominators}}$ or $\frac{1{,}000{,}000 + 20{,}000}{50 + 2}$ $= 19{,}615$ inhabitants per square mile.

APPENDIX VIII

(To Section 60)

Numerical Evaluation of the Three Elements of Cost..

(Contains considerable mathematics in the proof, but the results contain only elementary algebra.)

This method was published by the author for the first time in English in the ELECTRICAL REVIEW AND WESTERN ELECTRICIAN of Aug. 15, 1914. of which article the following is a portion (modified to tie in with the rest of this book.)

1. Calling c_1 the unknown customer cost per customer per annum,

c_2 the unknown energy cost per kw-hr. consumed,

c_3 the unknown demand cost per kw. of peak responsibility or rather of "equivalent demand" (see Appendix VI).

d the peak responsibility or the equivalent demand, respectively, of a certain customer,

e the same customer's yearly energy consumption in kw-hr.,

k the (unknown) total cost of serving that customer per annum,

we get the following equation:

$k = c_1 + c_2e + c_3d$ .... (1)

Adding these equations for all customers of the central station (considering the central station itself as a customer also, as far as its home consumption of electric service is concerned) we get

$\Sigma\,(k) = c_1N_1 + c_2\,\Sigma\,(e) + c_3\,\Sigma\,(d)$ .... (2)

where N_1 is the total number of customers served in that year.

$\Sigma\,(k) = K_1 =$ total annual cost of the central station.

$\Sigma\,(e) = E_1 =$ total energy consumed annually in kw-hr.

$\Sigma\,(d) = D_1 =$ sum of the customers' peak responsibilities (which is equal to the sum of the customers' equivalent demands, see Appendix VI, and to the central station's peak load, neglecting, in the latter case, the losses in transmission, transformation and distribution).

We have thus

$$\left.\begin{array}{l} \qquad K_1 = c_1N_1 + c_2E_1 + c_3D_1 \\ \text{for the following year we find} \\ \qquad K_2 = c_1N_2 + c_2E_2 + c_3D_2 \\ \text{and for the third year} \\ \qquad K_3 = c_1N_3 + c_2E_3 + c_3D_3 \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In these equations the values of K, N, E and D are known and we have, therefore, three linear equations for determining the three unknowns, c_1, c_2 and c_3. On account of the small number of the unknowns, any elementary method, such as the method of substitution or elimination, could be employed for solving the equations.

2. The following investigation will show, however, that the equations (3) can generally not be employed without further changes. The results of this investigation are expressed in simple algebra and readers who do not care to follow the author into the details of the investigation will find the simple results summarized in Section 4 *et seq.* of this Appendix.

These investigations are carried out here with the help of the theory of determinants. Readers who are not familiar with determinants are referred to the author's article in the ELECTRICAL REVIEW AND WESTERN ELECTRICIAN of Aug. 15, 1914, where the same investigation is also carried out graphically with the same results.

3. The values of the three knowns are given by

$$c_1 = R_1/R, \quad c_2 = R_2/R, \quad c_3 = R_3/R$$

where R, R_1, R_2 and R_3 are the following determinants:

$$R = \begin{vmatrix} N_1E_1D_1 \\ N_2E_2D_2 \\ N_3E_3D_3 \end{vmatrix} \qquad R_1 = \begin{vmatrix} K_1E_1D_1 \\ K_2E_2D_2 \\ K_3E_3D_3 \end{vmatrix} \qquad R_2 = \begin{vmatrix} N_1K_1D_1 \\ N_2K_2D_2 \\ N_3K_3D_3 \end{vmatrix} \text{ etc.}$$

The theory of determinants shows that equations (3) cannot be solved by finite values if

$$N_1 : N_2 : N_3 = E_1 : E_2 : E_3 = D_1 : D_2 : D_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

because in that case determinant R becomes zero and the equations will be fulfilled either by $c_1 = 0/0$, $c_2 = 0/0$, $c_3 = 0/0$, or by $c_1 = \infty$, $c_2 = \infty$, and $c_3 = \infty$. In the first case the three equations are identical amongst one another, the problem is underdetermined, and in the second case the equations contradict each other.

The latter case (contradiction of the equations) cannot occur in practice if we take the correct figures, but the first case may take place. It is most improbable that equation (4) will be fulfilled accurately, that the number of customers will grow in exactly the same ratio as the energy consumption and the central station's peak load grows; this means in other words, it is highly improbable that the central station's load-factor E/D will remain absolutely constant and also the average energy per consumer E/N and the average peak responsibility per consumer D/N. But it may happen and it may even be expected to happen that the changes of these three ratios from one year to the next will be very small. This means that the values of the determinants R, R_1, R_2 and R_3 are small as compared to the values of the factors K, N. E and D of which the determinants are composed. The determinants consist of a number of positive and negative members, which will very nearly cancel if equation (4) is very nearly fulfilled; for instance, the determinant will be figured as $1857 - 1854 = 3$. A small percentage inaccuracy of one or more of the constituent members (for instance, 1860 instead of 1854) may, and generally will, result in a very large error of possibly several hundred per cent of the value of the determinant and may even result in negative values for c_1, c_2 and c_3.

Such inaccuracies are inevitable. For instance, the figures we get from the company's books for the annual cost are the amounts *paid* for materials, wages, etc., during the respective year, whereas they ought to be for this computation the cost of materials, etc., actually *used* during that period. That is not exactly the same. The company may have bought a certain amount of fuel, for instance, in one year and used not all of it in the same year, and, moreover, paid for it only in the following year. Or in a certain year a repair has occurred which is particularly expensive and is due to extraordinary circumstances or accidents, for instance a boiler explosion, etc.

4. The requirements of a sufficient variation of the load-factor, the energy consumption per consumer and the average demand per consumer are generally better fulfilled if, instead of three successive years, three months are chosen at different seasons since the load-factor in summer and winter, for instance, reaches entirely different values. The result is further freed from the influence of casual irregularities by choosing not, as assumed heretofore, only three such periods, but by extending the calculation over a large number of periods, such as 24 months, with the result that one obtains average values of a larger number of periods.

This results in 24 linear equations with three unknowns.

There is, of course, in general no solution possible which satisfies all the 24 equations, but it is possible to find those values of the unknowns which satisfy the equations in the most accurate way, that is, with which the sum of the squares of the errors becomes a minimum, or in other words the most probable value. This is done by the method of Gauss which consists in the following:

Let the 24, or in general n, equations be (denoting the unknowns in the customary way as x, y and z):

$$\left.\begin{aligned} N_1x + E_1y + D_1z &= K_1 \\ N_2x + E_2y + D_2z &= K_2 \\ N_nx + E_ny + D_nz &= K_n \end{aligned}\right\} \quad \dots\dots\dots\dots\dots\dots (5)$$

Multiply both sides of the first equation by N_1, those of the second equation by N_2, etc., and add the equations which are obtained thereby:

This results in a new equation:

$$\begin{aligned} &(N^2_1 + N^2_2 + \dots + N^2_n)\,x + (E_1N_1 + E_2N_2 + \dots + E_nN_n)\,y \\ &\quad + (D_1N_1 + D_2N_2 + \dots + D_nN_n)\,z \\ &\quad = N_1K_1 + N_2K_2 + \dots + N_nK_n \quad \dots\dots\dots\dots\dots\dots (\mathrm{I}) \end{aligned}$$

This equation (I) is called the first normal equation. Now if we multiply the first one of equations (5) by E_1, the second one by E_2, ... etc., and add them again we obtain the second normal equation:

$$\begin{aligned} &(E_1N_1 + E_2N_2 + \dots + E_nN_n)\,x + (E^2_1 + E^2_2 + \dots + E^2_n)\,y \\ &\quad + (E_1D_1 + E_2D_2 \dots + E_nD_n)\,z \\ &\quad = E_1K_1 + E_2K_2 + \dots + E_nK_n \quad \dots\dots\dots\dots\dots\dots (\mathrm{II}) \end{aligned}$$

In an analogous manner we get the third normal equation by multiplying all equations (5) by $D_1, D_2 \dots D_n$, and adding them thus obtaining

$$\begin{aligned} &(D_1N_1 + D_2N_2 + \dots + D_nN_n)\,x \\ &\quad + (E_1D_1 + E_2D_2 + \dots + E_nD_n)\,y \\ &\quad + (D^2_1 + D^2_2 + \dots + D^2_n)\,z \\ &\quad = D_1K_1 + D_2K_2 + \dots + D_nK_n \quad \dots\dots\dots\dots\dots\dots (\mathrm{III}) \end{aligned}$$

The three normal equations can be written in a shorter and more perspicuous way as follows:

$$\begin{aligned} \Sigma\,(N^2_m)\,x + \Sigma\,(E_mN_m)\,y + \Sigma\,(D_mN_m)\,z &= \Sigma\,(N_mK_m) \dots (\mathrm{I}) \\ \Sigma\,(E_mN_m)\,x + \Sigma\,(E^2_m)\,y + \Sigma\,(E_mD_m)\,z &= \Sigma\,(E_mK_m) \,..\, (\mathrm{II}) \\ \Sigma\,(D_mN_m)\,x + \Sigma\,(E_mD_m)\,y + \Sigma\,(D^2_m)\,z &= \Sigma\,(D_mK_m)\ .\,(\mathrm{III}) \end{aligned}$$

It is seen that the 9 co-efficients of the unknowns are partly identical with each other in couples according to a law of symmetry which can be easily recognized. This makes their computation easier.

These three normal equations solved for the three unknowns x, y and z yield directly the most probable values for the latter, that is, the values which approximate most closely the simultaneous fulfillment of all the n original equations.

5. The charges found in the foregoing manner require a correction.

The kilowatt-hour charge must be increased in the ratio of kilowatt-hours generated to kilowatt-hours sold, since only the kilowatt-hours sold are paid for and not, as has been assumed heretofore, the kilowatt-hours generated.

On the other hand, the demand charge must be reduced because in practice it is based not on the "equivalent demand" of the customer or on his peak responsibility but on his individual maximum demand. The customers are classified and the ratio $\frac{\text{equivalent demand}}{\text{maximum demand}}$ or $\frac{\text{peak responsibility}}{\text{maximum demand}}$ is then found by measurements or by estimating to our best ability. The average of this value prevailing within every class of customers is then the reduction factor prevailing within that class.

6. The method described above is, of course, not restricted to the number of three charges as one can also aim from the beginning at the application of two charges only, for instance, a kilowatt-hour charge and a demand charge, and then one obtains only two unknowns y and z (c_2 and c_3) in the equations. With such central stations which are also in the electric railroad operating business one could add two more unknowns to the three original ones, namely, u (that is, that part of the cost of railroad operation which is proportional to the number of car-miles) and v (that is, that part which is proportional to the length of track). It should be understood, however, that the calculation of five unknows out of about 24 equations is a rather lengthy operation even with the use of computing machines and it is probably preferable in all those cases to separate the railway cost from the beginning entirely from the rest of the cost, if possible. The expenses which are common to both branches, such as general expenses, must then be distributed arbitrarily to our best judgment between the two branches of the business. Each one of the two branches is then to be resolved into its parts according to the analytic method just shown.

7. As regards the length of the period over which the computation should be extended, it must not be too small so that incidentals do not exert a disturbing influence and *that a* proper average can be found. On the other hand, it *must not* be too large since the amount of the cost for the *different* charges is slowly changing in the course of the

years in consequence of the development of the central station (for instance, the use of more economical generators, etc.). Twenty-four monthly periods are about the proper figure under ordinary circumstances. If during that time a change of rates has occurred, this influences favorably the accuracy of the computation since thereby the character of the use of the current by the customers is changed. The load-factor or the average number of kilowatts or kilowatt-hours, or all of these factors, are liable to change and this, as shown above, makes possible a more exact computation of the unknowns.

APPENDIX IX

(To Section 80 et Seq.)

Relations Between Selling Prices and Earnings.

(For readers who are familiar with mathematics.)

I. PRICES INDEPENDENT OF THE CUSTOMER'S VALUATION OF THE COMMODITY (COST-OF-SERVICE PRINCIPLE).

A. THE FOUR FUNDAMENTAL FACTS.

1. The deductions in this Appendix are based on four facts which are partly so self-evident and partly so generally known that they need no proof and can be accepted as axioms. These four facts are:

Fact No. 1.—Although we do not know just how the quantity m sold of a certain commodity varies with varying unit prices, it is obvious that with increasing prices the quantity sold will steadily decrease—other conditions remaining equal—and vice versa.

Fact No. 2.—If the price p per unit is zero, the quantity m will not be infinity, and with rising prices the quantity m will become zero at a finite value of p.

From Nos. 1 and 2 follows: If we plot the unit prices p as abscissae and the quantities m as ordinates ("Sales Curve" in Fig. A, compare also Fig. 2 of the main text) we will get a curve which will steadily fall from the left to the right, whatever its shape may be otherwise, and it will intersect the axes at finite distances from the origin O. This curve will be called hereafter the "sales curve."

Fact No. 3.—Let s designate the total cost of producing a certain quantity of the commodity per year including those capital charges which are, theoretically at least, independent of the earnings (bond interest, depreciation, etc.) but excluding the net return of the capital invested (dividend). Then s is a function $f(m)$ of the quantity m produced; it is in general not simply proportional to m, but consists of a constant part and of a part which is approximately proportional to m, so that the increment cost per unit produced is approximately constant and $f(m)$ is a straight line. Stating the law of cost in more accurate terms, we will have to recognize that a large manufacturing enterprise (central station), on account of larger and more economical machinery and for other reasons, will be able to produce an incremental unit more cheaply than a small enterprise. The increment cost per unit is therefore not independent of the amount produced m, but it decreases continually with increasing m, from a maximum value for $m=0$ to a minimum value for $m=\infty$, this minimum being still positive. The curve of cost $s=f(m)$ plotted against m will therefore start from the axis of ordinates ($m=0$) at a certain distance OS_0 from the origin O (see Fig. A[1]) so that OS_0 is the constant part of the cost and then the curve will steadily rise with increasing abscissae m, displaying a certain concavity towards the axis of m. If we want to go into further details, we can even say that the curvature of the line will decrease (radius of curvature increase) with increasing m because the increment of cost per unit produced converges towards a certain limit for $m=\infty$. The curve will asymptotically approach from below a straight line which is slanting upwards.

It is not essential for the investigations of this Appendix to assume that the curve of cost $s=f(m)$, whether it be a straight line or a curved line with the concave side pointing downwards, intersects the axis of ordinates at a certain distance above the origin. The curve of cost may also pass through the origin, which is only a special case ($OS_0=0$); it may, for instance, be a parabolic function of the amount m, of the form $s=\gamma m^2$.

From the left part of Fig. A we get the following two relations.

$$\text{Average cost per unit} = \frac{s}{m} = \frac{f(m)}{m} = \tan \sigma_0 \ldots\ldots\ldots (1)$$

$$\text{Increment cost per unit} = \frac{ds}{dm} = \frac{df(m)}{dm} = \tan \sigma \ldots\ldots (2)$$

Fact No. 4.—The curve $k=F(m)$ of the capital invested k plotted against the quantity sold (or rather produced) m, in the same manner as has just been shown for the cost s, has the same character as the curve $f(m)$ and everything said about the characteristics of the curve of cost

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[1] As the quantity m is stepped off in Fig. A on the vertical axis of co-ordinates, the cost $f(m)$ must be stepped off in horizontal direction so that this axis of $f(m)$ coincides with the axis of p (except that in order to keep the diagram free from confusion it has been stepped off to the left of O whereas the axis of p runs to the right).

applies to the curve of the capital also. See Fig. A curve $F(m)$, which is drawn to another scale than $f(m)$, that is a dollar per year in the s-curve is not represented by the same length as a dollar in the capital curve.

B. THE THREE SPECIES OF EARNINGS.

1. Gross Income.

2. We now assume first that we have only one product (no by-products) and a system of charging according to the cost-of-service principle, which means every unit is sold at the same price p as every other unit. The number of units sold is then m and the total gross income b is given by the product of the unit price and the quantity sold:

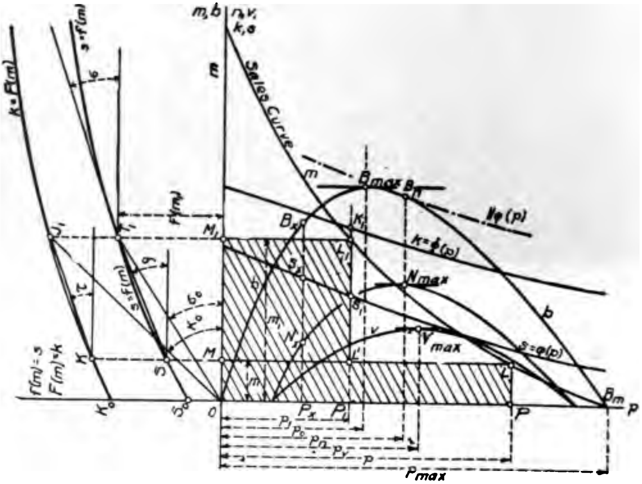

Fig. A.

$$b = mp = \text{area } OMLP \text{ (Fig. A)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

A certain gross income b will belong to every price p and if we step off the gross incomes b as ordinates over the abscissae p we get the curve b, which will intersect the axis of abscissae (p) in the origin of the system of co-ordinates because there one factor of the product mp is zero; on the other hand, for $p = p_{max}$, that is if the price has become so high that the last sales have just dropped out, the gross income will also become zero because then nothing will be sold ($m = 0$). The curve b must therefore necessarily have at least one maximum between these two points[2]. This maximum is b_{max} with the corresponding price* p_b as abscissa[3].

2. Net Income.

3. As regards the net income we have to deduct the cost s from the gross income b and in order to do this we have to transform the curve $s = f(m)$ (see left-hand portion of Fig. A) from the abscissae m to the abscissae p (right-hand portion of Fig. A) so that $s = \phi(p)$. A definite value of s corresponds to every price p. We can therefore construct the curve $s = \phi(p)$ with p as abscissa, as is done in the right-hand portion of Fig. A. For instance, for the abscissa $p_1 = OP_1$, we find m_1 from the sales curve $= P_1L_1$ and read from the $f(m)$ curve the corresponding value $s_1 = f(m_1) = M_1T_1$. If we step this off vertically upwards from the point P_1 we get the point S_1 as a point of the curve $s = \phi(p)$.

Deducting now the ordinates of the curve $s = \phi(p)$ from those of the curve b leaves the ordinates of the curve of the net income n. The ordinates of the area between the curves b and $s = \phi(p)$ (for instance S_xB_x) indicate by their length the net income n in dollars per year. These ordinates are also stepped off in Fig. A from the axis of abscissae upwards, as $P_xN_x = S_xB_x$, thus furnishing the curve of net income n. If we shift the curve $s = \phi(p)$ parallel to itself upwards until it just touches the curve b (at point B_n) the abscissa p_n of the tangential point gives the price for which the net income is a maximum; the corresponding ordinate of the area between the b and $\phi(p)$ curves gives the amount of this maximum N_{max} of the net income. Now curve $\phi(p)$ necessarily shows a steady decline with increasing abscissae[4]. The price p_n which furnishes a maximum of net income is therefore greater than that which furnishes a maximum of gross

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[2] Whether we have one or more maxima depends on the shape of the sales curve. The footnote of Section 22 of this appendix gives a condition for this shape under which condition the gross-income curve can have only one maximum.

*Erratum: In Fig. A designated by mistake as p_o.

[3] The case that the income curve has more than one maximum is treated later in this appendix, Section 22 et seq.

[4] The slope of the curve $s = \phi(p)$ is given by the derivative ds/dp and this can be written in the form $ds/dm \; dm/dp$. Owing to fact No. 3, Section 1 of this appendix ds/dm is always positive, and owing to fact No. 1 the term dm/dp is always negative. Consequently ds/dp, being the product of the two, must always be negative; this means that the curve $s = \phi(p)$ must be continually sloping downwards with increasing abscissae p.

The same can be proven in exactly the same manner for the curve $k = \Phi(p)$, see Sections 4 and 6 of this Appendix.

income because, owing to the slope of the $\phi(p)$ curve, the tangential point just mentioned must lie to the right of the point B_{max} which indicates the maximum of b-curve.

3. Rate of Return.

4. Proceeding to the rate of return (interest, dividend), we first enter the values of the capital invested as a function of the prices in the same manner as this has been shown above for the net income and thus get a curve $k=\Phi(p)$ which has a similar character as the ϕ (p) curve[5].

By entering for every single abscissa p the ratio n/k in per cent we get the curve of the rate of return v in Fig. A. This curve again has a maximum because its ordinates must be zero where the ordinates of n are zero. This maximum occurs at the point V_{max} and the abscissa p_v belonging to that point is the price which brings about the best rate of return under the assumed conditions.

By placing equal to zero the first derivative with respect to p of the term $v=\frac{n}{k}$ we arrive at the result that v becomes a maximum if $\frac{dn}{dk}=\frac{n}{k}$, that is, if $\frac{dn}{dp}=\frac{n}{k}\frac{dk}{dp}$. On the right side of this equation n and k are essentially positive values and dk/dp is essentially negative (see footnote to Section 3 of this Appendix). If therefore v becomes a maximum, dn/dp is negative. This means that at that point the n-curve is already declining; it has gone through its maximum. *The price p_v which furnishes a maximum of the rate of return is therefore higher than that which furnishes a maximum of net income and the latter price,* as has been shown above, *is higher than the price which furnishes a maximum of gross income;* that is

$$p_v > p_n > p_b$$

4. Summary.

5. Each one of the three curves of "earnings" (gross income, net income and rate of return, see Section 70 of the main text) will therefore rise from zero value with increasing prices to at least one maximum and then drop to zero again.

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[5] The capital invested depends on the selling price. If the selling price is lowered, a larger amount of the commodity (for instance, electrical energy) will be sold, therefore a larger amount must be produced. The manufacturing plant (central station, also transmission lines, etc.) must be extended and this requires in its turn an enlargement of the capital invested.

We thus have two branches of each curve, a rising left-hand one and a falling right-hand one. Consequently we can attain the same amount of earnings with at least two different prices, one on the left and the other one on the right branch. The former price is smaller than the price at which a maximum of the respective earnings is obtained and the latter larger. As long as we are on the left-hand branch an increase of the price will result in an increase of the earnings, whereas on the right side every increase of the price will reduce the earnings. If we cannot hit the price where the maximum earnings are obtained, we will of course generally prefer the price on the left branch of the curve to the corresponding price on the descending right-hand branch.

C. THE FINANCIAL EFFECT ON THE PRODUCER AND THE CONSUMER OF AN INCREASE (OR DECREASE) OF THE COST OF PRODUCTION.

6. The above methods of representing sales, cost and earnings give a good insight into the problem of finding to what extent the producer and the consumer relatively are the losers in case the cost of production is increased, for instance by taxation, a rise of the cost of labor or material, etc. The producer complains at such times about the high cost of production, whereas the consumer is liable to retort more or less hotly that the producer anyway simply loads the burden of increased cost on the consumer's shoulders, thus leaving his own profit unimpaired, if he does not use the increase of cost as a pretext for even raising the prices so high that his profit is increased. A little investigation into the situation may therefore be of interest.

If the unit cost is increased by a certain fixed amount, the total cost $s = f(m)$ is raised by an amount which is proportional to the quantity m; in other words, the curve s in the right-hand part of Fig. A will not only move higher up but it will also become steeper. The net income will become smaller than it was before for every price and consequently the maximum net income will also be reduced from its original value. Moreover, the maximum of the net income will take place at a higher price than before the cost was raised, because the point at which the s-curve, if shifted upwards parallel to itself (see Section 3 of this Appendix), just touches the gross income curve, will lie more to the right than before on account of the greater slope of the s-curve.

The price we have to charge in order to obtain the maximum of the rate of return will also be higher than it was originally, as can be shown in the following way. If c_0 is the amount by which the unit cost has been raised, the total cost will be increased to $s+c_0m$ and the net income $b-s$ reduced to $b-s-c_0m$. The rate of return $v=\frac{b-s}{k}$ is reduced to $v_m=\frac{b-s-c_0m}{k}$. The slope of the curves v and v_m referred to the axis of abscissae p is determined by the first derivatives of v and v_m with respect to p:

$$\frac{dv}{dp}=\frac{k\left[\frac{db}{dp}-\frac{ds}{dp}\right]-\left[b-s\right]\frac{dk}{dp}}{k^2}$$

$$\frac{dv_m}{dp}=\frac{k\left[\frac{db}{dp}-\frac{ds}{dp}-c_0\frac{dm}{dp}\right]-\left[b-s-c_0m\right]\frac{dk}{dp}}{k^2}$$

If now we let $dv/dp=0$, that is, if we choose the abscissa p so that v becomes a maximum, we find, by substituting dv/dp from the first one of these two equations into the second one, the following special value for

$$\left(\frac{dv_m}{dp}\right)_{v=\max}=\frac{-kc_0\frac{dm}{dp}+c_0m\frac{dk}{dp}}{k^2}=-c_0\frac{k\frac{dm}{dp}-m\frac{dk}{dp}}{k^2}$$

$$=-c_0\frac{d\left(\frac{m}{k}\right)}{dp}$$

$$=-c_0\frac{d\left(\frac{m}{k}\right)}{dm}\cdot\frac{dm}{dp}$$

Now k is a function $[F(m)]$ of m and m/k is the cotangent of the angle k_0 between the axis of abscissae and a straight line drawn from the origin of co-ordinates to a point on the curve $k=F(m)$, see left-hand portion of Fig. A. From the properties of the $F(m)$ curve (see fact No. 4 of Section 1 of this Insert) it follows that this angle k_0 is continuously decreasing for increasing m and therefore the cotangent of the angle $=m/k$ is continuously increasing. This means $\frac{d\,(m/k)}{dm}$ is necessarily always positive; dm/dp, on the other hand, owing to fact No. 1 of Section 1 of this Appendix, is always negative so that $\left(\frac{dv_m}{dp}\right)_{v=\max} = -c_0 \frac{d\left(\frac{m}{k}\right)}{dm} \cdot \frac{dm}{dp}$ has necessarily the same sign as c_0. If c_0 is positive, that is, if the cost of manufacturing increases, then the term $\left(\frac{dv_m}{dp}\right)_{v=\max}$ is also positive, which means that where v is a maximum the v_m-curve (plotted over the abscissae p) is sloping upwards, and it reaches its maximum at a higher price than v does.[6]

We see from this—using the term "profit" to include both net income and rate of return—the following to be true:

An increase in the cost of production will raise the price which produces a maximum of profit (and conversely a reduction will lower it). On the other hand, the value of the maximum itself is reduced by an increase in the cost of production (and conversely increased by a reduction), as can be easily understood from Fig. A.

A. Supposing now first that the producer have no other considerations to follow in determining his prices than to always obtain a maximum of profit, and supposing further that he had a full knowledge of the sales, cost, and capital-curves as given in Fig. A, then he will as a consequence of an increase of cost raise his unit price from the amount which resulted in a maximum profit under the original conditions of cost to the price which results in a maximum of profit under the new conditions of cost. The unpleasant effect of the advance of the cost of production will therefore be divided between the producer and the consumer in some ratio; the consumer will have to pay a higher price and the producer will get a lower profit. It is impossible for the producer to shift the entire burden of the increase of the cost onto the shoulders of the consumer. Moreover, the producer sees his chances reduced of investing new capital

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[6] A decrease of the quantity produced does not release any portion of the capital k tied up in the enterprise. The rate of return will therefore in this case change theoretically the same as the net return, since the capital will be constant. We can assume, however, at least in the central-station business with which we are chiefly concerned here, that the natural growth of the business will take up the decrease of the necessary capacity; in other words, that the effect of the increase of the cost of production, as far as the capacity is concerned, is simply a retardation of the growth of the capacity. If c_0 is negati
that is, if the cost of production is reduced, the above form
for the rate of return apply without any restricting assumpt

in his growing business. Conversely, if the cost of production is reduced, the producer cannot reap all the benefit alone, it is to his best interest to let the consumer participate by reducing the prices.

B. In practice the details of the shapes of the sales, cost, and capital-curves as represented in Fig. A are not known and we can make only rough guesses at the course these curves take. This and other reasons make it possible or even probable that the vendor (producer) has fixed the price not just at that amount which would yield him a maximum profit. Generally, as pointed out in the preceding section (5) the tendency will be to keep the price below that optimum value rather than above it. In that case it will be possible to shift a larger portion of the cost increase onto the shoulders of the consumer than under *A* above, and if the price has been sufficiently below the amount which resulted in a maximum it will be even possible for the producer to maintain his profit at the original level. It may even happen that the result is an increase of the profits. Owing to our ignorance of the details of the curves it will obviously always be a matter of chance which one of these contingencies will take place.

C. If the price should have been higher than that which results in a maximum of profit from the beginning, it is possible that even after an increase of cost a price reduction remains advisable from the producer's standpoint so that both the producer and the consumer would be benefited by an intelligent downward revision of the price. But it should not be overlooked that this is the case not *on account of* the raise of the cost but *in spite of* it and that the benefit would have been greater to both parties if the increase of the cost of production had not occurred. Moreover, the raise of the cost of production will in no way reveal the fact that the price has been (and still is) too high. The remedy for the decrease of profit will be sought in an increase of the prices instead of in a reduction.

II. PRICES DEPEND ON THE CUSTOMER'S VALUATION OF THE COMMODITY (VALUE-OF-SERVICE PRINCIPLE).

1. THE VALUATION AFFECTS THE PRICE OF EVERY PARTICLE OF SERVICE.

7.. We will change over now from the cost-of-service principle to the value-of-service principle in such a way that, starting with the unit price p, we add one or more other prices *lower than* p, for instance first the unit price p_1, in such a manner that this lower price applies to those parts of the commodity (service) *only* for which the respective *customers* are not able or willing to pay the original price p, whereas we continue to charge the original price p for all those parts of the commodity which had been sold heretofore under the original system of charging. We will in practice, of course, not be able to fulfill this theoretical requirement entirely, but there are several ways of approaching it with a reasonable degree of accuracy (see Sections 94-102 of the main text and Section 14 of this Appendix).

This operation of applying two or more different prices to the same commodity according to the valuation by the purchaser will be called *"price splitting"* and the prices formed in this manner will be called *"conjugated prices"* or a *"price combination."* If the prices split off in this manner are all lower than the original price (as has been assumed so far), the operation will be called *"price splitting downwards"* and the prices split off are *"lower conjugated prices."* If prices are added higher than the original price p in such a manner that they apply only for such parts of the commodity for which the customers would be ready to pay these higher prices instead of the original lower price, the operation is called *"price splitting upwards."* Price splitting upwards and downwards can be combined with each other.

8. As a result of the splitting of the prices we will sell the same total amount of the commodity as if we would offer every unit at the lower unit price p_1, that is, we will sell the quantity m_1, which is greater than m. The gross income b_c of the price combination is composed of two parts: (1) The gross income mp of the original customers who are charged the original price p and therefore buy the original quantity m and (2) the gross income of the new customers who buy the balance, that is $m_1 - m$ at the price p_1, therefore

$$b_c = mp + (m_1 - m)p_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$(m_1 - m)\,p_1$ is given by the rectangle MM_1L_1L' and b_c is therefore represented by the shaded area in Fig. A (page 309). No matter how small the lower conjugated price p_1 is, it will always increase the gross income.[1]

9. The total cost of production is determined only by the quantity sold (produced) and not by the prices at which the commodity is being sold. The total cost under the combination of prices is therefore $M_1T_1 = P_1S_1 = s_1$, just as if the whole commodity were offered for sale uniformly at

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[1] It may be stated here incidentally that we can, of course, add more than one conjugated lower price, in which case the gross income will become greater and the shaded area will contain more than two steps. We can, in fact, add an infinitely large number of conjugated lower prices which will change the shaded area, representing the gross income, into the total area under the sales curve between ordinates given by the highest and lowest prices.

the lower price p_1. Likewise the capital will be $MJ_1 = P_1K_1 = k_1$.

If the price combination applies, we will therefore have the net income given by

$$n_c = b_c - s_1 = mp + (m_1 - m)p_1 - s_1 \quad\ldots\ldots\ldots\ldots(5)$$

and the dividend (interest) $v_c = \frac{n_c}{k_1} = \frac{b_c - s_1}{k_1}$ ....(6)

10. **The question with which we have to deal here is the following:** When is it possible to add one or more conjugated *lower* prices to the original price p in such a manner that the "earnings" (that is, the gross income b_c, the net income n_c, or the interest v_c, respectively, as the case may be) are *increased* over the original earnings b, n and v?[2]

11. As regards the gross income b we have seen from equation (4) of this Appendix (preceding page) that it is always possible to increase it by price splitting downwards, no matter how large or small the original price p or the conjugated lower price p_1 is. There are no lower or upper limits for the prices.

12. As regards the net income we find the following: The original net income is given by

$$n = b - s = mp - s \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Subtracting this equation from equation (5) we get

$$n_c - n = (m_1 - m)p_1 - (s_1 - s)$$

Using the symbol Δ for differences in the customary way so that $\Delta m = m_1 - m$ (positive value) and $\Delta s = s_1 - s$ (also positive value) and $\Delta n = n_c - n$ (to be investigated whether positive or negative value) we can write the above equation in the following form:

$$\Delta n = p_1 \Delta m - \Delta s$$

Δn will be >0, that is n_c will be $>n$, if $p_1 > \frac{\Delta s}{\Delta m}$.

This means, the net income is necessarily increased by the addition of a lower conjugated price in all cases where that price is higher than the cost increment per unit.[3] Now the original price from which the lower conjugated price has

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[2] The importance of this problem is based on the following considerations: If it is possible to add the lower prices with the result mentioned, this means that we will be able to increase the earnings by price reductions to some customers without price increases to others. Or with constant earnings we can reduce the prices to all customers as a consequence of the change from the cost-of-service system to the value-of-service system. (See Sections 78 and 88-91 of the main text). The solution of this problem is therefore at the bottom of the question of the superiority of the cost-of-service or the value-*of-service* system of charging.

[3] *See footnote* to Section 75 of the main text.

been split off must, of course, according to the definition of a *lower* conjugated price (Section 7 of this Appendix) be greater than that lower price. On the other hand, the original price upon which the original cost-of-service system was based must have been greater not only than the cost increment per unit, but even greater than the average cost (see Section 2 of the main text) given by equation (1) of this Appendix, otherwise the enterprise would have been a losing one from the outset. We will therefore always be able to add lower conjugated prices which increase the net income. The lower limit of such profitable conjugated prices is the cost increment per unit.

13. As regards the interest (dividend) v we have the original interest v given by

$$v=\frac{n}{k}=\frac{b-s}{k}=\frac{mp-s}{k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Subtracting this from equation (6) we get

$$v_c-v=\frac{b_c-s_1}{k_1}-\frac{mp-s}{k}$$

or, substituting the value of b_c from (4),

$$v_c-v=\frac{mp+(m_1-m)\,p_1-s_1}{k_1}-\frac{mp-s}{k} \ldots\ldots (9)$$

Now, again introducing the symbol Δ in the same way as in Section 12 above (also $\Delta k=k_1-k$), we find from (9) that v_c will be $>v$ if

$$[mp+p_1\Delta m-(s+\Delta s)]k>(mp-s)\,(k+\Delta k)$$

or if $(p_1\Delta m-\Delta s)\,k>\ (mp-s)\Delta k$

$$\text{or if } \frac{p_1\Delta m-\Delta s}{\Delta k}>\frac{mp-s}{k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The fraction on the left-hand side of this relation may be called the "rate of return of the increments" or "interest of the increments," inasmuch as it is the increment net income divided by the increment capital (see Section 27 of this Appendix). We can say, therefore, that the interest will always be increased by the addition of a lower conjugated price if the "interest of the increments" is *greater* than the original interest.

It remains to be shown that such a lower conjugated price can be found in all cases which are to be considered in practice. This is proved in the following way:

If we make the lower price just a trifle, a differential, smaller than the original or upper price, that is if Δp becomes *dp*, and if we then should find that the differential of v *must* be positive for all upper prices which would be considered under the cost-of-service system, that would mean that the adding of lower prices, at least in a certain range below the upper price, *must* increase the interest (rate of return). It will be shown in the following that this is actually the case.

Calling $v_c - v = \Delta v$, we get from equation (9)

$$\Delta v = \frac{mp + (m_1 - m)p_1 - s_1}{k_1} - \frac{mp - s}{k}$$

Substituting now $p_1 - p = -\Delta p^*$ or $p_1 = p - \Delta p$
and $m_1 - m = \Delta m$
and $s_1 - s = \Delta s$ or $s_1 = s + \Delta s$
and $k_1 - k = \Delta k$ or $k_1 = k + \Delta k$

we get Δv

$$= \frac{mpk + \Delta m(p - \Delta p)k - (s + \Delta s)k - mp(k + \Delta k) + s(k + \Delta k)}{k(k + \Delta k)}$$

$$= \frac{pk\Delta m - k(\Delta m \Delta p + \Delta s) - (mp - s)\Delta k}{k(k + \Delta k)}$$

If we now choose Δp smaller and smaller and let it converge towards *dp* with a consequent change of the other differences into differentials, magnitudes of the second order will vanish beside those of the first order and we get

$$dv = \frac{pkdm - k\,ds - (mp - s)dk}{k^2} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

If now dv shall be >0 the numerator of (11) has to be positive or $p(k\,dm - m\,dk)$ has to be $> k\,ds - s\,dk$ or

$$p \text{ has to be}^{4} > \frac{k\,ds - s\,dk}{k\,dm - m\,dk} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

This is the condition for a positive dv, or in other words the condition that there is a certain range of prices in existence within which the conjugation of lower prices raises the rate of return v.

Now, from Fig. A and equation (1) of this Appendix we see

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[*] Note that $p_1 - p$ has a negative value because the lower price p_1 is smaller than p, whereas with the values m, b, s, and k the subscript 1 indicates an increase: $m_1 - m = +\Delta m$, etc., because the amount m, etc., which belongs to the lower price p_1 is greater than the amount which belongs to the original price p.

[4] We could also arrive at the same result by direct differentiation of v, n and b and subsequent substitution of the results, *but the* above method seems to be both shorter and clearer.

that the average unit output cost $s/m = \tan \sigma_0$. According to fact No. 3 of Section 1 of this Appendix, $\tan \sigma_0$ must continually decrease if m increases (see Fig. A) which means that

$$\frac{d\,(s/m)}{dm} \text{ is always} < 0$$

$$\text{or } \frac{m\,ds - s\,dm}{m^2} < 0$$

$$\text{or } m\,ds \text{ is always} < s\,dm$$

Multiplying by k and then subtracting $sm\,dk$ from both sides: $m(k\,ds - s\,dk)$ is always $< s(k\,dm - m\,dk)$

We can divide both sides of this relation by $m(k\,dm - m\,dk)$ without reversing the sign of inequality because this product is essentially positive[5] and we arrive in this manner finally at the following relation for s/m:

$$\frac{s}{m} \text{ is always} > \frac{k\,ds - s\,dk}{k\,dm - m\,dk} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The right sides of (12) and (13) are identical. If therefore $p = s/m$ the condition (12) will be fulfilled and if $p > s/m$ the condition (12) must be all the more fulfilled. As we have seen in Section 12 of this Appendix[6] prices which are fit for a practical system of charges under the cost-of-service principle will always be greater than s/m and therefore there will always exist a certain range below the original price within which the conjugation of lower prices raises the rate of return.

B. THE VALUATION AFFECTS THE PRICES OF GROUPS OF SERVICE ONLY, BUT NOT OF EVERY PARTICLE.

14. The above deductions are based on the assump-

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[5] The factor m is obviously always positive and as regards $(k\,dm - m\,dk)$ this can be written as $k^2 \frac{d(m/k)}{dm}\,dm$. Now fact No. 4 of Section 1 of this Appendix shows that m/k (which is given by $\cot \kappa_0$, see left-hand portion of Fig. A) increases with increasing m so that $\frac{d(m/k)}{dm}$ is essentially positive; dm is also positive because we start with a *reduction* of the price p by the differential dp, which (with reference to the shape of the sales curve Fig. A and fact No. 1 of Section 1 of this Appendix) necessitates dm being positive. The product of all these factors is therefore positive.

[6] This can also be seen from equation (7) which shows that the net income (and consequently also the rate of return) is positive as long as $mp > s$, which means as long as $p > s/m$.

tion that we can charge every particle of the service according to the valuation it meets from the customer, which means, not only are different services charged differently to the same customer, and different customers differently for the same service, but different units of the same service will be charged differently even though they are sold to the same customer (see Section 73 of the main text).

We can, of course, never carry out this assumption strictly in practice. We must group large numbers of units into one class of service or of customers, and then charge the whole class under a uniform rate or rate schedule. We may, for illustration, charge all kilowatt-hours for heating service at the same price and differently from the kilowatt-hours for other classes of service. The same may apply to the kilowatts demand for heating service. Or we may charge the first 500 kw-hr. of wholesale power service at a separate price, etc. We will thus select a certain limited number of unit prices and the problem is to choose these prices at such amounts as produce the maximum of earnings which is possible with the chosen number and distribution of prices. This grouping of the individual units into classes means of course a deviation from the theoretical system of charging, inasmuch as some of the units will be charged at lower prices than they could fetch at the utmost, whereas for others there will be demanded too high a price so that they cannot be sold. Therefore this grouping will reduce the earnings below the earnings of the theoretical system which charges every particle of the service individually.

As the units within each group are charged at the same price, this grouping amounts to resolving the value-of-service system into a number of cost-of-service systems. The relative prices of the various groups are determined according to the value-of-service principle. We have therefore in this *practical value-of-service system* a hybrid between the cost-of-service system and the real value-of-service system.

15. We have now a problem parallel to the problem of Section 10 et seq. of this Appendix as follows: Can we increase the "earnings" in this practical system by lowering the prices of one or more of the groups without raising the prices to any other group? To investigate this matter we will start with two groups of service only, called g and G (for instance, electric service for heating purposes only and for all other purposes) and the conclusions drawn from this investigation will then be extended to the case of a subdivision into more than two groups.

Each one of these groups has its own sales curve, M *and* m, respectively. The total-sales curve M_t is formed by the addition of the ordinates of *M* and *m* (see Fig. B). Under the original cost-of-service system both groups are charged the unit price p_0. This results in the sale of M_0 units of group *G* and of m_0 units of group *g*.

We shall now investigate the behavior of the three species of "earnings" (gross income, net income, and rate of return) if we split the prices between the two groups.

1. Gross Income.

a. Shape of the Gross Income Curve.

16. The question is: When can we lower the unit price in any one or both of the two groups, for instance in *g*, in such a manner that the aggregate gross income is increased thereby? We lower the price in group *g* from p_0 to p_1, thereby increasing the number of units sold in that group from m_0

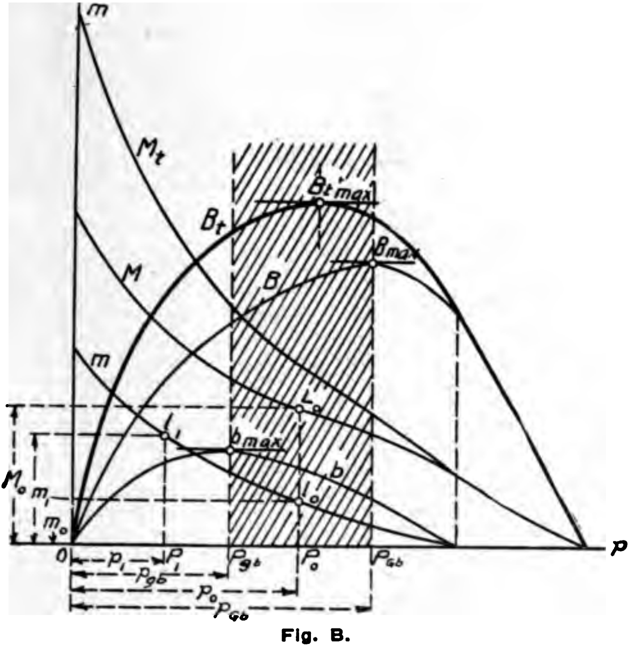

Fig. B.

to m_1. The original aggregate gross income B_{t_0}, which is given by $(M_0 + m_0)p_0$ changes into $B_{t_1} = M_0 p_0 + m_1 p_1$. If this change shall be an increase, that is, if B_{t_1} shall be $> B_{t_0}$, it follows from the above equations for B_{t_0} and B_{t_1} that

$$m_1 p_1 \text{ has to be} > m_0 p_0 \ldots\ldots\ldots\ldots (14)$$

Now $m_1 p_1$ and $m_0 p_0$ are the gross incomes from group g alone without any reference to its combination with group G and relation (14) means therefore the following: The necessary and sufficing condition for an increase of the aggregate gross income from the two groups as a result of a price reduction within one of the groups is that the income b from that group alone—group g without regard to the other group G—is *increased* by the price reduction. With reference to Fig. B, which shows the sales curves M and m, the gross income curves B and b, and their summation curves M_t and B_t, we see that this is possible only if the original price p_0 is greater than the price $p_{gb} = OP_{gb}$ which produces a maximum of gross income b from that group g alone for which the price is to be reduced; otherwise any price reduction for group g would necessarily result in a reduction of the gross income[7] b. (See Section 2 of this Appendix.)

17. This condition means, in other words, that the original price must be high enough so that the corresponding point on the gross income curve b of group g is situated on the descending (right hand) branch of that curve. This wording of the condition is sometimes more convenient for application than that of the preceding paragraph.

18. We can now distinguish the following three cases:

(a) The original price is lower than the lower one of the two respective prices which make the income of one of the two groups a maximum, that is, according to the assumptions made in Fig. B, it is lower than p_{gb}. The original price lies in the range to the left of the shaded range in Fig. B.

(b) The original price is lower than the higher one of the prices which make the incomes of one of the two groups a maximum (p_{Gb}) but higher than the lower one of these prices (p_{gb}). This means it is situated in the shaded range in Fig. B.

(c) The original price is higher than either of the above named prices p_{gb} and p_{Gb}; it lies to the right of the shaded range in Fig. B.

In case (a) any price reduction in any group will always reduce the aggregate gross income.

In case (b) any price reduction in group g to a price not

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[7] *This wording is based on the preliminary assumption that curve of the gross income has only one maximum.*

lower than p_{gb} will always increase the aggregate gross income, any price reduction in group G will reduce it.

In case (c) any price reduction in group g to a price not lower than p_{gb} and any price reduction in group G to a price not lower than p_{Gb} will always result in an increase of the gross income. Although therefore a price reduction in either one of the groups will be of advantage in this case, we will prefer an initial general reduction of all prices, as this is of greater advantage until the price p_{Gb} has been reached which results in the maximum gross income B_{max} of group G alone. After this has been done, this case necessarily reduces to case (b)[8], thus making advisable a further price reduction to one of the groups only.

It may be pointed out here that case (c) will not occur in a well designed cost-of-service system of charging. As the same gross income, and even a greater one, can in this case be obtained with a lower uniform price, case (c) would mean that the price has been chosen too high altogether from the producer's point of view and consequently also from the consumer's point of view. (Compare the closing sentence of Section 5 of this Appendix.)

19. Wherever we find that a price reduction in any group is of advantage to the aggregate gross income it is obvious that, in order to obtain the best results which can be obtained by a variation of the prices in the respective group, this price reduction must be continued so long and not longer until we have reached the price which results in a maximum of the gross income of the respective group, for the following reason: If we stop reducing the price earlier there will still exist the possibility of further increasing the gross income by a continued price reduction, because in that case we are still on the descending branch of the income curve[9]. Conversely by pushing the reduction beyond the mentioned point we will

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[8] An exception is the special case when the shapes of all the sales curves should happen to be such that the maxima of the income curves of all groups occur at the same price, in which case $\frac{dB}{dp} = \frac{db}{dp} = \dots = 0$ for $\frac{dB_t}{dp} = 0$. The width of the range within which price splitting is of advantage becomes zero in this limiting case.

[9] If there should occur more than one maximum of the group's gross income curve in the range of prices below the original price (wavy income curve, see footnote to Section 22 of this Appendix) we must obviously push the price down so far and not farther until we have reached the highest one of these maxima, because if we stop reducing earlier, that is, before reaching the maximum or on reaching a maximum which is lower than the highest one, there is still the possibility of increasing the income from that group, according to relation (14) of this Appendix, by a price reduction of that group.

come onto the ascending branch of the income curve which shows that an increase of the price would be proper in its place and that we have therefore gone too far with the reduction.

20. Relation (14) can be written

$$\frac{m_1}{m_0} > \frac{p_0}{p_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

This means in words: A reduction of the price in a certain group or class of service *must* have the effect of increasing the gross income if the sales curve has such a shape that the number of units sold of that group rises more rapidly than the price is reduced. If, for instance, the price of a certain class of service is reduced to 3/4 of its original amount, the number of units sold would have to rise to more than 4/3 of the original amount; $\frac{p_1}{p_0} = \frac{3}{4}$, $\frac{p_0}{p_1} = \frac{4}{3}$, $\frac{m_1}{m_0} > \frac{4}{3}$.

The above is another way of expressing the law contained in Section 17 of this Appendix. If the price corresponds to the descending branch of the income curve the number of units sold at a price in the immediate neighborhood of the original price increases at a more rapid rate than the price reduction indicates, and vice versa. This is demonstrated by relations (14) and (15).

21. It is easy to extend this line of reasoning to more than two groups (and to income curves with more than one maximum). As a result we can express the law laid down in Section 18 of this Appendix in different words and with more general validity as follows:

Where it is intended to bring about an increase of the gross income by means of price reductions to one or more groups of service without any price increase to others, this is always possible if the sales curves of these groups have such a shape that a ratio of price reduction can be found which makes the number of units sold in that group or groups rise at a more rapid rate than the rate at which the prices decrease; (if, for instance, at a reduction of the price to one-half its original value, the number of units sold would rise to more than twice their original amount). Whether this means a general reduction of the prices or a splitting off of prices for a certain number of groups only, depends on the number of groups which have the above quality. If they *all* have sales curves of the shape mentioned [Section 18—case (c)] a general lowering is the most efficient means of easing the income, and according to Section 19 of this

Appendix, the price of every group will generally be lowered to a different level, which means price splitting downwards. Otherwise [case (b)], the prices will be lowered to some, but not to all, of the groups (again generally to a different level in every group, if the reduction takes place in more than one group.) This again means price splitting downwards.

b. Shape of the Sales Curve.

22. A graphical representation will make the above clearer and at the same time pave the way for the subsequent deductions. If, instead of the relation (14) $m_1p_1 > m_0p_0$, which states the condition for an increase of the gross income by price splitting downwards, we write the equation

$$m_1p_1 = m_0p_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

this represents the limiting case of relation (14), and consequently also of (15). The graphical expression of this equation is an equilateral hyperbola with the axes of coordinates m and p as asymptotes (Fig. C). If the value of $m = P_0N_0$, which belongs to p_0, is assumed as variable (P_0x_1, P_0x_2 ..., etc., see Fig. C) a family of such hyperbolas, as indicated by the thin lines in Fig. C, will result. With sales curves having the shape as determined by facts Nos. 1 and 2 (stated at beginning of this Appendix), every sales curve must from left to right intersect higher and higher situated hyperbolas of this family until it just touches one of them from below (dash-and-dot line, point N_t) indicating that the maximum gross income has been reached and then it intersects a second time all the hyperbolas in the reverse order.[10]

Where the sales curve intersects the hyperbola in such a

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[10] We can see from this that if the sales curve has no point of inflection the income curve cannot have more than one maximum, because then obviously the above mentioned point at which the sales curve just touches one of the hyperboles from below, cannot be repeated in the course of the sales curve. As will become clear from the latter part of this Appendix (Sections 37-38 and 41-42) this applies not only to the curve of the gross income, but also to those of the net income and of the rate of return. On the other hand, one or more points of inflection in the sales curve do not necessarily mean that the curve of earnings must have more than one maximum.

It also follows that a sales curve with the concave side facing upwards, such as the sales curve in Fig. A, results in a flatter peak of the gross income curve (and of the other earnings curves) than a sales curve with the concave side facing downwards and that the choice of the right price is of greater importance in the latter case. If the sales curve runs along one of the hyperbolas in Fig. C through a certain range of prices the curve of the gross income will be entirely horizontal for that entire range and it will make no difference on [illegible] income which price we choose as long as we remain w[illegible] that range.

manner that it is lower than the hyperbola to the left of the point of intersection and higher to the right, the gross income curve is therefore in its ascending (left hand) branch, and vice versa. The two points of intersection with the same hyperbola, for instance H_1 and H_2, indicate prices which produce the same income (because mp is constant along the whole course of every individual hyperbola of the entire family).

23. From this follows another way of wording the condition that price splitting downwards will increase the gross income. We reduce the ordinates of the sales curve to percentages of the ordinates of the respective curve at the price p_0, choosing, for instance, for all curves the ordinate M_0 of group G at abscissa p_0 as 100%. Thus, for illustration, the sales curve M of group G has been transferred unchanged from Fig. B to Fig. C and the sales curve m of group g has

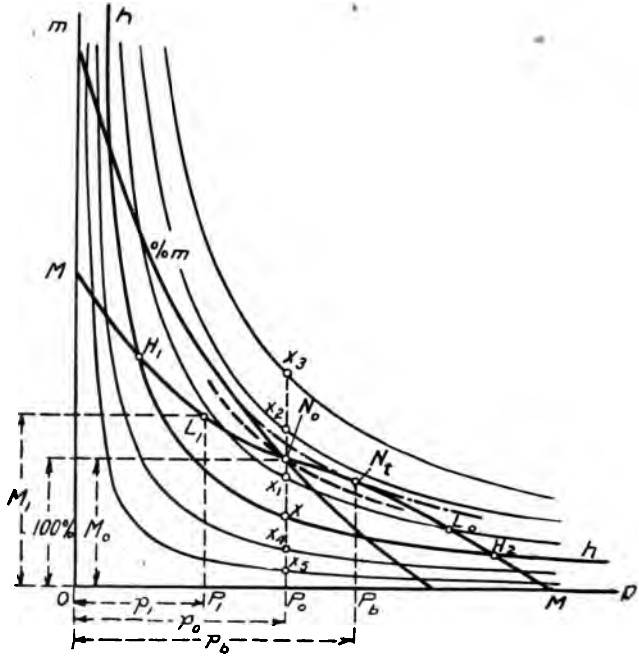

Fig. C.

been entered with ordinates increased in proportion so as to reach the percentage $M_o = 100\%$ at abscissa p_o. Consequently this curve, which has been called $\%m$ in Fig. C, will intersect the sales curve M (which now might also be called $\%M$) at the point N_o (abscissa p_o). If we have more than two groups, the curves of the remaining groups will be treated like curve *m* so that the percentage curves of all the groups will pass through point N_o. A hyperbola of the family will of course also pass through the same point N_o in which all the percentage sales curves intersect. If now at least one of the percentage curves has a greater slope than the hyperbola (in other words, if the hyperbola is not the steepest one of the bundle of curves passing through the point N_o) then it *must* be possible, within a certain range of prices $< p_o$, to improve the gross income by price splitting downwards. The prices are to be lowered for that group (or those groups) which has a greater slope than the hyperbola.

It is also clear, and needs no further explanation *per longum et latum*, how far down the range reaches within which price splitting downwards is of advantage in every group and for which price the maximum amount of income is reached.

2. *Net Income.*

1. Definitions.

24. The same lines of reasoning as have been applied in the above, beginning with Section 16, for the gross income, can also be applied *mutatis mutandis* to the net income and to the rate of return (dividend) with entirely analogous results.

In the following will be shown in what respects the methods and the results of the investigation differ from those for the gross income.

25 In the preceding Sections 16-23 it has been shown how the conditions for an increase of the aggregate gross income as a consequence of a lowering of the price to certain groups of service can be reduced to certain qualities of the gross-income curve which belongs to the respective group (or groups) in which the price is to be lowered. If we try to extend this method to the net income (and to the rate of return) we must first decide what is to be called "the net income" or the "rate of return," respectively, of a particular group. We can determine the gross income of a certain group of service, but we cannot say without certain additional defining assumptions how large the net income or the rate of return is which results from that group because the aggregate cost (total cost) and the total capital invested cannot be assessed to the individual groups in a definite way unless we make certain assumptions.

26. We can now make various assumptions about what we want to call the "net income" and the "rate of return" of a certain group for the present purpose. The most obvious two methods of defining the net income and return of a group are (a) the incremental net income (return of the increments) and (b) what might be called the group's "segregate" net income (segregate rate of return); (compare also Section 3 of the main text).

27. The *"incremental net income"* is the difference: Increment of the gross income caused by the addition of that group to the other groups minus the increment of the combined cost[1] over the combined cost (aggregate cost) of all other groups. In other words, it is the difference of gross income derived from that group minus the increment of the combined (aggregate) cost of all other groups. The *"rate of return of the increments"* is the incremental net income, as just defined, divided by the increment of the capital.

28. The group's *"segregate net income"* is the difference of that group's gross income minus the group's segregate cost (see Section 3 of the main text; the "group's segregate cost" is the cost of serving that group alone if no other groups were in existence). Likewise the group's *"segregate rate of return"* is the quotient of the segregate net income, as just defined, divided by the capital which would be necessary to establish a manufacturing plant (central station, etc.) just large enough to provide for that group alone.

2. Incremental Net Income of the Group. (Sufficient and Necessary Condition.)

29. Starting again with the assumption that we have two groups of service only, G and g, we call N_{t_0} and N_{t_1} the

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[1] The increment of the cost is the amount by which the aggregate cost is raised in consequence of the fact that the respective group is being supplied with service. The sum of the increment costs of all groups will be smaller than the total aggregate cost and the sum of the segregate costs of all groups will be larger than the total aggregate cost. A balance will remain after summing up the increment costs of all groups which has not been assigned to any one of the groups (overhead cost). If we regard the total service as one group, the increment cost of that group is the excess of the aggregate cost over the cost at the production 0, and the cost at production $0 = OS_0$ (on the left-hand part of Fig. A) will be left over, and not accounted for, so to speak. The same will apply if we have more than one group, provided the line of cost $s = f(m)$ or S_0ST, is a straight line, otherwise the unaccounted portion will be greater (always taking the incremental cost of every group over the combined cost of all other groups).

The same considerations prevail for the increment of the capital.

aggregate net incomes from both groups, corresponding to the aggregate gross income B_{t_0} and B_{t_1} above (Section 16 of this Appendix). N_{t_0} is therefore the aggregate net income if the unit prices charged in both groups are the same and equal to p_0, resulting in the sales M_0 and m_0 units in the two groups, respectively, whereas N_{t_1} is the aggregate net income if in group G the price is maintained at p_0 and in group g it is lowered to some other amount p_1, increasing thereby the sales in that group from m_0 to m_1. Let the symbol $f(m)$ denote again the function which determines the cost (excluding the capital cost) if the amount produced is given by m (variable). We have then

$N_{t_0} = (M_0 + m_0)p_0 - f(M_0 + m_0)$...Uniform price p_0. (17)
$N_{t_1} = M_0 p_0 + m_1 p_1 - f(M_0 + m_1)$.........Price splitting downwards (lowering price in group g)............(18)

If price splitting downwards shall improve the net income, that is, if N_{t_1} shall be $> N_{t_0}$, then

$m_1 p_1 - f(M_0 + m_1)$ has to be $> m_0 p_0 - f(M_0 + m_0)$ ...(19)

or using, as above, the symbol b (gross income) for the product mp so that $b_0 = m_0 p_0$ and $b_1 = m_1 p_1$

$(b_1 - b_0) - [f(M_0 + m_1) - f(M_0 + m_0)]$ must be > 0..(20)

Writing this in the form

$$\{ b_1 - [f(M_0 + m_1) - f(M_0)] \} - \{ b_0 - [f(M_0 + m_0) - f(M_0)] \} > 0 \quad \text{..........(21)}$$

we have in the first couple of braces the incremental net income of the group if the price is p_1 and in the second couple of braces the incremental net income over the other group if the price is at its original amount p_0. This means: *Lowering the price in a certain group is of advantage to the aggregate net income, if the incremental net income of that group is improved thereby.*

Such an improvement of the group's incremental net income according to relation (21) is the *necessary and sufficient* condition for an improvement of the aggregate net income by price splitting downwards, which means, if the group's incremental net income is improved by price splitting downwards, the latter is always profitable and conversely, if price splitting downwards is profitable, the group's incremental net income must be improved thereby.

3. Segregate Income of the Group. (Sufficient But Not Necessary Condition.)

30. Can we establish a similar relation between the aggregate income of all groups and the *segregate* income of the group in which the price is to be lowered?

The segregate net income of group g at the lowered pr

p_1 is greater than at the original price p_0 if

$$b_1 - f(m_1) > b_0 - f(m_0)$$

or $(b_1 - b_0) - [f(m_1) - f(m_0)] > 0$ ....(22)

How does this compare with relation (21)? A glimpse at Fig. D will make this clear. We have here

$$OA' = m_0 \quad \dots\dots\dots\dots\dots\dots \quad A'A = f(m_0)$$
$$OB' = m_1 \quad \dots\dots\dots\dots\dots\dots \quad B'B = f(m_1)$$

where m_1 is greater than m_0 because the price p_1 is obtained from p_0 by price splitting downwards so that $p_1 < p_0$, consesequently according to fact No. 1 of Section 1 of this Appendix m_1 must be $> m_0$.

We have further in Fig. D

$$A'C' = B'D' = M_0 = \text{constant}$$
$$OC' = m_0 + M_0 \quad \dots\dots\dots \quad C'C = f(m_0 + M_0)$$
$$OD' = m_1 + M_0 \quad \dots\dots\dots \quad D'D = f(m_1 + M_0)$$
$$\left.\begin{array}{l} B''B = f(m_1) - f(m_0) \\ D''D = f(M_0 + m_1) - f(M_0 + m_0) \end{array}\right\} \dots\dots\dots\dots (23)$$

If now the curve $f(m)$ complies with fact No. 3 of Section 1 of this Appendix, that is, if it rises continuously with increasing abscissae, turning the concave side towards the axis of abscissae and approaching with increasing abscissae asymptotically a straight line, this means that $B''B > D''D$, because $A'B' = m_1 - m_0$ and $C'D' = (m_1 + M_0) - (m_0 + M_0) = m_1 - m_0$ and consequently $A'B' = C'D'$. For the limiting case where the $f(m)$ line is a straight line $B''B = D''D$.

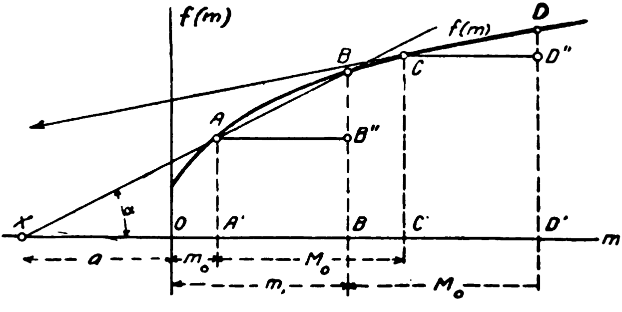

Fig. D.

This means from (23)

$$f(m_1) - f(m_0) \geqq f(M_0 + m_1) - f(M_0 + m_0) \quad ..(24)$$

Multiplying by (-1) and adding $(b_1 - b_0)$ to both sides (24) becomes*

$$(b_1 - b_0) - [f(m_1) - f(m_0)] \leqq$$
$$(b_1 - b_0) - [f(M_0 + m_1) - f(M_0 + m_0)]$$

The left side of relation (22) is therefore always $\leqq$ than the left side of (20), or if relation (22) is true, (20) and (21) must of necessity also be true, because if the term of (22) is positive, the term of (20) which is equal or greater, must of course also be positive. But we can reverse the statement only in case the $f(m)$ line is a straight line, which means: Unless $f(m)$ is a straight line there will exist cases in which (21) is true, but (22) is not.

In other words: An increase of the segregate group income for a certain price reduction is a sufficient condition for an improvement of the aggregate net income by price splitting downwards by means of that price reduction, but it is not a necessary one, unless the $f(m)$ curve is a straight line.

4. General Remarks.

31. What has been proved in Sections 18 and 19 of this Appendix for the gross income can be demonstrated in exactly the same manner for the group's incremental net income and the group's segregate net income. It is unnecessary to repeat in detail all these results in their application to the net income (maximum of the aggregate net income, more than one maximum of the group's net income curve, increase of the net income either by price reductions in *all* or in some of the groups, increase of the net income if the original system of prices has been so designed as to result in the highest possible net income attainable with a uniform price). We can also without difficulty extend the conclusions to more than two groups.

32. Which condition is more strict, that for an increase of the gross income or of the net income by price splitting downwards? The gross income can be raised by price splitting downwards if, according to relation (14), $m_1p_1 > m_0p_0$, that is $b_1 > b_0$. The corresponding condition for the net income is, according to relation (20), $b_1 > b_0 + [f(M_0 + m_1) - f(M_0 + m_0)]$. Now since $p_0 > p_1$, it follows, according to fact No. 1 of Section 1 of this Appendix, that $m_0 < m_1$ and, according to fact No. 3, that $f(M_0 + m_0) < f(M_0 + m_1)$. Therefore $[f(M_0 + m_1) - f(M_0 + m_0)]$ is always positive. Consequently with a given sales curve for group g and with a given upper price p_0, high enough so that price splitting downwards increases the aggregate *gross* in-

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*Note that multiplication by (-1) results in a reversal of the inequality sign.

come, there will always exist certain values of the price p_1 which, although making $b_1 > b_0$, and therefore improving the aggregate *gross* income B_t if added as lower prices, still will reduce the aggregate *net* income N_t because b_1 does not reach the value $b_0 + f(M_0 + m_1) - f(M_0 + m_0)$.

For a graphical representation of the fact set forth in this Section see Section 33 et seq. of this Appendix.

33. It is of interest to investigate again, as has been done in the case of the gross income (Sections 20-23 of this Insert) how the condition for the possibility of increasing the net income without raising the prices to anybody is expressed in the shape of the sales curve. Calling

$$f(M_0 + m_1) - f(M_0 + m_0) = \Delta f(M_0 + m_0)$$

we can write relation (19) as

$$m_1 p_1 > m_0 p_0 + \Delta f(M_0 + m_0) \quad \ldots\ldots (25)$$

If we now call $r = \dfrac{\Delta f(M_0 + m_0)}{m_1 - m_0} = \dfrac{\Delta f(M_0 + m_0)}{\Delta m_0} \quad \ldots\ldots (26)$

this means r is the average cost increment per unit[2] if the quantity produced rises from $(M_0 + m_0)$ to $(M_0 + m_1)$, or we can say $r = \tan \gamma$ in Fig. A. If we substitute the value for $\Delta f(M_0 + m_0) = r(m_1 - m_0)$ from equation (26) into (25) we get

$$m_1 p_1 > m_0 p_0 + r(m_1 - m_0)$$

or $m_1(p_1 - r) > m_0(p_0 - r) \quad \ldots\ldots (27)$

The limiting case of this relation is given by changing the inequality sign into an equality sign:

$$m_1(p_1 - r) = m_0(p_0 - r) \quad \ldots\ldots (28)$$

34. Assuming now for the present that curve $s = f(m)$ be a straight line, the cost increment r per unit will then be constant for any values of m_0 and m_1 (or p_0 and p_1, respectively). Equation (28) is then represented by a family of equilateral hyperbolas which is identical with that of equation (16) of this Insert, except that the vertical asymptote does no longer coincide with axis of ordinates m. It runs now at the distance r to the right of that axis (Fig. E). The reasoning of Sections 22 and 23 of this Appendix can now be applied to these hyperbolas.

The net-income hyperbola passing through a certain point will be steeper than the gross-income hyperbola passing through the same point. This is illustrated by comparing the family of net-income hyperbolas with the dash-and-dot gross-income hyperbola in Fig. E which has been selected at ran-

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[2] See the first footnote to Section 75 of the main text.

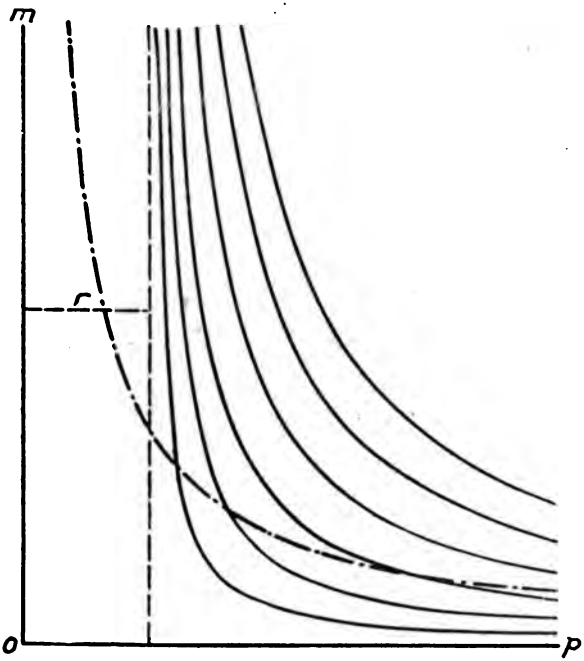

Fig. E.

dom.[3] Consequently, considering Section 23 of this Appendix, the lowest limiting slope of the sales curve of group *g* which still allows an increase of the earnings by lowering the price in that group will be greater in case of the net income than

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[3] It is hardly necessary to prove this in a more scientific way by comparing the angles of the slope of the two curves as given by the first derivative dm/dp in both cases:

Gross-income hyperbola....... $mp = \text{constant}$...... $\frac{dm}{dp} = -\frac{m}{p}$

Net-income hyperbola.. $m(p-r) = \text{constant}$...... $\frac{dm}{dp} = -\frac{m}{p-r}$

Therefore dm/dp is smaller for the net-income hyperbola than for the gross-income hyperbola, but since dm/dp is essentially negative in both cases, the absolute value of the angle of inclination of the net-income hyperbola is greater than that of the gross-income hyperbola passing through the same point.

in case of the gross income. In other words, the conditions for an increase of the net income by price splitting downwards are more strict than the corresponding conditions for the gross income. (Compare Section 32 of this Appendix.)

35. If now the curve $s = f(m)$ is no longer a straight line, but curved, in the only way in which it can be curved, that is with the concave side pointing steadily downwards, this means that r, see equation (26), will no longer be a constant for every m_1, but it will (for a given m_0 and M_0) grow smaller as m_1 increases and vice versa. It will, however, for very large amounts m_1—mathematically expressed, for $m_1 = \infty$—still have a finite positive value, viz.: the limiting value $c_{1\infty}$ of the increment cost c_1 per unit, if very large quantities of the "commodity" are manufactured (very large central stations). We will get, therefore, a different set of hyperbolas for every value of m_1 in such a manner that the vertical asymptote will move nearer and nearer to the vertical axis of co-ordinates with increasing m_1 without ever getting closer to it than a certain minimum $c_{1\infty}$. Consequently we can say that in case of a curved $f(m)$ line the conditions for increasing the net income by price-splitting downwards will be less strict for larger quantities m_1 than for smaller ones, but they will always be stricter than the corresponding conditions for the gross income.

The results of this graphical investigation are therefore entirely in harmony with the results of the analytical investigation carried out in Section 32 of this Appendix.

3. Rate of Return.

a. Shape of the Rate-of-Return Curve.

1. Total Rate of Return of All Groups.

36. Proceeding along the same general lines of reasoning as in Sections 16-24 and 25-35, respectively, of this Appendix, we can investigate the criterion for an increase of the rate of return either analytically or graphically and arrive at the same results with either method.

Using the same symbols as heretofore, we can write

$$v_0 = \frac{(M_0 + m_0)p_0 - f(M_0 + m_0)}{F(M_0 + m_0)} \quad \ldots\ldots\ldots\ldots (29)$$

and

$$v_1 = \frac{M_0 p_0 + m_1 p_1 - f(M_0 + m_1)}{F(M_0 + m_1)} \quad \ldots\ldots\ldots\ldots\ldots (30)$$

where F is the symbol for the function of m which represents the capital invested (see left side of Fig. A) so that

$F(m_x)$ is the capital necessary as an investment for the annual production of m_x units; v_0 and v_1 are then the rates of return corresponding to the gross incomes B_{t_0} and B_{t_1}, or to the net incomes N_{t_0} and N_{t_1}, respectively, in the preceding Sections of this Appendix, in such a manner that v_0 is the rate of return obtained if the price p_0 is charged to both groups G and g, and v_1 is the rate of return if the price is lowered to p_1 in group g and maintained at p_0 in G (price splitting downwards).

2. Return of the Increments. (Sufficient and Necessary Condition.)

37. In analogy with the investigations of "Net Income" (Sections 25 to 35 of this Appendix) we shall investigate whether and how the criterion for an increase of the rate of return as a consequence of the lowering of the price in group g alone can be expressed in terms of

(a) the return of the increments of group g (see Section 27 of this Appendix);

(b) the segregate rate of return of group g (see Section 28 of this Appendix).

The condition that the rate of return is raised by the change of the price p_0 to p_1, in group g only is: $v_1 > v_0$ or $v_1 - v_0 > \theta$, that is from (29) and (30):

$$\frac{[M_0p_0 + m_1p_1 - f(M_0 + m_1)]\,F(M_0 + m_0)}{F(M_0 + m_0)\,F(M_0 + m_1)} - \frac{[(M_0 + m_0)p_0 - f(M_0 + m_0)]\,F(M_0 + m_1)}{F(M_0 + m_0)\,F(M_0 + m_1)} > 0$$

This fraction is greater than 0 if the numerator is greater than 0 because the denominator is essentially positive. The condition can therefore be written as

$$[M_0p_0 + m_1p_1 - f(M_0 + m_1)]\,F(M_0 + m_0) - [(M_0 + m_0)p_0 - f(M_0 + m_0)]\,F(M_0 + m_1) > 0 \quad\ldots\ldots(31)$$

Substituting into this, for the sake of brevity, the following short symbols: B_0 for M_0p_0, b_0 for m_0p_0, and b_1 for m_1p_1 we get

$$[B_0 + b_1 - f(M_0 + m_1)]\,F(M_0 + m_0) > [B_0 + b_0 - f(M_0 + m_0)]\,F(M_0 + m_1) \quad\ldots\ldots\ldots\ldots\ldots\ldots(31^*)$$

Setting $f(M_0 + m_1) - f(M_0 + m_0) = \xi$
and $F(M_0 + m_1) - F(M_0 + m_0) = \eta$

we can write (31*) as

$$[B_0 + b_1 - f(M_0 + m_0) - \xi]\,F(M_0 + m_0) > [B_0 + b_0 - f(M_0 + m_0)]\,[F(M_0 + m_0) + \eta]$$

from which $(b_1 - b_0 - \xi)\,F(M_0 + m_0) > [B_0 + b_0 - f(M_0 + m_0)]\,\eta$

$$\text{or}\quad \frac{b_1 - b_0 - \xi}{\eta} > \frac{B_0 + b_0 - f(M_0 + m_0)}{F(M_0 + m_0)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

The fraction on the left side of this relation is the "rate of return of the increments," as will be readily seen by comparing it with the definition of that term as given in Section 27 of this Appendix. The right side is the actual rate of return of both groups under the original charges v_0 [equation (29)], that is when the price p_0 is charged in both groups. It is easily seen that this can be extended to more than two groups and we arrive thus at the following law:

Price splitting downwards by groups is of advantage to the rate of return as long as the rate of return of the increments in the respective group where the price is lowered is greater than the original rate of return. (Sufficient and necessary condition.)

3. Segregate Rate of Return. (Sufficient But Not Necessary Condition.)

38. In an attempt to introduce, in analogy with what has been shown previously (Section 30 of this Appendix) about the net income, the more convenient term of the segregate rate of return $\frac{b_1 - f(m_1)}{F(m_1)}$ we start from the sufficient and necessary condition for an increase by price splitting downwards as given in relation (31*):

$$[B_0 + b_1 - f(M_0 + m_1)]\, F(M_0 + m_0) \text{ has to be}$$
$$> [B_0 + b_0 - f(M_0 + m_0)]\, F(M_0 + m_1)$$

Calling the two sides of this relation X and Y, respectively, so that $X > Y$, we can replace X by a term $X_1 < X$ and the new relation $X_1 > Y$ will still be a condition for an improvement of the rate of return by price splitting downwards because, if even the smaller value X_1 is greater than Y, the larger value X will all the more be greater than Y. We can thus say that any change in relation (31*) which reduces the left side relatively to the right side will leave the changed relation (31*) a condition for the improvement of the rate of return. But it will no longer be a *necessary* condition after that change because there will be cases where, although the substitute relation $X_1 > Y$ does not hold true, still the real criterion $X > Y$ will be fulfilled. On the other hand, we are never justified in *increasing* the left side relatively to the right side.

Let us now after this introductory remark turn back to relation (24), which may be written in the following form:

$$f(M_0+m_1)-f(m_1) \text{ is always } \leqq f(M_0+m_0)-f(m_0) \quad \text{..(33)}$$

From fact No. 4 of Section 1 of this Insert we have further

$$F(M_0+m_0) \text{ is always} < F(M_0+m_1) \quad \text{..............(34)}$$

because $m_0 < m_1$.

Multiplying the left and right sides of (33) and (34) with one another, respectively, we get

$$[f(M_0+m_1)-f(m_1)]\,F(M_0+m_0) \text{ is always}$$
$$< [f(M_0+m_0)-f(m_0)]\,F(M_0+m_1) \quad \text{........(35)}$$

Adding (35) and (31*) will increase the right side of (31*) more than the left side, and is therefore permissible, according to what has just been found out above. (31*) then changes into

$$[B_0+b_1-f(m_1)]\,F(M_0+m_0) \text{ has to be}$$
$$> [B_0+b_0-f(m_0)]\,F(M_0+m_1) \quad \text{.............(36)}$$

Returning now to Fig. D of this Appendix it can be shown[4] that

$$\frac{A'A}{C'C} < \frac{B'B}{D'D} \quad \text{..................................(37)}$$

Assume now the curve in Fig. D to represent not the $f(m)$ line but the $F(m)$ line, an assumption which is perfectly permissible since both curves have the same character. We can thus write (37) as

$$\frac{F(m_0)}{F(M_0+m_0)} < \frac{F(m_1)}{F(M_0+m_1)} \quad \text{..................(38)}$$

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[4] The secant AB, produced to the left, intersects the axis of abscissae at the distance a to the left of the origin of coordinates and the secant CD intersects it at the distance c where $a \leqq c$. The slope of the secant AB will be called α and that of CD will be called γ. We have now (see Fig. D):

$$\frac{A'A}{C'C} = \frac{(a+m_0)\tan\alpha}{(c+M_0+m_0)\tan\gamma} \text{ and } \frac{B'B}{D'D} = \frac{(a+m_1)\tan\alpha}{(c+M_0+m_1)\tan\gamma}$$

For a comparison of the sizes of the two terms we can suppress the factor $\tan\alpha/\tan\gamma$ in each term and write the remaining part of $B'B/D'D$ in the form

$$\frac{(a+m_0)+(m_1-m_0)}{(c+M_0+m_0)+(m_1-m_0)}$$

W[illegible] that this fraction which represents $B'B/D'D$ corresponds to the fraction that represents $A'A/C'C$ except that both numerator and denominator are enhanced by the same amount (m_1-m_0). If we add a constant amount to both numerator and denominator of a fraction the value of that fraction will approach closer to unity and since $\frac{a+m_0}{c+M_0+m_0}$ is smaller than unity (because $a \leqq c$) the value of the fraction will increase, that means $B'B/D'D > A'A/C'C$.

It is now again permissible to multiply (36) by (38) because this will relatively reduce the left side of (36) and we get thus the requirement as

$$[B_0+b_1-f(m_1)]\,F(m_0) \text{ has to be} > [B_0+b_0-f(m_0)]F(m_1)$$

or

$$\frac{B_0+b_1-f(m_1)}{F(m_1)} \text{ has to be} > \frac{B_0+b_0-f(m_0)}{F(m_0)} \quad ..(39)$$

or

$$\frac{B_0+n_{s1}}{F(m_1)} \text{ has to be} > \frac{B_0+n_{s0}}{F(m_0)} \quad ....(40)$$

where n_{s1} and n_{s0} are the segregate net incomes of group g for the prices p_1 and p_0, respectively.

These last two relations (39) and (40) are as close as we can get to basing a sufficient but not necessary condition on the segregate income and segregate capital of the group g. It is impossible to further simplify this relation by the elimination of B_0 for the following reason. We have obviously

$$\frac{B_0}{F(m_1)} < \frac{B_0}{F(m_0)} \quad ....(41)$$

because $F(m_1) > F(m_0)$. If it were permissible to subtract (41) from (40) this would eliminate B_0 but it would mean relatively increasing the left side of (40) with respect to the right side and that is just what is *not* permissible, as shown above.

We can therefore formulate as follows a sufficient but not necessary requirement for an increase of the rate of return by price splitting in group g: *The lowering of the price must increase a modification of the segregate rate of return of group g which is formed by adding the gross income of all other groups G to that of group g before the deduction of the segregate cost of group g and the division by the capital necessary for the group g.*

4. General Remarks.

39. Here again everything that has been said in Sections 18 and 19 of this Appendix for the gross income can be proved for the group's rate of return of the increments and for the modification of the group's segregate rate of return, the former as necessary and sufficient requirement, the latter as sufficient requirement. This refers, for instance, to the maximum of the rate of return obtainable by price splitting downwards, etc., as per Section 31 of this Appendix.

b. Shape of the Sales Curve.

40. We can get a clearer mental picture of the conditions under which a lowering of the price in group g can

improve the rate of return from all groups if we apply to the rate of return (dividend) the graphical method which has been shown with reference to the net income in Sections 33-35 of this Appendix. This is done in the following:

Call $$\left.\begin{array}{l}\Delta m_0 = m_1 - m_0 \\ \Delta p_0 = p_1 - p_0\end{array}\right\} \qquad (42)$$

and $$\left.\begin{array}{l}\Delta f(M_0+m_0) = f(M_0+m_1) - f(M_0+m_0) \\ \Delta F(M_0+m_0) = F(M_0+m_1) - F(M_0+m_0)\end{array}\right\} \qquad (43)$$

Substituting this in relation (31), which is the condition for an increase of the rate of return by price splitting downwards in group *g*, we get

$$[M_0p_0 + (m_0+\Delta m_0)(p_0+\Delta p_0) - f(M_0+m_0) - \Delta f(M_0+m_0)]\,[F(M_0+m_0)] - [(M_0+m_0)p_0 - f(M_0+m_0)]\,[F(M_0+m_0) + \Delta F(M_0+m_0)] > 0$$

or after proper reduction

$$[m_0\Delta p_0 + p_0\Delta m_0 + \Delta m_0\Delta p_0 - \Delta f(M_0+m_0)]\,F(M_0+m_0)] > [(M_0+m_0)p_0 - f(M_0+m_0)]\,[\Delta F(M_0+m_0)] \qquad (44)$$

We call now in analogy to equation (26) of this Appendix

$$t = \frac{\Delta F(M_0+m_0)}{\Delta m_0} \qquad (45)$$

$t = \tan \tau$ (see left part of Fig. A) represents the same value with respect to the capital-curve *k* as *r* does with respect to the cost-curve *s*. We get by substituting (26) and (45) into (44):

$$[m_0\Delta p_0 + (p_0 + \Delta p_0 - r)\,\Delta m_0]\,F(M_0+m_0) > [(M_0+m_0)p_0 - f(M_0+m_0)]\,t\Delta m_0$$

Divide both sides by $F(M_0+m_0)$ and substitute v_0 from (29)

$$m_0\Delta p_0 + (p_0+\Delta p_0 - r)\Delta m_0 > v_0t\Delta m_0$$

$$(p_0 + \Delta p_0 - r - v_0t)\,\Delta m_0 > m_0(-\Delta p_0)$$

or $$\frac{\Delta m_0}{m_0} > \frac{-\Delta p_0}{p_0+\Delta p_0 - r - v_0t}$$

Adding the denominator to the numerator on both sides and substituting m_1 for $(m_0 + \Delta m_0)$ and p_1 for $(p_0 + \Delta p_0)$ from (42) we get

$$\frac{m_1}{m_0} > \frac{p_0 - (r + v_0t)}{p_1 - (r + v_0t)}$$

$$m_1[p_1 - (r + v_0t)] > m_0[p_0 - (r + v_0t)] \qquad (46)$$

This formula is entirely analogous to (27) of this Appendix except that, instead of deducting the unit increment cost *r* of the service from the price, we have to deduct the sum of the unit increment cost plus the interest v_0t on the unit increment capital t, the interest being computed at the same rate of return v_0 at which the original capital has been bearing interest before the price reduction.

41. Everything that has been said in Sections 33 to 35 of this Appendix regarding the effect of price splitting downwards on the net income applies also to the rate of return, except that the distance of the vertical asymptote from the axis of ordinates (Fig. E) is not r (increment cost) but $r + v_0 t$ (increment cost plus interest on the increment capital). This makes the condition for an increase of the rate of return stricter than that for the net income (compare Section 34 of this Appendix).

4. *Resumé.*

42. A resumé in non-mathematical terms of those of the above laws which are concerned with the advantageousness of price splitting downwards is contained in Sections 81 to 86 of the main text. This resumé brings out a few points of view not touched upon in this Appendix.

APPENDIX X

(To Section 114)

Reasons for the Desirability of Graduation of the Energy Charges With Pure Meter Rates.

(Graphic Interpretation.)

The problem of how and why the energy charge of a pure meter rate should vary with the energy consumption is much more readily interpreted by the curve which shows the total charges (customer's bill) as a function of the energy consumption than by the customary curve of the average charges per kilowatt-hour and the former curve gives a much better and more direct insight into the problem than the latter. It is easy to reduce the curve of the total charges to the curve of the average charges per kw-hr. (see Appendix XIV) should this be desired, but it is not at all necessary. The total bill is really the primary function of the energy consumption and the average charge per kw-hr. is a secondary function, being the total bill divided by the energy consumption. (The only exception to this is the straight meter rate where both functions may be considered to be of equal importance.)

As regards the shape of the curve of total charges, we have to remember that the total charge, i. e. the bill, is composed of cost plus profit.

The cost, as has been shown in Part I, is composed of the energy cost, the demand cost, and the customer cost.

The energy cost is proportional to the energy consumption and the curve representing this part of the cost with relation to the consumption is therefore a straight line $OX_{\circ}$ (Fig. A) rising from the origin O of co-ordinates.[1]

As we are dealing at present with such rates only as do

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[1] The angle of elevation μ is obviously the larger, the higher the energy cost per kilowatt-hour is (kw-hr. cost per kw-hr.). Those of our readers who are familiar with elementary mathematics will understand that tan μ is proportional to the kilowatt-hour cost per kilowatt-hour.

not contain any demand charge, we will have to base the entire cost of calculation on the kw-hr. as unit, that is, we will have to average the demand cost in some manner between the kilowatt-hours consumed. Assuming as an approximation that all customers have the same demand for every kilowatt-hour consumed (in other words, the same load-factor) the demand cost will also be defined by a straight line rising from the origin, OX_{dem} in Fig. A. The size of the angle ν at which it rises, depends upon the average load-factor; a large load-factor results in a small angle and

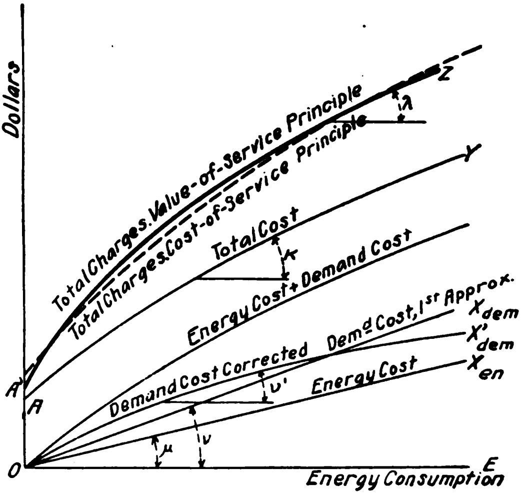

Fig. A.

vice versa. Now it has been explained (Section 115 of the main text) that large energy users on the average have a larger load-factor than small ones. Therefore the demand cost is not strictly proportional to the amount of energy consumed, but it increases less and less rapidly as we proceed on the curve from the small energy consumers to the medium and large ones. The demand cost will rise more rapidly for the small energy consumers than indicated by the straight line OX_{dem} and less and less rapidly as we proceed to the d large ones. The straight line OX_{dem} will therefore have to be corrected to a shape[2] of the type OX'_{dem}, steadily curving downwards.

Adding the ordinates OX_{en} and OX'_{dem} for every abscissa and adding further the constant customer cost as given by OA results in the curve AY as the curve of the total cost. For the energy consumption 0 this curve indicates a certain minimum value of cost—equal to the customer cost OA. The curve rises from the point A and the angle of elevation χ becomes smaller and smaller as the energy consumption increases.[3]

From the curve of the cost we arrive at the curve of the price (charges or total bill) by adding the profit as a certain percentage of the cost. Now it has been shown (Sections 97-99) that the value-of-service principle requires this percentage to be smaller at either end of the curve (that is, for the large and for the small consumers) than in the middle (medium-sized consumers). A constant percentage of profit added would result in the dotted curve[4] for the total charges (bills), whereas these charges under the value-of-service principle would be typified by the heavy line A'Z[5].

---

[2] The angle of elevation ν of OX'_{dem} towards the horizontal becomes steadily smaller as the energy consumption increases and it converges towards a certain minimum ν'_{lim}, that means, although it always remains larger than that minimum, it approaches that minimum value steadily more and more the larger the energy consumption becomes.

[3] The angle of elevation K of the cost curve AY also converges towards a limiting value K_{lim} which is the larger the larger μ and the larger ν_{lim} is, $\tan K = \tan \mu + \tan \nu'$ and $\tan K_{lim} = \tan \mu + \tan \nu'_{lim}$.

[4] The angle of elevation λ is given by $\tan \lambda = (1 + p/100) \tan K$ if p is the constant percentage of profit added and it converges towards λ_{lim} where $\tan \lambda_{lim} = (1 + p/100) \tan K_{lim}$.

[5] The electric company is generally interested in an increase of a customer's consumption only as long as such an increase brings about an increase in the profit from that customer in dollars and cents. In accordance with the value-of-service principle the percentage may be reduced incidentally to the increase of the energy consumption, but if—roughly speaking—the reduction of that percentage goes so far that the amount of the profit in dollars and cents remains stationary—not to say decreases—the electric service company will refuse to be interested in such additional business. For that reason the $A'Z$ curve will in every part of its course be steeper than the corresponding part (that is, the part with the same abscissa) of the cost curve AY. In other words, the increment of charges will always have to be larger than the increment of cost.

APPENDIX XI

(To Section 121)

Theoretical Remarks About the Step Rate—Inaccuracy—Size of the Steps—Size of the Energy Charges—Design of a Step Rate.

The sudden reductions of the bill which take place with the step rate when the limit between two steps is exceeded, bring it about that, as we gradually proceed from the lower to the larger energy consumption, the bills are alternately larger and smaller than they ought to be. The same is indi-

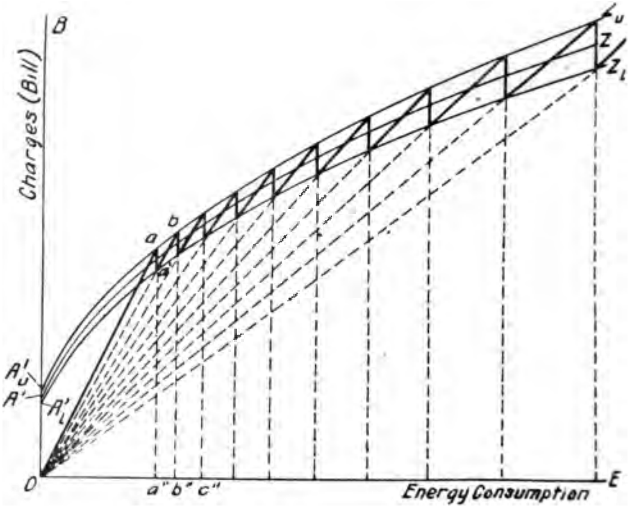

Fig. A.

cated by the zigzag form of the curve (Fig. 5 of the main text and Figs. A to C of this Appendix) which gives the actual amount of the bill (stepped off in vertical direction) for the varying energy consumptions (stepped off horizontally). This zigzag form which brings the curve of the actual bill alternately above and below the curve which shows what the bill ought to be theoretically (see the preceding Appendix X) persists even if we stipulate that the bill shall never be greater than what would be billed for any larger energy consumption (Figs. D and E).

It seems logical to require that these periodic positive and negative deviations or inaccuracies should by a proper choice of the rate constants be limited to a certain percentage of the total bill in such a manner that at each and every cycle—that is, at each step of the schedule—the inaccuracy just touches this maximum percentage on the positive and on the negative side.

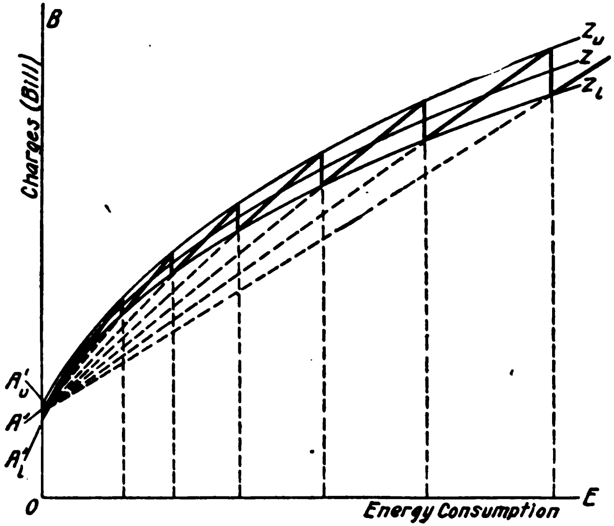

Fig. B.

Supposing, for illustration, that we have given the curve of total charges $A'Z$ as we want them to be (see Fig. A) and assuming further the permissible percentage of inaccuracy at ± 5%, we draw one curve A'_uZ_u, the ordinates of which are 5% greater than those of the given charge curve $A'Z$ (Fig. A) and another curve A'_lZ_l with ordinates 5% smaller. Starting with a certain initial straight meter rate given by Oa, we drop a vertical line from the intersection point of that line with the A'_uZ_u curve. This vertical intersects the curve A'_lZ_l at a'; $a''a'$ is now practically 10% smaller than $a''a$. Connecting a' with O gives us the second step of the step rate $Oa'b$, etc.

The drawing Fig. A shows that with the typical shape of the curve of charges $A'Z$ the increments of the steps $a''b''$, $b''c''$, etc., become steadily greater and greater as we advance into the higher steps.

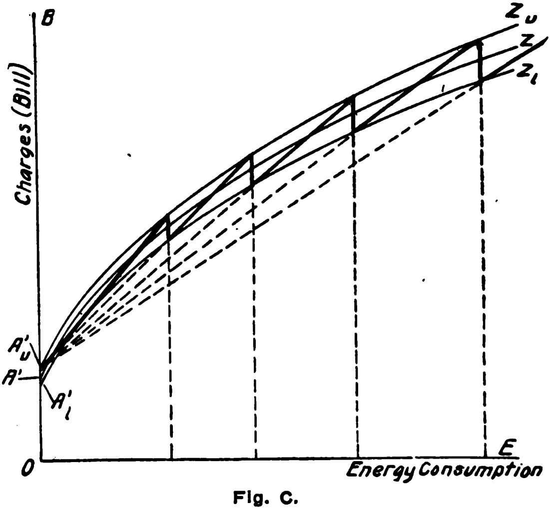

Fig. C.

This does not apply, however, to the first step Oa'', as can be seen from Fig. A. Counting *backwards* from the larger energy consumptions to the smaller ones, we see that in order to satisfy the requirement of a constant percentage change of the bill between steps, the steps would have to become smaller and smaller and finally to become infinitely small. In other words, we would never reach a first step. Therefore, the first step must of necessity exceed the limit iven percentage of inacuracy, inasmuch as in the lower portions of that step the amount of the bill will be much smaller than permitted by that requirement. (For the energy consumption 0 the percentage inaccuracy will even become infinitely large, since the amount of the bill is 0, whereas it ought to be equal to the customer charge OA'). The size of the first step is not governed by the same law as that of the other steps.[1]

As regards the unit charges per kilowatt-hour it is easy to show that under the above requirement of a constant percentage reduction of the bill between steps the unit charges must decrease in a geometrical relation, which means that the ratio of the kilowatt-hour charge in a given step to the kilowatt-hour charge in the preceding step is constant or, in other words, the kilowatt-hour charge in any step can be found from the kilowatt-hour charge in the preceding step by multiplying the latter by a given constant percentage.[2]

To summarize: *The gradation of the energy charges is given by the percentage drop of the bill permitted between steps and the size of the steps is determined by the shape of the theoretical rate curve which is to be approximated by the step rate.*

Since the percentage should be constant by which the kilowatt-hour charge is to be reduced between steps, the numerical amounts of the reductions of the kilowatt-hour charges (in cents or fractions of cents) from one step to the next should become smaller as the kilowatt-hour charge

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[1] Where we have the customer charge OA' explicitly made in addition to the step meter rate, we can draw a line for the first step as a tangent from A' to the lower curve (Fig. B) and where this tangent intersects the upper curve we get the point a as the end of the first step. Then the rate in its first step will never deviate from the theoretical value by more than the percentage allowed for the inaccuracy. The subsequent construction from Fig. B is self-explanatory. The increments of the steps will be larger than if the customer charge is absent. Larger increments of the steps result in a smaller number of steps necessary to cover a given total range of kilowatt-hours and consequently—*ceteris paribus*—in a greater simplicity of the rate schedule.

We can further slightly increase the size of the first step and of the step increments (and thereby reduce the necessary number of the steps) if we take the amount of the customer charge not at its correct value OA' but enhanced by the permissible percentage of inaccuracy, that is, if we take the customer charge at OA'_u. A comparison of Figs. B and C will explain this without further words. In practice this means that it is better for the simplicity of a step rate with a customer charge to choose the latter too large than too small.

A stipulation that the charges shall never be larger for a smaller energy consumption than for a larger one (Section 120) will also increase the size of the increments of the steps; consequently the number of steps which are necessary to cover a certain range of kilowatt-hours will become smaller. This is shown in Fig. D, which is also self-explanatory. Fig. E finally shows a combination of that stipulation with a customer charge and demonstrates plainly the reduction in the number of steps.

[3] To explain this, let us suppose, for instance, that we have a rate where some one of the steps charges 6 cents per kw-hr. up to 800 kw-hr. and that the drop of the bill between every two steps is required to be 5% of the bill. The drop will then be 5% of (6 × 800); that means the bill for 800 kw-hr. calculated with the charge of the next following step (to be quite exact we would have to say: the bill for 801 kw-hr.) must be 95% of (6 × 800). The charge per kilowatt-hour in this following step will then be

$$\frac{95\% \text{ of } (6 \times 800)}{800}$$

or 95% of 6 cents, that is 5% less than the charge in the original step.

For readers who are familiar with algebra: Call S_n the number of kilowatt-hours at the end of the n^{th} step and p the percentage by which the bill is to be reduced when we exceed the limit of the $S_n{}^{th}$ kilowatt-hour, that is the limit between the n^{th} and the $(n+1)^{st}$ step. Let further c_n and c_{n+1} denote the kilowatt-hour charge in the n^{th} and $(n+1)^{st}$ step, respectively. The bill at the end of the n^{th} step is c_nS_n and at the beginning of the $(n+1)^{st}$ step it is $c_{n+1}S_n$. (To be quite exact we would have to say $c_{n+1}(S_n+1)$ but S_n is always very much greater than 1 so that we can set S_n for S_n+1). We have thus

$$c_{n+1}S_n = c_nS_n(1 - p/100)$$

S_n cancels out, indicating that neither the number of kilowatt-

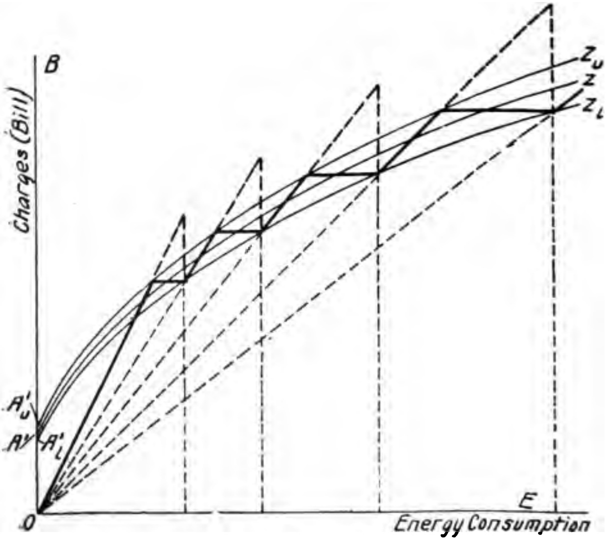

Fig. D.

itself becomes smaller, that is as the number of kilowatthours becomes greater.

However, for obvious reasons simple figures are desirable for the unit charges (either gross or net, see Section 110), that is either whole cents or multiples of half or quarter cents, or decimal figures of cents with one or two digits. In the latter case the second decimal is preferably a figure 5. In consequence of these limitations we have to be satisfied with graduating the unit charges by identical intervals, for instance half cents, over a number of steps and then change to a graduation of, let us say, quarter cents for the next few steps, etc. This will, of course, have the effect that the balance between the percentage drops at the various step limits will be more or less disturbed.

The following table is an example, taken from practice, of a power step rate (1st and 2nd columns). The units of

1	2	3	4	5	6	7	8
					Suggested improvement.		
			Interval to next energy charge.			Interval to next energy charge.	
Cents per hp-hr.	For hp-hrs.	Intervals, hp-hrs.	c/hp-hr.	%	c/hp-hr.	c/hp-hr.	%
5	99 or less		0.5	10			
4.5	100 to 199	100	0.25	[1]5.55			
4.25....	200 to 399	200	0.25	5.88			
4	400 to 599	200	0.25	6.25			
3.75....	600 to 799	200	0.25	6.67			
3.5	800 to 1,199	400	0.2	5.71	3.5	0.25	7.14
3.3	1,200 to 1,599	400	0.2	6.06	3.25	0.25	7.69
3.1	1,600 to 2,399	800	0.2	6.45	3.00	0.25	8.33
2.9	2,400 to 3,599	1,200	0.2	6.90	2.75	0.25	9.09
2.7	3,600 to 4,799	1,200	0.2	7.41	2.50	0.2	8.00
2.5	4,800 to 5,999	1,200	0.2	8.00	2.30	0.2	8.70
2.3	6,000 to 7,999	2,000	0.2	8.70	2.10	0.15	7.04
2.1	8,000 to 9,999	2,000	0.2	9.52	1.95	0.15	7.69
1.9	10,000 to 11,999	2,000	0.15	7.90	1.80	0.15	8.33
1.75....	12,000 to 14,999	3,000	0.15	8.57	1.65	0.15	9.09
1.6	15,000 to 19,999	5,000	0.15	9.37	1.50	0.1	6.67
1.45....	20,000 to 29,999	10.000	0.15	[2]10.34	1.40	0.1	7.14
1.3	30,000 to 39,999	10,000	0.1	7.69			
1.2	40,000 to 59,999	20,000	0.1	8.33			
1.1	60,000 to 79,999	20,000	0.1	9.09			
1.0	80,000 and over						

[1] Minimum. [2] Maximum

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hours at which the step limit is located nor the number of steps is of any consequence, and we get

$$\frac{c_{n+1}}{c_n} = 1 - \frac{p}{100}$$

in other words, c_n and c_{n+1} have the same ratio to each other as the respective bills. The decrease from c_n to c_{n+1} is the same percentage as the decrease of the bills.

energy happen to be in this case horsepower-hours and not, as usual, kilowatt-hours. The third column gives the increment in the length of the respective step to the next following one, showing how these increments increase in size. The fourth and fifth columns show the decrements of the energy charges from the respective step to the next, in cents per horsepower-hour and per cent, respectively. In accordance with what has been said above, the percentage of the decrement of the unit energy charge is equal to the percentage decrement of the bill between the same two steps. The table shows that this percentage varies between the limits 5.55 and 10.34%. By strictly carrying out the principles outlined in this Appendix, the percentage can be made never to exceed the limit of 9.09% with the same number of steps and no finer gradation of the unit charges (multiples of 0.05 cent per hp-hr.). This is indicated in the three last columns of the table and is done by extending the 0.25 cent per hp-hr. and 0.15 cent per hp-hr. decrements of the unit charge a little further at the expense of the 0.2 cent per hp-hr. decrement.

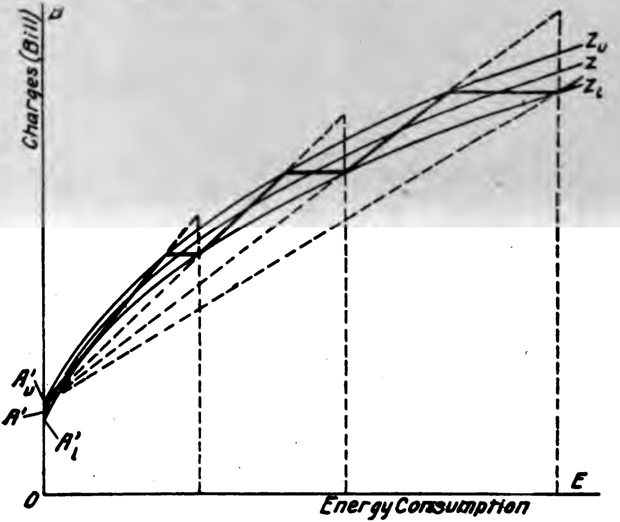

Fig. E.

APPENDIX XII

(To Section 122)

Graphical Representation and Analysis of the Block Meter Rate.

1. A graphical representation of the block meter rate gives a most vivid insight into its nature. Let us take as an example the Retail Power schedule of Fort Wayne, Ind.:

The first 50 kw-hr. are charged at 5 cents per kw-hr.
the next 100 kw-hr. are charged at 4 cents per kw-hr.
the excess over 150 kw-hr. at 2 cents per kw-hr.

We choose as horizontal axis the number of kilowatt-hours consumed and step off vertically corresponding the amounts of the customer's bill (Fig. A). A vertical line *I* drawn at the distance 50 kw-hr. from the vertical axis and another one *II* 100 kw-hr. further to the right will then give us the various blocks. In the first block we have a straight meter rate of 5 cents per kw-hr., which is represented by a straight line *Ox*, where *x* is, for instance, the point corresponding to 100 kw-hr. and $5. From the point *A*, where that line crosses the dividing line between the first and the second blocks, every additional kilowatt-hour is charged at 4 cents only and consequently we have to proceed from this point along a line with a smaller angle of elevation, corresponding to 4 cents per kw-hr. We can get this angle by joining *O* with, for instance, the point *y* (100 kw-hr., $4). Drawing a parallel to this line *Oy* through *A* will give us the amount of the bills in the second block, line *AB*. It is evident, if we produce this line backwards towards A_0 that the rate in the second block corresponds to a straight 4-cent meter rate increased by a constant amount OA_0, that is by a customer charge. It is obvious from the drawing that with every successive block this equivalent customer charge increases as the kw-hr. charge of the block rate decreases. It is also evident that the kilowatt-hour charge of the straight meter component of the equivalent rate is identical with the kilowatt-hour charge of the block meter rate in the respective block.

2. [illegible] numerical value of the equivalent customer charge in a given case can be found either by measuring the values OA_0, OB_0, etc., in the chart, or by purely arithmetical operations in the following way. The customer charge of the second block in Fig. A is OA_0, which obviously equals aa_0. Now $aa_0 = aA - a_0A =$ (50 kw-hr. × 5 c/kw-hr.) — (50 kw-hr. × 4 c/kw-hr.) = 50 cents. Similarly we get the customer charge for the third block as: (50 kw-hr. × 5 c/kw-hr.) + (100 kw-hr. × 4 c/kw-hr.) — (150 kw-hr. × 2 c/kw-hr.) = 350 cents or $3.50.

In general, to find the equivalent customer charge of the nth block, figure the products of each preceding range in kilowatt-hours by the energy charge in the respective range and from the sum of all these products subtract the product of the sum of all ranges in kilowatt-hours, preceding the nth range, in kilowatt-hours, by the kilowatt-hour charge of the nth range.

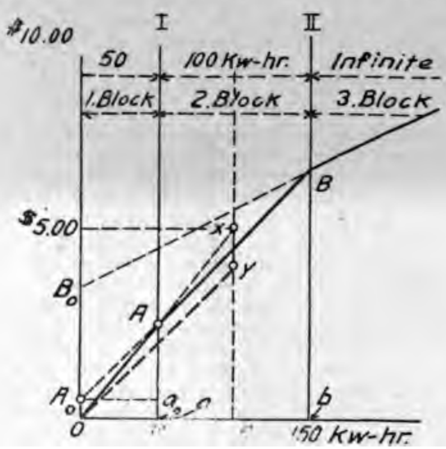

Fig. A.—Block Rate.

An example will make this clear. Take, for instance, the Commercial Light rate in Sacramento, which is a block meter rate charging:

[illegible] cents per kilowatt-hour for the first 50 kw-hr.

5 cents per kilowatt-hour for the next 400 kw-hr.
4 cents per kilowatt-hour for the next 600 kw-hr.
3 cents per kilowatt-hour for the excess over 1050 kw-hr.

How large is the equivalent customer charge in the 4th block? Answer: (6 × 50) + (5 × 400) + (4 × 600) — 3(50 + 400 + 600) = 1550 cents or $15.50. We can thus figure the bill of any customer with more than 1050 kw-hr. simply as $15.50 plus 3 times the total kilowatt-hour consumption of that customer. For instance, if the customer has used 2000 kw-hr. his bill will be $15.50 + (3 × 2000 cents) = $75.50.

This method is a simplification of the obvious method of computing the bills by figuring out the charge for the energy in each block and adding them, as this method does away with the subtraction of 1050 from 2000.

We can therefore make up a little table as follows:

Range in kw-hr.	0 — 50	51 — 450	451 — 1050	1051 and over
Customer charge ...	$0	$0.50	$5.00	$15.50
Energy charge (cts. per kw-hr.)	.06	.05	.04	.03

From this table we can calculate any bill with not more than one multiplication and one summation, without any subtraction.

Algebraic Analysis of the Block Meter Rate.

3. An algebraical investigation for those readers who are familiar with algebra will supplement this. Calling c_1 the energy charge per kilowatt-hour in the first block, that is from 0 to e_1 kw-hr., e_2 the energy charge per kilowatt-hour in the second block, that is between e_1 and e_2 kw-hr., etc., we get the amount of the bill in the first block (less than e_1 kw-hr.) as

$$a_1 = c_1 e \quad \dots\dots (1)$$

where e is the number of kilowatt-hours consumed. If the customer's energy consumption e reaches into the second block we have to pay for the first e_1 kilowatt-hours at the rate of c_1 cents per kw-hr. and for the balance of $(e - e_1)$ kw-hr. at the rate of c_2 cents per kw-hr., so that the whole bill a_2 becomes

$$\begin{aligned} a_2 &= c_1 e_1 + c_2(e - e_1) \\ &= (c_1 - c_2)\, e_1 + c_2 e \quad \dots\dots (2) \end{aligned}$$

The constant term $(c_1 - c_2)e_1$ is obviously the customer charge and the part which is proportional to e is a straight meter rate. The unit charge of that straight meter rate is the factor by which e is to be multiplied, that is c_2.

If the energy consumption e becomes greater than $(e_1 + e_2)$ and therefore reaches into the third block we have first to pay for $(e_1 + e_2)$ kw-hr. as per equation (2), that is $(c_1 - c_2)\, e_1 + c_2\, (e_1 + e_2)$ and for the remainder of $e - e_1 - e_2$ we have to pay at the rate of c_3 cents per kw-hr., in other words

$$\begin{aligned} a_3 &= (c_1 - c_2)e_1 + c_2(e_1 + e_2) + c_3(e - e_1 - e_2) \\ &= (c_1 - c_3)e_1 + (c_2 - c_3)e_2 + c_3 e \end{aligned}$$

In the same manner we get

$$a_4 = (c_1 - c_4)e_1 + (c_2 - c_4)e_2 + (c_3 - c_4)e_3 + c_4 e$$

. . . . . etc. . . . . .

The terms in brackets are constant and represent always the customer charge in every one of these equations, whereas the co-efficient of e (that is c_1, c_2, c_3, . . . etc.), is the energy charge of the straight meter rate component of the equivalent combinations of rates. If we write this in the form

$$a_4 = c_1e_1 + c_2e_2 + c_3e_3 - (e_1 + e_2 + e_3)\, c_4 + c_4 e$$

we see that the result is the same as the one obtained above for a special numerical case. The results can also easily be verified graphically.

4. We see from the figures of Appendix XI on the one hand, and from a comparison of Fig. A of this Appendix with Fig. A of Appendix X on the other hand, that the block meter rate follows the theoretical requirements of the pure meter rate more closely than the step meter rate. Especially if we add an explicit customer charge to the block meter rate[1] (which would mean that the duct of straight lines of Fig. A of this Appendix will have to be raised parallel to itself by the amount of the customer charge) we see that we can with a very limited number of blocks approach the theoretical curve to any degree of approximation which is desirable in practice.

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[1] See Section 124 of the main text.

APPENDIX XIII

(To Section 123)

Combinations of Block Meter Rates With Straight Meter Rates and Step Meter Rates.

These combinations do not occur frequently. An example of the combination of a block meter rate with a straight meter rate is the General Lighting schedule of Cambridge, Mass. This rate charges

9 cents per kw-hr. (net) for the first 500 kw-hr.
8 cents per kw-hr. (net) for the next 500 kw-hr.

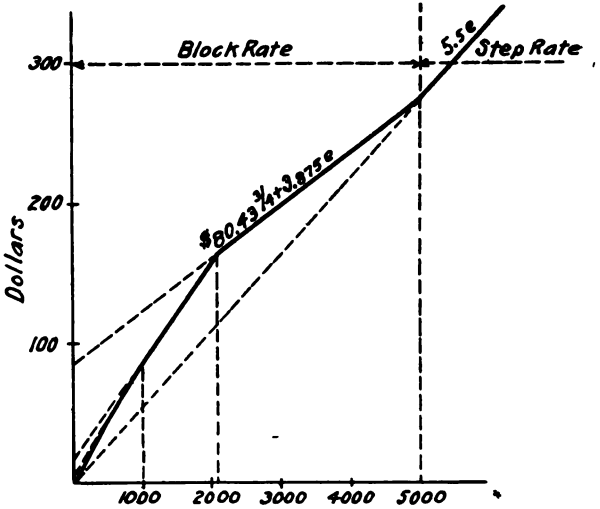

Fig. A.—Block Rate Followed by Step Rate.

7 cents per kw-hr. (net) for the next 1094 kw-hr.
3.875 cents per kw-hr. (net) for the next 2856 kw-hr.
5.5 cents per kw-hr. (net) for 4950 kw-hr. or over.

The last line of this table is a straight meter rate. The price per kilowatt-hour in this rate is chosen in such a way that we arrive at the same bill for the limiting number (4950 kw-hr.) of kilowatt-hours, whether we figure it according to the last, or to the last but one line. The curve of the rate (see also Appendix XII) is shown in Fig. A.

The Municipal Wholesale Power rate of Fort Wayne, Ind., is a combination of a block rate in the lower ranges of energy consumption and a straight meter rate in the higher ones. Steps are worked into the rate by discounts which extend back into the range of the block rate as follows:

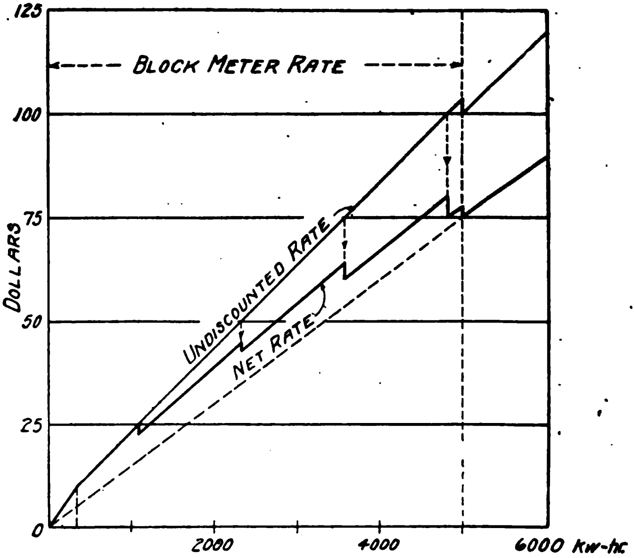

Fig. B.—Combination of Block and Step Meter Rates.

The rate charges: 3 cents per kw-hr. for the first 350 kw-hr.
2 cents per kw-hr. for the excess over 350 kw-hr.

If over 5000 kw-hr., 2 cents per kw-hr. for
all the energy consumption.
Quantity discounts: 10% on bill of $ 25
15% on bill of $ 50
20% on bill of $ 75
25% on bill of $100
30% on bill of $150

Fig. B shows the curve of this rate and Fig. C that of the Retail Power rate of New Britain, Conn., which is the reverse of the rate just discussed, inasmuch as it contains a step rate in the lower, and a block rate in the higher ranges. The last one of the 15 steps charges 2.5 cents per kw-hr. in the range from 4000 to 5000 kw-hr. The block rate begins nominally at 5000 kw-hr., charging in the first block (5000-10,000 kw-hr.) the same unit rate of 2.5 cents per kw-hr. as the last step of the step rate. Therefore the curve is an unbroken straight line from 4000 to 10,000 kw-hr. and we might choose any point between 4000 and 10,000 kw-hr. as the end of the step rate and the beginning of the block rate. The block rate has very large blocks (5000 kw-hr. each) and the difference between the unit charges of successive blocks is only 0.1 cent, so that the curve of the block rate is very nearly an unbroken straight line.

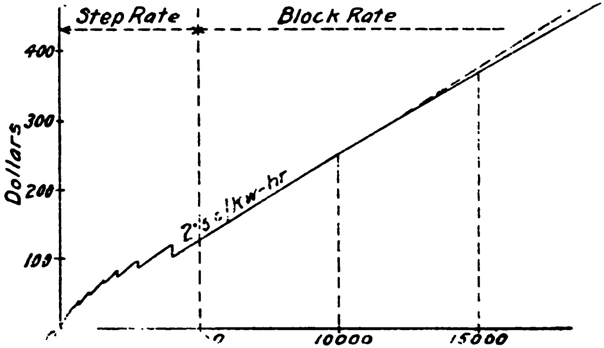

Fig. C.—Step Rate Followed by Block Rate.

APPENDIX XIV

(To Section 125)

Average Charges per Kilowatt-Hour with the Pure Meter Rates.

The only graphical representation of pure meter rates used so far was that of showing the amount of the customer's bill with varying energy consumptions (Appendices X-XIII and Figs. 4-6 of the main text). In this method of representation we get ducts of straight lines for the block and the step meter rate, and each of these straight lines is the equivalent of a combination of a straight meter rate with a customer charge. The energy charge of the schedule in cents per kilowatt-hour is given by the angle of elevation ϵ of the respective straight line from the horizontal[1]. (Fig. A)

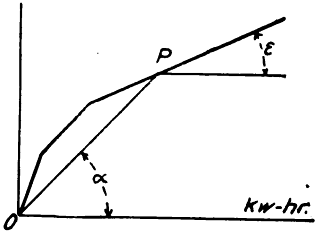

Fig. A.

The larger the energy charge, the steeper the line will be. This energy charge can also be defined as the increment cost (to the consumer) per kilowatt-hour, that is, the price of every additional kilowatt-hour and must not be confounded with the average price per kilowatt-hour. This average price

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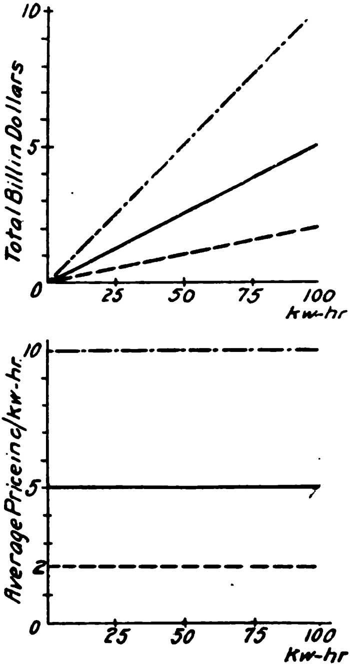
[1] Speaking accurately and in mathematical terms: By the tan ϵ.

is given by the total bill divided by the number of kilowatt-hours consumed. If we connect a point P of the rate curve (duct of straight lines) with the origin O (Fig. A) the angle α which this connection line includes with the horizontal is a measure of the average price per kilowatt-hour. The steeper that angle is the higher is the average price[2].

We can thus by a glance at the rate curve easily form an



Figs. B$_a$ and B$_b$.

idea how large the average price per kilowatt-hour is for every number of kilowatt-hours and how it varies. We need only estimate the variation of the angle α.

In many cases we find employed curves of the average

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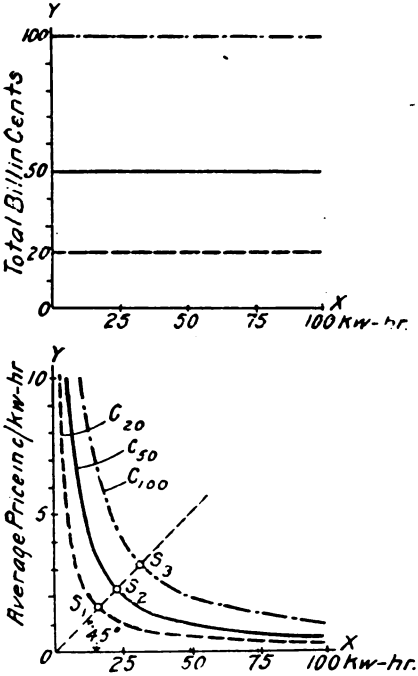
[2] Speaking accurately and in mathematical terms: The average price is given by tan α.

price per kilowatt-hour rather than of the total bill. The curve of the total bill is, however, not only more instructive as regards the nature of the rate, but generally also simpler, because it almost always consists of straight lines only. Nevertheless, since, in spite of this, the curve showing the average



Figs. C_a and C_b.

price per kilowatt-hour is still frequently being employed, it will be shown in the following how the average price per kilowatt-hour varies in case of the block rate and of the step meter rate.

It has been shown that both the block rate and the step meter rate are composed of the two elements of the straight meter charge and the customer charge, and no other elements. *These two* elementary charges will be treated first and their combination afterwards. The total bill under a straight meter charge is represented by Fig. B_a. The average price per kw-hr. is, of course, constant. It is therefore represented by a straight horizontal line (Fig. B_b) the height of which above the axis is given by the kilowatt-hour charge. (The full lines in Figs. B_a and B_b belong to one another, likewise the dotted lines and the dash-and-dot lines.) In the other one of the two elementary rates, consisting of a pure customer charge, the curve which represents the customer's bill

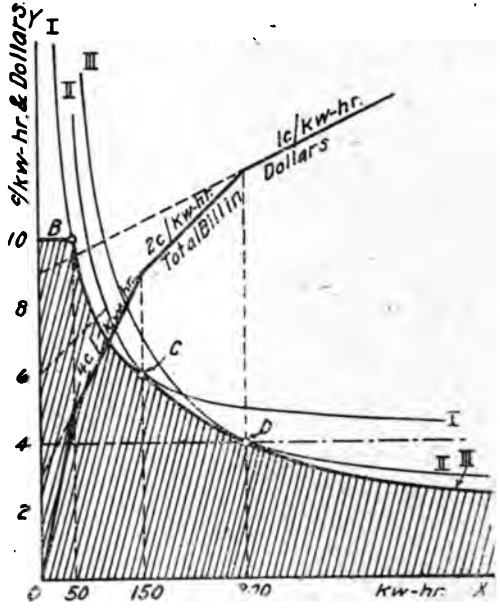


Fig. D.

is a straight horizontal line (Fig. C_a), the height of which above the horizontal axis corresponds to the amount of the customer charge in dollars or cents. The curve for the average charge per kilowatt-hour is less simple (Fig. C_b). If the bill is constant, for instance 50 cents (full line) the average price per kilowatt-hour will be

10 cents per kw-hr., if energy consumption is 5 kw-hr.
5 cents per kw-hr., if energy consumption is 10 kw-hr.
2 cents per kw-hr., if energy consumption is 25 kw-hr.
1 cent per kw-hr., if energy consumption is 50 kw-hr.

The curve which results from plotting these values is shown as curve C_{50} in Fig. C_b. The other curves for $1 and for 20 cents customer charge are also shown in the same figure. All these curves are equilateral hyperbolas with asymptotes OX and OY; this means that the curve steadily

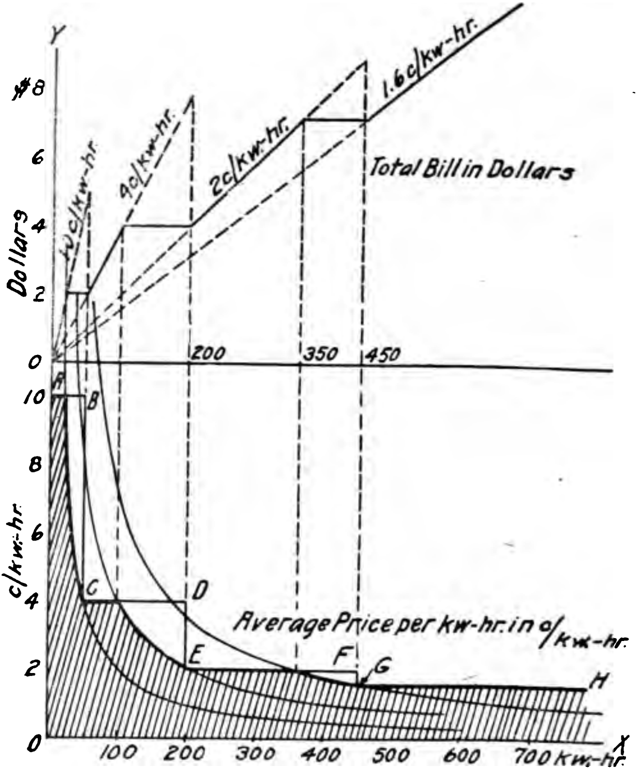


Fig. E.

straightens out from *O* towards *X* and towards *Y* and it approaches the axes *OX* and *OY* more and more the further out we extend it, but it never touches them.

If we now add these two elements, the straight meter charge and the customer charge, as in the block rate, the average-price curve will be a summation of the respective curves of the two elements, Fig. B_b and C_b, this means we will get the hyperbola in Fig. C_b, but the horizontal line of Fig. B_b will express itself by raising the hyperbola into a higher position. We thus get the curve of the average price per kilowatt-hour for a block rate (Fig. D, shaded area). It consists in the first range of the block rate of a horizontal straight line as the block rate itself in that range is nothing but a straight meter rate. In the following ranges a series of hyperbolas will result (*I*, *II*, *III*), one for each range. The axis of ordinates is the common vertical asymptote for them all and the respective horizontal asymptotes are located at distances above the horizontal axis which are given by the energy charge in the respective block. Thus, for instance, the horizontal asymptote of the hyperbola *I* (dash-and-dot line) is located at 4 cents per kw-hr. above the axis *OX*[3].

As regards the unmodified step rate, the average charge per kilowatt-hour is evidently indicated by the curve (duct) *ABCDEFGH* (Fig. E) resembling steps on a flight of stairs (hence probably the name "step rate"). Where we modify the step rate by the requirement that no greater total charge shall be made for a smaller number of kilowatt-hours than for a larger one the corners at *B*, *D* and *F* are cut off by hyperbolas, the asymptotes of which are always given by the axes *OX* and *OY*, as will be easily understood from the foregoing.

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[3] It could be easily shown with a little mathematics that the length of the "axis" of the hyperbola (OS_1, OS_2, OS_3, respectively, in Fig. C_b) is proportional to the square root of the customer charge. This means that as the shape of the hyperbola is the more rounded off, the larger is the equivalent customer charge.

APPENDIX XV

(To Sections 53 and 137)

Explanation of the Term Power-Factor and Allied Terms.

(For the Non-Technical Reader.)

In case of purely heating and incandescent lighting loads the power in watts consumed by the customer is given by the product volts × amperes. The number of the kilowatts is of course only 1/1000 of this value, as the unit of kilowatt is 1000 times as great as the unit watt. The power in kilowatts with such loads is therefore $\frac{\text{volts} \times \text{amperes}}{1000}$.[1] With certain other loads, the so-called inductive or reactive loads, notably with most motor loads, the physical conditions in alternating-current circuits are such that the power which is produced by a certain current at a given voltage is smaller than the term $\frac{\text{volts} \times \text{amperes}}{1000}$ indicates. A larger current is therefore necessary with these loads to produce a certain power in a given system (that is at a given voltage).

Now the "capacity" or "size" of generators and transformers is determined by the product of volts × amperes and since in a given electric system of the most common type the voltage is constant, a generator or transformer of a given size can supply less power with inductive load than with noninductive load.[2] Also the copper sections of trans-

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[1] This applies to direct-current and single-phase alternating-current circuits. In case of three-phase circuits this product must be multiplied by 1.732 here and in the following.

[2] This does not mean that power is lost (except in a secondary way). The generator is simply unable to deliver as much power without undue heating and it can, therefore, not receive as many horsepower from the prime mover. It is designed and built as a larger "generator" with corresponding cost but can operate only as a smaller one if the load is inductive. Transformers are affected in the same way.

mission and distribution lines depend on the current they have to carry and a given line will therefore not be able to carry as much inductive load as noninductive load. We see from this that it is cheaper to supply noninductive load because the electrical supply equipment is smaller and therefore cheaper. Moreover, the maintenance of proper voltage is more difficult with inductive load.

The term $\frac{\text{volts} \times \text{amperes}}{1000}$ is called the "apparent kilowatts" or the "apparent power." The apparent power is equal to the real power only in case of noninductive (nonreactive) load, otherwise it is always larger. The unit of apparent power is the "kilovolt-ampere," which is 1000 times the apparent power delivered by one ampere at one volt, with single-phase current, just as the kilowatt is 1000 times the real power delivered by one ampere at one volt on noninductive load with single-phase current. A kilovolt-ampere furnishes one kilowatt in case of noninductive load, otherwise less.

The ratio of the real power to the apparent power is called the power-factor. In central-station practice the power-factor is expressed in per cent; "80% power-factor" means that the real power is 80% of the apparent power. The power-factor can never be larger than 100%. In case of noninductive load it is equal to 100%, while for inductive load it is smaller. The power-factor of small motors is smaller than that of larger motors, and for a given individual motor it decreases as the load on the motor decreases. This refers to asynchronous (induction) motors. In synchronous motors the power-factor can be regulated by the exciting current.

All our readers will probably be familiar with the fact that in alternating-current circuits the direction in which the current is flowing is continually reversing and at a very rapid rate too, in most of our central-station systems 120 times a second. It increases from zero to its maximum value in 1/240 second, then decreases in the next 1/240 second again to zero, reverses its direction, reaches its maximum in the opposite direction after the third 1/240 second (negative maximum), again becomes zero and thus after 4/240 or 1/60 second begins the same cycle as described. The same applies to the voltage. The speed with which the voltage varies is exactly the same as that of the variations of the current (namely 120 reversals of the direction every second), but the current does not in every case reach its maximum (or its zero value) at the same instant as the voltage. It may be, for instance, always 1/500 second ahead of the voltage, or it may be a little behind the voltage. In the former case we say the current is "leading," in the latter it is "lagging." If the current and the voltage both reach their maximum at exactly the same instant, that is, if the current is neither leading nor lagging, we say current and voltage are "in phase."

The machinery in the central station determines the number of reversals per second, the character of the load determines whether the current is leading, lagging, or in phase.

Now noninductive load means nothing else than that current and voltage are exactly in phase. Then the power-factor is 100%. If they are out of phase we have a power-factor of less than 100%. We see from the above that if the power-factor is smaller than 100% this may be due either to a leading or to a lagging current.

As may be expected, the lag of the current of one customer may be balanced partly or wholly by the lead of the current of another customer, so that the supply of the two together may combine into a current in phase or at least less out of phase. The generators, transformers, etc., will then have to supply current which is less out of phase, the power-factor will be raised and the undesirable effect of a low power-factor on the central station and lines will be reduced.

Now the ordinary asynchronous or induction motor has such characteristics that it can only draw a lagging current from the system. Conditions of the customer's load which make the current lead the voltage are practically found only in the so-called synchronous motors under certain methods of operation (overexcitation.) These synchronous motors are practicable in exceptional cases only, for large sizes and under certain other conditions. This explains why in Sioux City (see footnote 11, Section 137) the increase of the demand charges in case of small power-factors is restricted to lagging currents.

APPENDIX XVI

(To Section 140)

Instruments for Measuring the Maximum Demand.

One type of demand-measuring instruments is working in connection with the ordinary service watt-hour meter of the consumer. A contact mounted on the wormwheel of the watt-hour meter makes and breaks a circuit at a rate proportional to the speed at which the meter disk revolves, that is, proportional to the power which passes through the meter. Every time the contact is made, a member in the demand-indicating instrument is pushed forward a small fixed amount by means of an electromagnet and a ratchet. This member may be either a stylus (if a graphic record is desired) or a driving dog which pushes a friction pointer forward on a dial. After a prearranged interval over which the demand is to be averaged has elapsed (5, 10 or 30 minutes, one hour, etc.) the above mentioned stylus or driving dog is disengaged by the action of a clock and drawn back to zero position by spring action. The stylus in the recording type of instrument then begins a new mark on the sheet or disk of paper which is creeping or revolving underneath the stylus. The driving dog in the other type of instrument on its withdrawal leaves the friction pointer lying in the position to which it had been pushed previously, so that the pointer always indicates the highest position which the dog has ever reached since the pointer was reset the last time by hand. From the chart of the recording instrument (called "demand indicator" by the manufacturers in contradistinction to the dial instrument, which is called "demand meter") we have then to select the highest mark as indicating how large the highest demand was and when it occurred, whereas in the dial instrument we can read directly from the position of the pointer how large the maximum demand has been since the last resetting of the instrument, but not when it occurred.

Another type of instrument for measuring the demand is the "Wright maximum-demand meter." It consists of a

U-shaped glass tube partly filled with liquid and sealed at both ends. At each end (both of which in the normal position point vertically upwards) the tube is widened into a bulb. Just below one of these two bulbs the vessel communicates with the upper end of a graduated vertical overflow glass tube, which is sealed at its lower end. (The whole arrangement is therefore hermetically sealed on all sides.) The bulb on the other leg is surrounded by a coil of resistance wire connected in series with the consumer's installation. The heat generated in the coil by the current expands the air in that bulb and forces a portion of the liquid out of that leg, making the liquid overflow at the other leg into the vertical overflow pipe. The liquid will remain in the latter even after the coil cools off and thus the amount of liquid contained in the graduated tube is a measure of the maximum current, which has been passing through the coil since the last resetting of the instrument. This resetting is made by tilting the instrument so that the liquid can run back into the U-shaped tube. The whole instrument is enclosed in a cast-iron casing and hinged, so that the electric light company's inspector can easily tilt the instrument for resetting after he has made his periodic reading.

As this instrument acts on the heating capacity of the current it will at fluctuating loads register differently from the electromagnetic instruments previously described, which base their registry on the energy delivered. The Wright demand meter is rated to record as follows (from Foster, "Electrical Engineer's Pocket Book"):

If the maximum load lasts 5 minutes 80% will register.

If the maximum load lasts 10 minutes 95% will register.

If the maximum load lasts 30 minutes 100% will register.

If we have, for instance, a load of 1 kw. (1000 watts) for 5 minutes and no load for the following hour this combination of loads will register on a Wright demand meter as 800 watts, whereas an electromagnetic instrument will record the following maximum demands:[1]

If the instrument is set for 5 minutes or less: 1000 watts.

If the instrument is set for 10 minutes: 1000 × 5/10 = 500 watts.

If the instrument is set for 30 minutes: 1000 × 5/30 = 167 watts.

As the Wright instrument records 100% of the demand after 30 minutes duration of the full demand, the comparison

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[1] Provided that this 5-minute interval of 1000 watts starts at the same moment when the electromagnetic instrument starts its own interval of readjustment, that is, at the moment of the disengagement by the clockwork.

might be made with an electromagnetic instrument set for a 30-minute interval. Supposing now the full load of 1000 watts to be turned on for a number of minutes, as shown in the first column of the following table, and no load for the rest of the time, the registries will be:

Duration of 1000-watts Load	Electromagnetic Instrument Set for 30 minutes	Wright Demand Meter
5 minutes.....	167 watts	800 watts
10 minutes.....	333 watts	950 watts
30 minutes.....	1,000 watts	1,000 watts

We see from these comparisons that the Wright demand meter is more sensitive to high demands of short duration than the electromagnetic devices. This will not surprise the engineers among our readers as they know that the heating effect increases with the square of the current.

It seems reasonable to assume that the heating effect of various percentages of load acting for various periods of time on the air in the bulb of the Wright instrument is more or less similar to the heating effects on electric machinery and equipment; the way in which the Wright instrument responds to various loads may therefore to a certain degree express the effect of an overload on the necessary size of the capacity of the power-house machinery, etc., and with that on a certain part of the capital invested and the corresponding demand charges.

We cannot, however, by any means say that the way in which the Wright demand meter responds to various durations of overload is preferable to that of the electromagnetic instruments. (Compare also Sections 36 and 37). In the first place a certain part of the electric light company's equipment, notably the distribution lines, have no time element in the way they respond to overloads; this means a short overload on the distribution lines will at once show its full effect on the quality of the service (by an excessive voltage drop). In the second place the heating and cooling curves of the various parts are different from one another and different from that of the Wright instrument. In the third place we have moved so far away from the exact theoretical requirements by the various successive approximations made[2] that these nice distinctions lose their importance alongside these approximations.

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[2] Compare Appendix V and the additional assumption that the consumer's maximum demand is proportional to his peak responsibility, and that the latter is the correct portion of the central station's capacity for which the customer should be charged; see Appendix VI.

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